Manipulated Electorates and Information Aggregation*

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Abstract

We study information aggregation with a biased election organizer who recruits voters at some cost. Voters are symmetric ex-ante and prefer policy a in state A and policy b in state B, but the organizer prefers policy a regardless of the state. Each recruited voter observes a private signal that is imperfectly informative about the unknown state, but does not learn the size of the electorate. In contrast to existing results for large elections, there are equilibria in which information aggregation fails: As the voter recruitment cost disappears, a perfectly informed organizer can ensure that policy a is implemented independent of the state by appropriately choosing the number of recruited voters in each state.

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1 Introduction

Voting is considered to be an effective mechanism for aggregating information that is dispersed among voters about which of the available policies or candidates is better for the society. Indeed, Feddersen and Pesendorfer (1997) showed that in large elections, the majority decision will often be *as if* there was no uncertainty. Hence, simple majority rules not only allow voters to express their preferences, but also allow the society to better aggregate dispersed information, provided that the electorate is sufficiently large.

Elections, however, take place in larger contexts in which interested parties try to influence election outcomes. For instance, one often sees interested parties spend resources to affect voter turnout. Examples of such activities include the bussing of voters to polls in elections or in referenda, the activities of a CEO directed at increasing participation in shareholder voting, or the prodding of colleagues by a department chair. Hence, it is natural to ask the extent to which an election organizer's ability to manipulate the voter participation rate might allow him to influence the outcomes of elections.

To this end, we analyze a model in which voters have to decide between two policies: policy a and policy b. Voters prefer policy a in state A and policy b in state B; i.e., voters prefer that the implemented policy matches the state of the world. However, no individual voter knows the state, and hence voters are uncertain about the correct policy. Although uncertain about the state, each voter has a small piece of information in the form of a noisy signal.

The key element in our model is that the number of voters is chosen by an *election organizer* who privately observes the realization of the state of the world and recruits an odd number of voters after observing the state. Recruitment is a costly activity, and the total recruitment cost is increasing in the number of recruited voters. Each voter then has an equal chance of being selected to participate in the election. The election organizer is biased —has a conflict of interest with respect to the voters— in the sense that he always prefers that policy a be implemented independently of the state. We assume that the number of recruited voters is not observed by the voters. However, the voters will make Bayesian inferences about the state from being recruited, since the organizer may choose different participation rates in different states.

Therefore, a voter receives some information about the state from the fact that he is selected.

To fix ideas, consider a referendum to build a bridge in a town. The cost of the bridge is unknown, and the voters prefer that the bridge be built only if the cost of building the bridge is low. The governor knows the cost and chooses how much resource to spend in order to mobilize the voters, which in turn affects voter turnout. The governor prefers the bridge to be built no matter what the cost is, may be because it will increase his popularity, and hence his re-election chances in the next election, or because he will benefit from doing business with the construction company in charge of building the bridge. Our paper then explores how much the ability to influence voter turnout can translate into the ability to influence the policy outcome.

We show in our main result that the ability to manipulate turnout significantly affects the performance of elections. There are equilibria in which the majority chooses policy a almost always, independently of the state, when the recruitment cost is almost zero, and when the potential number of voters is arbitrarily large.

Our result has implications for the electorate's ability to make correct choices, i.e., the extent to which elections aggregate information. Elections fully aggregate information if the correct policy is chosen with probability one, and elections aggregate no information if the same policy is chosen in both states. Our main result implies that the presence of the biased organizer reduces information aggregation. When the recruitment costs are small and the electorate is large, there are symmetric equilibria in which the selected outcome is independent of the state of the world; thus information aggregation completely fails. Moreover, in such equilibria the organizer's favorite policy is implemented with a probability that approaches one in the limit; hence, the organizer ensures that information aggregation fails in the most drastic way that is favorable for him. This result is in stark contrast to earlier papers on voting and information aggregation. For example, Feddersen and Pesendorfer (1997) showed that in a large electorate without an organizer, the outcome of the election coincides with the outcome of a voting model without uncertainty.

Driving this negative result is a recruitment effect. The intensity of the organizer's recruitment activity depends on his private information. This introduces an endogenous relationship between the number of voters and the state. In particular, the number of participating voters is state dependent. A voter's vote affects the outcome of the election only when there is a tie in the number of votes cast in favor of each alternative, i.e., when the vote is *pivotal*. Therefore, a voter votes as if his vote is pivotal. All else equal, a voter is more likely to be pivotal in the state where participation is lower, i.e., the state in which the organizer recruits fewer voters. The organizer manipulates the outcome of the election via recruiting more voters in state B and fewer voters in state A. This strategy works, because a voter believes that when his vote is pivotal, state A is more likely to be the realized state and hence votes for policy A with a probability larger than one half.

In our second set of results (Theorems 2 and 3), we characterize the equilibrium behavior across *all equilibria* when the recruitment costs are small and when the population is large. First and foremost, there is no equilibrium that fully aggregates information. Moreover, apart from equilibria in which the organizer recruits no additional voters[1]—we assume that there is always one voter present independent of the organizer's recruitment activity to avoid off-equilibrium problems with zero voters—there is only one limit equilibrium outcome with a different outcome. In this type of equilibrium, the expected vote shares in state A for each policy approach a half, and hence there is a close race in state A. The election outcome in state A is deterministic — policy a is implemented with probability approaching one—if the number of voters grows without bound and is not deterministic otherwise. In state B, the organizer recruits no additional voters, and hence, the implemented policy is not deterministic in state B.

In the final part of our analysis, we tackle an *election design* question and show via two examples how the extent of manipulation can be diminished using policy tools. Theorem 4 presents our results when there is a participation requirement, i.e., when there is a requirement that the number of voters participating be above a certain threshold. Since we are interested in the limit outcomes of large elections, we analyze equilibria of a sequence of elections in which the required threshold weakly increases as well as the organizer's recruit-

[1]Such an equilibrium exists for some parameters, and does not exist if the number of voters present independent of the organizer's recruitment activity is sufficiently large.

ment cost disappears. If the required threshold increases without bound, then there is always an equilibrium sequence that aggregates information. Moreover, if the threshold increases sufficiently fast relative to the rate at which the recruitment cost disappears, then all symmetric equilibria aggregate information. However, if the threshold increases at a rate that is too slow relative to the rate at which the recruitment cost disappears, then a limit outcome exists in which the organizer ensures that policy a is implemented.

Another election design tool that we examine is the use of unanimity rule for policy a to be implemented. If the minimum number of voters who participate grows without bound, then the symmetric equilibrium outcomes do not fully aggregate information. However, the outcome is very different from the outcome of manipulated equilibria, and the amount of inefficiency that comes from the failure of information aggregation is smaller if the highest signal is more precise. Therefore, when the information content of the highest signal is sufficiently precise, then the worst equilibrium outcome of elections with unanimity rule is better for voter welfare than the worst equilibrium outcomes of elections with majority rules. This result stands in contrast to Feddersen and Pesendorfer (1998), who showed that unanimity rules are the only voting rules among all supermajority rules that fail to aggregate information in large elections.

2 Model

A finite number of potential voters, N, has to choose between two available policies, $\{a, b\}$. The voters have common interests but are uncertain which policy serves their interest better. In particular, there are two possible states of the world, denoted by $\omega \in \Omega := \{A, B\}$. Voters share the following utility function:

$$\begin{aligned} u(a, A) &= u(b, B) = 1, \\ u(a, B) &= u(b, A) = -1, \end{aligned}$$

where $u(x, \omega)$ denotes the utility if policy x is chosen in state ω. In other words, voters prefer the implemented policy to match the true state of the world, but they do not know what the state is.[2]

Information Structure:

There is a common prior belief $\pi \in (0,1)$ that the state is A. Each voter receives a private signal, $s \in S := [0,1]$. The signals are distributed according to a c.d.f. $F(s|\omega)$. Conditional on the state, the signals are independent across voters. The distribution F admits a continuous density function, denoted by $f(s|\omega)$. We assume a strict version of the Monotone Likelihood Ratio Property (MLRP).[3]

Assumption 1.

$$\frac{f(s|A)}{f(s|B)} \qquad \textit{is strictly decreasing in } s.$$

Assumption 1 implies that voters who receive higher signals attach a strictly larger probability to the state of the world being state B. Another implication of Assumption 1 is that signals carry some information about the state of the world; i.e., $f(s|A)$ is not identical to $f(s|B)$ for all $s \in S$. Our second assumption puts a bound on the informativeness of the signals.

Assumption 2. *There exists a number $\eta > 0$ such that*

$$\eta < f(s|\omega) < \frac{1}{\eta} \qquad \textit{for } \omega \in \Omega \textit{ and for } s \in S.$$

An implication of Assumption 2 is that there is no single voter type who has arbitrarily precise information about the state of the world.

Organizer's Actions and Preferences:

There is a single election organizer who observes the realization of the state of the world ω and recruits a number of voters who will participate in the election. He prefers that policy a be implemented irrespective of the state of the world. Recruitment is costly, and in particular, recruiting each additional

[2]Note that here we also make the simplifying assumption that $u(a,A) - u(b,A) = u(b,B) - u(a,B)$, but none of our results depends on this specification.

[3]Continuity of the densities $f(\cdot|\omega)$ and the strict version version of MLRP are for expositional simplicity. All of our results continue to hold without continuity of the density functions and also with the weak version of MLRP, together with a condition that states that "$f(s|A)$ is not everywhere identical to $f(s|B)$."

pair of voters costs $c > 0$ to the organizer. So, if the organizer recruits n pairs of voters, then the number of participants in the electorate is equal to

$$2n + 1 \in \{1, 3, 5..., N\}.$$

If the organizer recruits no one, $n = 0$, then a randomly chosen voter becomes the unique voter. The number of voters is always odd and therefore a "tie" in the vote count cannot occur.

The payoff of the organizer is:

$$\begin{aligned} u_O(a, n) &= 1 - cn, \\ u_O(b, n) &= -cn, \end{aligned}$$

where the first argument is the policy that the majority of the electorate chooses to implement, and the second argument is the number of pairs of voters the organizer recruits.

We make the following assumption on the relation of c and the size of the potential voters, N.[4]

Assumption 3.

$$N \geq \left\lfloor \frac{2}{c} \right\rfloor.$$

This assumption ensures that the size of the population is never a binding constraint for the organizer.[5]

Timing of the Voting Game:

Summarizing our description, the timing is as follows.

- The organizer learns the state.
- The organizer chooses n.
- Nature chooses (recruits) $2n + 1$ voters, each equally likely, from the population.

[4]The term $\lfloor x \rfloor$ refers to the largest integer not greater than x.

[5]Note that the assumption is a lower bound on the size of the population. Our analysis remains unchanged when the number of voters is infinite, in which case we could dispense this assumption. The advantage of a finite population is that being recruited is a positive probability event, which facilitates the application of Bayes' formula.

- Each recruited voter observes his private signal but does not observe the number of recruited voters, n.
- Only the recruited voters participate in the election and each recruited voter votes for policy a or policy b.
- The policy that receives more votes is implemented.

Strategies and Equilibrium:

A strategy for the organizer is a pair of distributions over integers,

$$\tilde{n} = (\tilde{n}_A, \tilde{n}_B) \in \Delta(\{0, 1, ..., (N-1)/2\})^2,$$

which denotes the recruitment choice of the organizer in states A and B, respectively.

A pure strategy[6] for voter i is a mapping

$$d : S \to \{a, b\},$$

that prescribes how the voter will vote as a function of his signal, conditional on being recruited. When a voter is not recruited, he does not have a ballot to cast.

A symmetric Nash equilibrium is a tuple $(\tilde{n}, d)$ in which the organizer's strategy $\tilde{n}$ is a best response to a voter strategy profile in which each voter uses the same strategy d, and the strategy d is a best response to the strategy profile in which the organizer's strategy is $\tilde{n}$ and all other voters are using the strategy d. From here on, equilibrium refers to symmetric Nash equilibrium.

For any given symmetric voter strategy d, let the expected vote share for policy a in state ω be:

$$q_\omega(d) := \Pr\left(d\left(s\right) = a|\omega\right) = \int_{s\in[0,1]} \mathbf{1}_{d(s)=a} f\left(s|\omega\right) ds.$$

Inference of Voters and Cutoff Strategies:

[6] As will become clear, voters' best replies will have a cutoff structure, and therefore, focusing on pure strategies for the voters is without loss of generality.

In our model voters are consequential; i.e., they care only about the implemented policy and not directly about how they vote. In other words, a single vote will make an impact on the outcome of the election only when the number of votes cast for either alternative without that single vote is exactly equal, i.e., when that vote is *pivotal.* More precisely, a voter votes as if his vote is pivotal, as is typical in voting models with incomplete information. The probability of being pivotal in state ω if the expected vote is q_ω and the number of recruited voters is n_ω, is given by

$$\binom{2n_\omega}{n_\omega}(q_\omega)^{n_\omega}(1-q_\omega)^{n_\omega}.$$

In our model, there is an additional source of information that the voters use in their inference about the state of the world, because there is some information carried in the event that a voter is recruited. This is because the number of recruited voters depends on the state of the world, and hence, a voter learns some information about the state of the world from being recruited. The probability of being recruited in state ω if the number of recruited voter pairs is n_ω, is given by

$$\frac{2n_\omega+1}{N}.$$

The posterior likelihood ratio that the state is A, conditional on being recruited and conditional on being pivotal for a voter who received signal s, when all other voters are using the strategy d, and the organizer is using a pure strategy $\tilde{n}=(n_A,n_B)$ is calculated as below:[7]

$$\beta(s,piv,rec;\tilde{n},d):=\underbrace{\frac{\pi}{1-\pi}}_{prior}\underbrace{\frac{f(s|A)}{f(s|B)}}_{signal}\underbrace{\frac{\frac{2n_A+1}{N}}{\frac{2n_B+1}{N}}}_{recruited}\underbrace{\frac{\binom{2n_A}{n_A}(q_A)^{n_A}(1-q_A)^{n_A}}{\binom{2n_B}{n_B}(q_B)^{n_B}(1-q_B)^{n_B}}}_{pivotal}, \tag{1}$$

where we omit the dependence of q_ω on the voter strategy d for ease of reading.

This likelihood ratio, which we refer to as *the critical likelihood ratio*, guides

[7]Note that the extension of the expression to the case in which the organizer is using a mixed strategy is rather straightforward, and we skip it in order to highlight the recruitment effect and the pivotal effect in the displayed expression. For completeness, we write the critical likelihood ratio when $\tilde{n}$ is a mixed strategy in Equation (7), which is in the Appendix.

a voter's voting decision. In particular, a voter with a signal s votes for policy a if his critical likelihood ratio is above 1, and he votes for policy b otherwise. Therefore, voters use cutoff strategies in all equilibria. This is a standard result in voting models and follows from the MLRP condition from Assumption 1.

Lemma 1. *Any equilibrium voting strategy has a cutoff structure. There is a signal $\hat{s}$ such that a recruited voter casts a vote for policy* b *if $s > \hat{s}$ and for policy* a *if $s < \hat{s}$.*

From here on we will use $\hat{s} \in S$ to denote a generic cutoff strategy and $q_\omega(\hat{s})$ to denote the expected vote share for policy a in state ω when voters use a cutoff strategy $\hat{s}$. Lemma 1 allows us to conveniently express the expected vote share for policy a as:

$$q_\omega\left(\hat{s}\right) = F\left(\hat{s}|\omega\right).$$

In an equilibrium with an interior cutoff—$0 < s^* < 1$—the cutoff type is indifferent between voting for either option, so

$$\beta(s^*, piv, rec; \tilde{n}, s^*) = 1.$$

Remark 1. *The critical likelihood ratio is found by using the information in the voter's prior belief, in his signal, conditioning on the events that he is recruited, and that his vote is pivotal. The recruitment strategy enters in two places. Holding everything else constant, if the number of recruited voters in state ω increases, then the recruitment effect will push the critical likelihood ratio of the voter toward state ω. However (unless the outcome is deterministic), the pivotality probability in state ω decreases if the number of recruited voters in state ω increases.*

Organizer's Best Reply:

The organizer chooses n in order to maximize the probability with which policy a is implemented less recruitment cost. Recall that the voters do not observe the choice n of the organizer, and hence the organizer's choice does not affect voter behavior directly. Therefore, in state ω the organizer's (pure) best reply correspondence to a given cutoff strategy $\hat{s}$ of the voters is:

$$\underset{n\in\left\{0,1,\ldots,\frac{1}{2}(N-1)\right\}}{\arg\max} \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n+1-i} - nc. \qquad (2)$$

The first term in the objective function of the organizer is the probability that policy a is implemented when the probability that a randomly selected voter votes for policy a is $q_\omega(\hat{s})$, and when the size of participation is $2n+1$. The second term is the cost of choosing a participation size of $2n+1$.

Remark 2. *The number of potential voters, N, appears in the recruitment effect in Equation* (1) *and in the organizer's best reply in Equation* (2)*. However, the term N cancels out in the recruitment effect. Hence, N has an impact on equilibrium behavior only if it is sufficiently small that it becomes a binding constraint in the organizer's best reply in Equation* (2)*, which we rule out by Assumption* 3*. Therefore, N plays no further role in the analysis.*

To get more insight into the organizer's best reply, we calculate the increase in the probability that policy a gets selected when the organizer recruits an additional pair of voters, that is, the *marginal benefit* of increasing n:

$$\begin{aligned}\Delta\left(n-1,\omega,\hat{s}\right) \quad := \quad & \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n+1-i} \\ & - \sum_{i=n}^{2n-1} \binom{2n-1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1-q_\omega\left(\hat{s}\right)\right)^{2n-1-i}.\end{aligned}$$

This expression can be rewritten as:[8]

$$\Delta\left(n-1,\omega,\hat{s}\right) = \frac{1}{2}\binom{2n}{n} \left(q_\omega\right)^n \left(1-q_\omega\right)^n \left(2q_\omega - 1\right).$$

[8]Adding an additional pair of voters when there are initially $2n-1$ voters changes the outcome only if either exactly $n-1$ voters already support a, and both of the additional voters happen to vote for a, or exactly n voters already support a, and neither of the additional voters vote for a. Hence:

$$\begin{aligned}\Delta\left(n-1,\omega,\hat{s}\right) &= \binom{2n-1}{n-1} \left(q\right)^{n-1} \left(1-q\right)^n \left(q\right)^2 - \binom{2n-1}{n} \left(q\right)^n \left(1-q\right)^{n-1} \left(1-q\right)^2 \\ &= \frac{1}{2}\binom{2n}{n} \left(q\right)^n \left(1-q\right)^n \left(q-\left(1-q\right)\right)\end{aligned}$$

The increase in the probability that policy a is implemented when the size of the recruited voters increases from $2n-1$ to $2n+1$ is equal to the probability of a tie in the vote counts for policies a and b, multiplied by the term $\frac{1}{2}(2q-1)$.

If $q_\omega(\hat{s}) \leq 1/2$, then $\Delta(n-1, \omega, \hat{s}) \leq 0$ for every n; so the organizer recruits no additional voter, since recruitment is costly. Indeed, when the odds are against him, the organizer recruits as few people as possible in order to maximize the variance of the outcome of the election and also to save on recruitment costs.

On the other side, if $q_\omega(\hat{s}) > 1/2$, then $\Delta(n-1, \omega, \hat{s}) > 0$ and $\Delta(n-1, \omega, \hat{s}) > \Delta(n, \omega, \hat{s})$. Therefore, the objective function is strictly concave. An implication of this is that if $q_\omega(\hat{s}) > 1/2$, then there is a unique n such that $\Delta(n-1, \omega, \hat{s}) \geq c$ and $\Delta(n, \omega, \hat{s}) < c$. Notice that when $q > 1/2$, the odds are with the organizer, so he wants to minimize the variance of the election outcome by recruiting many people. For instance, if the organizer recruited an infinite number of voters, then by the law of large numbers, policy a would be implemented. However, there is a cost of recruiting voters, so the organizer recruits as many voters as possible until he reaches a point at which the marginal benefit of recruiting an additional pair of voters is not more than the marginal cost.

Therefore, in both cases the organizer's best reply will either be unique (meaning one integer for each state), or it will be a mixed strategy in which the support consists of two adjacent integers, for one or both of the states.

3 Manipulated Electorates

We study election outcomes when c is small. When c is small, the organizer may recruit many voters and so we can compare our result to those for exogenously large elections. To this end, we fix the common prior π and some information structure F that satisfies Assumptions 1 and 2. Let $\{G(c)\}_{c>0}$ be a collection of voting games in which for each game $G(c)$, the prior belief is π, the information structure is F, the recruitment cost to the organizer is c, and the number of potential voters, $N(c)$, is some integer that satisfies Assumption 3.

Theorem 1. *Let $\{c_k\}_{k=1,2,...}$ be a sequence of positive numbers that converge to zero. Then, there is a sequence of symmetric Nash equilibria of $G(c_k)$ such that in both states:*

1. *The probability that policy* a *is implemented converges to one.*
2. *The number of recruited voters grows without bound.*
3. *The organizer's payoff converges to one.*

Theorem 1 states that as the recruitment cost disappears and the number of potential voters becomes large, there are equilibria in which policy a is elected with a probability that is arbitrarily close to one in *both* states. Therefore, information aggregation fails in the most drastic way that is beneficial to the organizer. In other words, in such equilibria the organizer's favorite outcome is implemented regardless of the state. Moreover, in both states, the number of recruited voters becomes large and the organizer's expected payoff becomes one. Hence, an endogenously large electorate may lead to the failure of information aggregation, and in the limit, the organizer incurs no cost from the recruitment efforts despite recruiting an unbounded number of voters.

In such *manipulated equilibria*, a randomly selected voter votes for policy a with a probability strictly larger than $1/2$ in *both states of the world.* Therefore, when the recruitment cost is small, the organizer can ensure that the majority selects policy a by recruiting many voters.

The aggressive voter behavior in favor of policy a is a consequence of the *asymmetry* in the number of voters who are recruited in state A and state B. In such equilibria, the organizer recruits more voters in state B than in state A. Such asymmetry in the numbers of voters in different states affects voter behavior in two ways. First, because there are more voters in state B than in state A, a voter is more likely to be recruited in state B, and his posterior belief that the state is B goes up when he is recruited. This is the *recruitment effect.* The other effect that works in the opposite direction is the *pivotality effect.* Because there are more voters in state B than in state A, the pivotal probability in state A is larger than the pivotal probability in state B. Among the two effects, the pivotality effect is the more dominant one, and the overall net effect leads to the voting behavior in favor of policy a.

We now turn to the organizer's incentives. To provide some insights, we want to argue that whenever voters are using a cutoff s^* such that $q_A(s^*) > q_B(s^*) > 1/2$, then it will be the case that the organizer recruits more voters in state B than in state A, when the recruitment cost is small. To see why, recall that the marginal benefit of recruiting one more voter is:

$$\Delta(n-1, \omega, s^*) = \underbrace{\binom{2n}{n} q_\omega(s^*)^n (1 - q_\omega(s^*))^n}_{\text{pivot probability}} \frac{1}{2}(2q_\omega(s^*) - 1).$$

The first term reflects how likely it is that an additional voter changes the outcome. The second term reflects how likely it is that an additional voter swings the election in the organizer's favor (rather than against). Comparing the relative magnitude of the two terms in the two states shows that they go in opposite directions,

$$q_A(s^*)(1 - q_A(s^*)) < q_B(s^*)(1 - q_B(s^*)), \text{ whereas}$$
$$2q_A(s^*) - 1 > 2q_B(s^*) - 1.$$

In general, the relative marginal benefits of additional voters in the two states will depend on both terms. However, for sufficiently large n, the first term unambiguously dominates the second. Because the organizer recruits a growing number of voters when the recruitment cost is small,

$$\Delta(n^* - 1, A, s^*) < \Delta(n^* - 1, B, s^*),$$

for every integer n^* pair of voters he recruits in state A. Because the marginal benefit $\Delta(n, \omega, s)$ is decreasing in n, it follows that the organizer recruits strictly more voters in state B than in state A. Figure 1 depicts how the probability that the majority selects policy a changes with n for q_A and q_B. When n is large, then the curve given q_B is steeper than the curve given q_A, that is, the marginal benefit of an additional voter is larger in state B.

In order to prove the first part of the theorem, we show that there is an equilibrium sequence in which the probability that a randomly selected voter votes for policy a stays bounded above from a half in each state of the world.

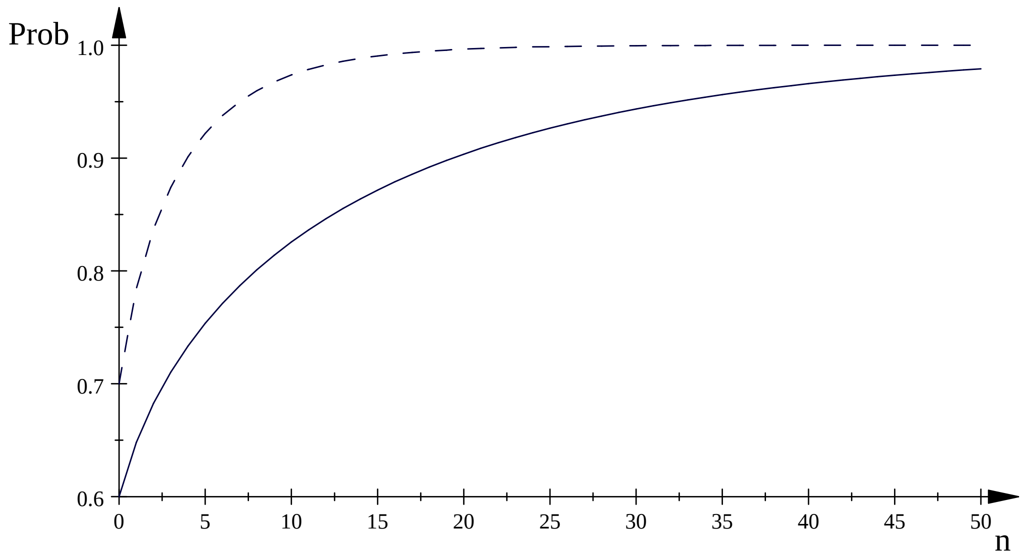

Figure 1: The probability that policy a receives a majority of votes given the number of recruited voters n for $q = 0.6$ (straight) and for $q = 0.7$ (dashed).

Define the "median-signal" in state ω as

$$s_\omega := q_\omega(s_\omega) = 1/2.$$

The main step is to prove that equilibrium cut points converge to a signal $s^* > s_B$. To see why this suffices to prove the theorem, first observe that because $s^* > s_B$, $q_\omega(s^*) > 1/2$ for each $\omega \in \{A, B\}$. This is because the expected vote share for policy a is increasing in the voters' cut point, and because the expected vote share in state B is weakly higher than that in state A. If the limit cut point $s^* = 1$, then the voters ignore their signals and vote for policy a almost independent of their signals, resulting in policy a being implemented for sure. If $s^* < 1$, then as c disappears, the organizer recruits an arbitrarily large number of voters in both states.

There may be multiple equilibrium sequences, with different limit cutoffs s^*, and with different organizer behavior. However, provided that the limit cutoff $s^* > s_B$, the policy that is implemented becomes deterministic and is independent of the state. Moreover, there is at least one equilibrium sequence along which the number of recruited voters grows without bound in both states. As we show in Lemma 7, located in the Appendix, and in the subsequent remark, the ratio of the number of recruited voters in states A and B stays bounded away from zero and infinity.

Contrast with Voting Models with State Independent Number of Voters:

The existence of equilibria with a limit cut point $s^* > s_B$ stands in sharp contrast to the results of Federsen and Pesendorfer (1997), who showed that in a voting model where the number of voters is state independent, all symmetric equilibrium cut points have limit points $s^* \in (s_A, s_B)$.[9] This then implies that all equilibria aggregate information. What drives Feddersen and Pesendorfers' result is that otherwise, for any s^* that is not in the interval (s_A, s_B), the ratio of pivot probabilities is degenerate in the limit. Note that the ratio of pivot probabilities is

$$\lim_{n\to\infty} \frac{\binom{2n}{n}(q_A(s^*))^n(1-q_A(s^*))^n}{\binom{2n}{n}(q_B(s^*))^n(1-q_B(s^*))^n}. \tag{3}$$

This expression goes to 0 if $s^* > s_B$ and to ∞ if $s^* < s_A$. Therefore, the critical likelihood ratio cannot be one and there cannot be an equilibrium with a limit cut point $s^* \notin (s_A, s_B)$.

In our model, the information contained in the pivotal event is shaped by both the expected vote shares in both states *and* the relative ratio of the number of participants in each state. This is because an equal split is less likely in the state with a larger number of participants. Indeed, the organizer's decision about how many voters to recruit is linked to the expected vote shares in a way that keeps the inference made by being pivotal relatively moderate, compared to the case in which the number of voters is exogenous. Hence, the existence of the organizer opens up the possibility that the majority votes for policy a.

The key to the result that $s^* > s_B$ can be a limit cut point is through the organizer's recruitment strategy and its interaction with the pivotal probabilities in different states. For a given probability $q > 1/2$, which represents the probability that a randomly selected voter votes for policy a, the organizer chooses the number of recruited voters, $2n + 1$, such that:

[9] They consider a setting with both private and common values. Large elections with pure common values are analyzed in Feddersen and Pesendorfer (1998) and Duggan and Martinelli (2001)

$$\binom{2n}{n} q^n (1-q)^n (2q-1) \approx 2c.$$

The approximation in the above statement represents the error that comes from ignoring the integer constraints. Therefore, if the voters' cut points are any $s > s_B$, then the ratio of pivot probabilities in each state of the world stays bounded away from 0 and infinity and is approximated as:

$$\frac{\binom{2n_A}{n_A}(q_A)^{n_A}(1-q_A)^{n_A}}{\binom{2n_B}{n_B}(q_B)^{n_B}(1-q_B)^{n_B}} \approx \frac{2q_B - 1}{2q_A - 1}. \tag{4}$$

Note that the right-hand side is independent of c. This is because the organizer's choice of the size of the electorate keeps the pivot probabilities in each state relatively at the same order, and when c disappears, the relative pivot probabilities stay bounded away from 0 and infinity. This is unlike the case in which the number of voters is state independent, as depicted by Equation (3).

4 All Limit Equilibria

In this section we characterize systematically the limiting equilibrium outcomes that can be generated by any equilibrium sequence, as the recruitment cost vanishes. The recruitment activity limits information aggregation in all symmetric equilibria. Recall that s_ω is the median signal in state ω, that is, $q_\omega = F(s_\omega|\omega) = 1/2$.

Trivial Equilibrium:

An equilibrium is a *trivial equilibrium* if: *i)* the organizer recruits no voter in either state, so that there is only one voter casting a ballot in both states, and *ii)* the selected voter votes for policy a with a probability not more than 1/2 in both states. In a trivial equilibrium, the organizer is passive and information is not aggregated because only one voter makes the decision on the implemented policy.

It is easy to see that a voting game G admits a trivial equilibrium for *all* recruitment costs c if and only if the distribution of signals, F, satisfies the following inequality:

$$\frac{\pi}{1-\pi}\frac{f(s_A|A)}{f(s_A|B)} \leq 1. \tag{5}$$

If inequality (5) holds, then in a trivial equilibrium, the voter who is selected to cast his vote will vote for policy a with a probability not more than 1/2 in both states of the worlds. This in turn justifies the organizer's behavior to recruit no additional voters. Of course, if c is large, a trivial equilibrium exist even if inequality (5) fails. However, if the inequality fails, then a trivial equilibrium does not exist when the recruitment cost c is sufficiently small. This is because, in a putative trivial equilibrium, the probability that the voter voting for policy a is strictly larger than 1/2 in state A, and hence if the recruitment cost is small, then the organizer's best reply is to recruit some voters.

All Non-Trivial Equilibria:

In Theorem 2 below, we argue that, fixing all parameters of the environment other than the recruitment cost, any limit point of a non-trivial equilibrium cutoff sequence, as the recruitment cost disappears, has to be either equal to s_A or strictly larger than s_B.

Theorem 2. *Let $\{c_k\}_{k=1,2,\ldots}$ be a sequence of positive numbers converging to zero.*

1. *For any s^* that is a limit point of non-trivial equilibrium cutoffs of the sequence of voting games $\{G(c_k)\}_{k=1,2,\ldots}$, either $s^* = s_A$ or $s^* > s_B$.*

2. *$\{G(c_k)\}_{k=1,2,\ldots}$ has a sequence of non-trivial equilibria with limit cutoff $s^* = s_A$, and another sequence with limit cutoff $s^* > s_B$.*

The theorem states that there are only 2 types of limit points of non-trivial equilibrium cutoffs, as the recruitment cost disappears. None of these equilibria aggregate information fully, so that information aggregation failure is inevitable in any equilibrium.

One type of limit equilibrium outcome is when $s^* > s_B$. These types of equilibria are essentially identical to the equilibrium outcomes of equilibria presented in Theorem 1. In such equilibria the majority selects policy a; i.e., there is *full manipulation* and a drastic failure of information aggregation.[10]

The second type of equilibrium sequences are those where there is a close race between the two policies in state A. This is because $s^* = s_A$, and hence, the probability that a randomly selected voter votes for policy a converges to $1/2$. On the other side, in state B, the organizer recruits no one, and policy b is implemented with a probability $1-F(s_A|B)$ in state B. In the next theorem, we identify the properties of the limit outcomes of such equilibrium sequences.

Theorem 3.

1. *If inequality* (5) *is not satisfied, then along all equilibrium sequences with limit cutoff* $s^* = s_A$*:*

    - *The number of voters recruited in state* A *grows without bound.*
    - *Policy* a *is implemented in state* A *with probability converging to one.*

2. *If inequality* (5) *is satisfied, then for each of the following outcomes, there exists a corresponding equilibrium sequence with limit cutoff* $s^* = s_A$ *such that:*

    - *The number of voters recruited in state* A *grows without bound, and policy* a *is implemented in state* A *with probability converging to one.*
    - *The number of voters recruited in state* A *stays bounded, and policy* a *is implemented with a probability between 0 and 1 in state* A.

In this type of equilibrium sequences with limit points $s^* = s_A$, whether policy a is implemented in state A for sure depends on the number of voters recruited. If inequality (5) fails, then all such equilibrium outcomes have the organizer recruiting a number of voters that grows without bound, and hence, even if there is a close race between the policies, policy a prevails as the winner in state A.

[10] The only difference is that the number of voters may remain bounded when $s^* = 1$.

Remark 3. *One may be tempted to think that equilibrium cut points that converge to s_A cannot be sustained, since in state* B *there is only one voter, and hence, conditional on being pivotal, the voters should believe that the state is* B *and hence vote for* B. *The main force that sustains this type of equilibrium is the recruitment effect. Because in state* A *the electorate gets very large, the recruitment effect pushes the belief toward state* A, *and the pivotality effect pushes it in the opposite direction. When the expected vote share is* $1/2$, *the pivotal probability decreases to zero at the rate at which* $\frac{1}{\sqrt{n}}$ *decreases to 0, where n is the number of voters recruited. Therefore, the recruitment effect, which is at the order of n, becomes stronger than the pivotal effect in the close neighborhood of the vote fraction* $1/2$.

Finally, we illustrate the ordering of the equilibrium cutoffs with Figure 2. It shows the median types in the two states, s_A and s_B. The cutoff corresponding to the limit equilibrium of a large election with exogenous n—FP (Information Aggregation)—must solve $q_A(s) - 1/2 = 1/2 - q_B(s)$, that is, in both states the election must be equally close to being tied; see Feddersen and Pesendorfer (1997). Otherwise, (3) would fail to be interior and would either be infinity or zero.

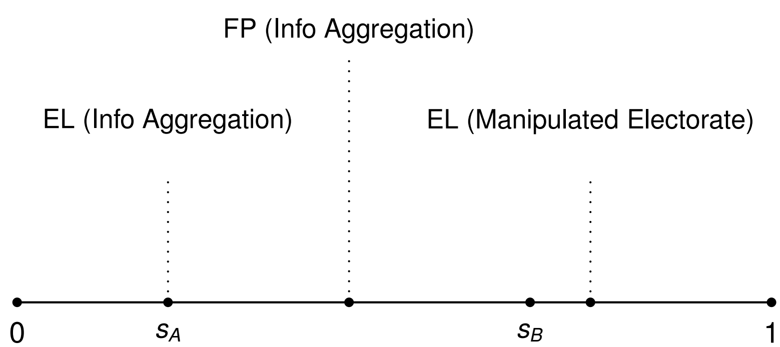

Figure 2: $F(s_A|A) = F(s_B|B) = 1/2$

5 Robust Election Design

In this section we explore whether electorate design can be a remedy for an organizer's ability to manipulate election outcomes. To this end, we analyze

2 election design tools that provide protections against manipulation, namely, quorum requirement and unanimity rule.

5.1 Participation Requirement

We start by relaxing the assumption that the minimum number of voters participating in the election when the organizer is passive is 1, and instead, we introduce the minimum number of voters that are participating as an election design tool, denoted by the parameter $2m+1$. Our main result in this setup is that if the number of participants who are present already without any recruitment activity grows large, then there exists an equilibrium sequence in which the majority votes for the correct policy; thus, information aggregates.

Specifically, suppose the number of voters is $2(m+n)+1$ for some integer m, where m is the number of required pairs of voters and n is the number of additional pairs of voters recruited by the organizer. We consider games parameterized by c (a real number) and m, denoted $G(c,m)$. We assume that the m required pairs are free to the organizer and the recruitment cost is only nc.[11] The number of potential voters is $N = \left\lfloor 2\left(m+\frac{1}{c}\right)+1\right\rfloor$. The following result shows that information can be aggregated in the limit, and if m grows sufficiently quickly, information will be aggregated.

Theorem 4. *Let $\{c_k, m_k\}_{k=1,2..}$ be a sequence of positive numbers c_k converging to zero and integers m_k diverging to infinity. We are interested in the symmetric equilibrium sequences of the voting games $\{G(c_k,m_k)\}_{k=1,2,...}$.*

1. *There exists a sequence of equilibria such that the probability with which the correct policy is implemented converges to one (that is, policy* a *in state* A *and policy* b *in state* B*).*
2. *If m_k diverges sufficiently fast relative to the rate at which c_k converges to 0, in all sequences of equilibria the probability with which the correct policy is implemented converges to one.*

[11] This assumption is for concreteness. Nothing changes if costs are $(m+n)c$.

3. *If m_k diverges sufficiently slowly relative to the rate at which c_k converges to 0, then there is a sequence of equilibria in which policy* a *is implemented with probability converging to one in both states (i.e., information aggregation fails and the outcome is manipulated).*

The first part of the result is related to Theorem 2. If m_k increases sufficiently slowly compared to the rate at which c_k disappears, then there is an equilibrium sequence along which the voter cutoffs converge to s_A. In the limit, there is a close race in state A, and the vote shares for the available policies are equal. However, despite the close race, in state A, policy a wins. In state B, the vote share for policy B is strictly above $1/2$ and because m_k diverges, policy b wins.

The second part of the result is an immediate consequence of the result for exogenously large elections (Feddersen and Pesendorfer (1997) and Duggan and Martinelli (2001)). When m_k increases very fast, relative to the speed at which c_k disappears, the organizer doesn't recruit anyone in either state, and hence, we get an election in which the number of participants is independent of the state of the world.

The third result is an immediate consequence of Theorem 1. Recall that there exists a sequence of equilibria along which the number of voters diverges in both states. Hence, if m_k diverges sufficiently slowly so as not to exceed that number, the originally identified equilibria stays an equilibrium.

One interpretation for the parameter m is that there is a "quorum" requirement, i.e., the minimal number of voters that the organizer must recruit is m. Another interpretation is a mandatory voting requirement that increases the participation rate even in the absence of the organizer's recruitment efforts. In the equilibria that aggregate information stated in the theorem, this constraint on the lower bound on the number of participating voters will bind in state B and, if m is sufficiently large, it will also bind in state A. Thus, the theorem suggests that when the number of voters can be manipulated, quorums are one instrument to improve the performance of elections.

Another such instrument one may consider is a unanimity rule, where policy a is elected only if all voters vote for a, which we analyze next.

5.2 Unanimity Rule

In this subsection, we analyze what happens with a unanimity rule; i.e., all of the participants' votes are required for policy a to be implemented. It is clear that the organizer will recruit no additional voter, because recruitment is costly, and the probability that policy a is selected weakly decreases with the electorate size. Because the organizer recruits no new person regardless of his cost, we drop c as a parameter and consider a participation requirement of $2m+1$ voters as in Section 5.1.

Because the organizer recruits no new voter, voters' participation rates will not depend on the state of the world. Therefore, our model corresponds to models analyzed by Duggan and Martinelli (2001) and Feddersen and Pesendorfer (1997), with a unanimity rule. It is well known by now that unanimity rule is the only supermajority rule that fails to aggregate information in large electorates. However, the extent to which information aggregation fails depends on the informativeness of the extreme signals. In our environment, it will depend only on the likelihood ratio of the highest signal which is

$$\frac{f(1|A)}{f(1|B)}.$$

For this section, we assume that

$$\frac{\pi}{1-\pi}\frac{f(1|A)}{f(1|B)} < 1. \tag{6}$$

If (6) fails, then there is a unique equilibrium in which voters vote for a for all signals. Given this assumption, let s_m be an interior equilibrium cutoff used by the voters in the game with $2m+1$ voters.[12] A voter is pivotal only if all other $2m$ voters vote for a. Hence, a voter is pivotal in state ω with probability $F(s_m|\omega)^{2m}$. The indifference condition of the cutoff signal delivers that

$$\frac{\pi}{1-\pi}\frac{f(s_m|A)}{f(s_m|B)}\left(\frac{F(s_m|A)}{F(s_m|B)}\right)^{2m} = 1.$$

Lemma 2. $\lim_{m\to\infty} s_m = 1$.

[12]Given (6), the existence of an interior equilibrium cutoff is guaranteed when the number of participating voters is large.

Proof. On the way to a contradiction, suppose that $s_m \to x \in (0,1)$. Then $F(x|A) > F(x|B)$ by the MLRP condition. However, then the indifference condition cannot be satisfied at the limit, which is a contradiction.

Suppose now that $s_m \to 0$. Then, $\lim_{s\to 0} \frac{F(s|A)}{F(s|B)} = \frac{f(0|A)}{f(0|B)} > 1$. But then the indifference condition cannot be satisfied at the limit, which is a contradiction. $\square$

In the following theorem, we show that the limit equilibrium probabilities that the correct policy is implemented depend only on the informativeness of the highest signal. This observation was made by Feddersen and Pesendorfer (1998) and Duggan and Martinelli (2001) in a related voting model. The authors show that unanimity voting rule fails to aggregate information and, therefore, unanimity rule is an inferior voting rule among other supermajority rules. In our model, however, simple majority rule gives rise to effective manipulation by a conflicted organizer, while unanimity rule mitigates this type of manipulation. Hence, when the highest signal is sufficiently informative, the inefficiency induced by the unanimity rule may be less than the inefficiency inflicted by the organizer's manipulation.[13]

Theorem 5. *Consider the unanimity rule with a quorum of m voters. Suppose (6) holds and $m \to \infty$. The probability that policy* a *is selected in state* A *converges to*

$$\left(\frac{\pi}{1-\pi}\frac{f(1|A)}{f(1|B)}\right)^{\frac{f(1|A)}{f(1|B)-f(1|A)}},$$

and the probability that policy a *is selected in state* B *converges to*

$$\left(\frac{\pi}{1-\pi}\frac{f(1|A)}{f(1|B)}\right)^{\frac{f(1|B)}{f(1|B)-f(1|A)}}.$$

As $\frac{f(1|A)}{f(1|B)} \to 0$, these probabilities converge to 1 and 0, respectively. Hence, information is arbitrarily close to being aggregated when signal 1 is very informative.

[13]The theorem restates Theorem 4 from Duggan and Martinelli (2001). We provide a proof in the appendix for completeness.

6 Robustness, Extensions and Discussion

In this section, we analyze various extensions of the model to highlight the robustness of our model to variations on our assumptions.

6.1 Multiple States

We now extend the model to incorporate a continuum of states. To this end, suppose that $\Omega := [0, 1]$, with $g(\omega)$ being the prior p.d.f. over the states of the world. Assume that $g(\omega)$ has a strictly positive lower bound on its support. We assume that the set of signals is $S := [0, 1]$, and that the MLRP condition is satisfied; i.e., for every pair of states $\omega_1 < \omega_2$, $\frac{f(s|\omega_1)}{f(s|\omega_2)}$ is strictly decreasing in s. Also, we assume that $f(s|\omega)$ has a uniform strictly positive lower and upper bounds for all $s \in S$ and $\omega \in \Omega$. Voters have identical preferences, where $v(\omega) := u(b, \omega) - u(a, \omega)$ is a strictly increasing function, and that $v(0) < 0$ and $v(1) > 0$.

As before, the organizer observes the true state ω and picks a number of pairs of voters to participate in the election. In this scenario, our main result, i.e., an appropriate version of the statements in Theorem 1, continues to hold. In particular, as c disappears, there is a sequence of equilibria in which the probability that policy a is implemented converges to 1 in *all* states of the world, the number of voters grows without bound, and the organizer's payoff converges to 1 in all states.

6.2 Heterogeneous Preferences and Private Values

Now we extend the model to accommodate heterogeneity in voter preferences. We still maintain the assumption that there are two states, $\Omega = \{A, B\}$, and that there is a finite number of voter types, $T := \{t_1, .., t_R\}$, where each $t_i \in (0, 1)$, and t_i is increasing in i. The type t_i represents the probability that a voter of type t_i has to attach to the state of the world being state A in order to be indifferent between the two outcomes a and b. Voters with higher preference types are more difficult to convince to vote for a than voters with higher preference types. However, it is feasible to persuade each voter to vote in favor of policy j by providing sufficiently informative evidence in favor of state j. Each voter's preference type is drawn according to a p.d.f. $h(\cdot)$ with full support, and independent of the state, and independent of the signal distribution.

In this environment, all of our results stated in Theorems 1-3 go through with minor but straightforward modifications. In particular, the manipulated equilibria are sustained in this setup. Also, the structure of all limit equilibria as identified in Theorem 2 and the information aggregation result stated in Theorem 4 continue to hold.

6.3 Competitive Organizers

In the paper we assume that there is a single organizer who makes all of the recruitment choices. Suppose that there is a second organizer, whom we refer to as O_1, who prefers that policy b be implemented regardless of the state, and he incurs the same marginal recruitment cost as the organizer, whom we refer to as O_0, who prefers that policy a be implemented independent of the state.

In this scenario, there is always a sequence of manipulated equilibria in which policy a is implemented with probability that converges to one in both states, and O_0 carries out all recruitment activity and O_1 is passive. Similarly, there is also a sequence of manipulated equilibria in which policy b is implemented with probabilities that converge to one in both states, and O_1 is active and O_0 is passive. There is, however, one more sequence of equilibria in which O_0 chooses to recruit many voters in state A, and O_1 chooses to recruit many voters in state B, and information gets aggregated; i.e., the correct policy is implemented with probability that converges to one. Therefore, competition among organizers opens up the possibility of information aggregation.

6.4 Role of Recruitment Cost

A strictly positive recruitment cost is essential for our main result in Theorem 1. This is because the organizer's ability to manipulate the election outcome comes from his ability to pick different voter participation rates in different states. In the manipulated equilibria, the organizer recruits more voters in state B than in state A. This is because, the probability that a randomly selected voter votes for policy a is strictly higher in state A then in state B, and also because *the marginal recruitment cost* is strictly positive. If, contrary to what we assume, recruitment is costless, the manipulated equilibria would not be sustained. In particular, when faced with an aggressive voter behavior in which voters vote for policy a with a probability strictly more than $1/2$— as is the case in manipulated equilibria— the organizer's best reply would be to recruit all potential voters in both states, which would take away his ability to recruit different numbers of voters in different states. Intuitively, strictly positive recruitment cost makes it optimal for the organizer to choose a recruitment strategy that sways voters' behavior aggressively in favor of policy a.

6.5 Role of Organizer's Private Information

The organizer's ability to manipulate the electorate relies on his private information about the state of the world. If we had assumed that he does not hold any further information than the common prior belief, then his recruitment strategy would be independent of the state of the world, and hence the voters would not be able to infer any new information about the state of the world from being recruited. As we argued before, the asymmetry in the number of voters recruited in different states is essential for manipulation, and such an asymmetry is not possible if the organizer has no private information. For instance, if the minimum number of people the organizer can choose to recruit is very large, then a result similar to Feddersen and Pesendorfer (1997) would hold, and the voters would select the correct policy with arbitrarily high accuracy.

6.6 Abstention, Costly Voting, and Subsidies

Abstention:

Feddersen and Pesendorfer (1996) observed that in an election in which voters have common interests, some voters who are not well informed may have strict incentives to abstain and their abstention has significant effects on the election outcome. In the equilibrium of our model, however, there is never a strict incentive to abstain: For all signals above the equilibrium cutoff, a voter strictly prefers to vote for policy b (rather than to abstain or vote differently) and she strictly prefers to vote for a for signals below that cutoff. With a signal exactly equal to the cutoff, a voter is just indifferent between each vote and abstaining.

The fact that there is no incentive to abstain in this equilibrium relies on the fact that the number of participating voters is odd, so that there are never any ties. However, once voters can abstain, there may be additional equilibria in which each voter does abstain with strictly positive probability, implying a positive probability of an even number of voters and hence a tie. Thus, our original equilibrium remains if voters can choose to abstain but additional equilibria may arise.

Costly Voting and Subsidies:

Suppose that, in contrast to our model, all citizens can vote but voting is costly. Here, recruitment may correspond to a subsidy by the organizer. Concretely, suppose that there are N citizens and each citizen can vote at a cost r. This cost may correspond to the cost of collecting information or to the physical act of going to the voting booth. The organizer can reduce the cost of voting to zero by paying c, for example, by bussing voters to the voting booth. If the voting costs r are sufficiently high, only those citizens who receive a subsidy will actually vote.[14]

Note that the cost r doesn't actually have to be very large. To see this point, suppose that voters receive their signal only after making the participation decision (as in Krishna and Morgan (2012)). Now note that in the original "manipulated equilibrium" of our model, we have $n_B > n_A$, that is, not being recruited is evidence in favor of state A. Since in that equilibrium in that state the election outcome is (correctly) a with high probability, this suggests that participation incentives in that equilibrium are small. Moreover, note that when a citizen contemplates participation, she compares her participation cost r to her private benefit from the correct decision *weighted* by the probability that she is pivotal. As we observed before, the number of voters the organizer recruits is determined by equating the recruitment cost c (the

[14]In this context, one can interpret the "participation requirement" m as the number of voters who have zero voting costs, while the cost of voting is r for the $N - m$ remaining voters.

cost of the subsidy) to his private benefit from policy zero, weighted by the probability that the election is tied. Thus, the probability of being pivotal may be small—and the participation incentives may be weak—whenever the organizer's private benefit is large relative to the individual voter's benefits and if the subsidy c is of a magnitude similar to r. Thus, there may be many scenarios where one may expect that many citizens who are not recruited will optimally choose to not participate even for intermediate voting costs.

Further analysis of costly voting with subsidies may be an interesting extension of the current model, and such analysis may yield a better understanding of exactly what such scenarios may be and when to expect voter subsidies to have substantial effects on voting behavior.

7 Literature Review

Information aggregation in elections with strategic voters has been studied by Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1996, 1997, 1998, 1999a,b), McLennan (1998), Myerson (1998a,b), Duggan and Martinelli (2001), among others.[15] These papers study equilibrium outcomes with an exogenously large number of voters.

In particular, Feddersen and Pesendorfer (1997) show that in a model with multiple states—and both private and common values—under all supermajority rules except the unanimity rule, large electorates aggregate information. Similar to us, they provide a complete characterization of all equilibria. The main difference from their model is that here the number of participating voters is selected by a conflicted organizer, and hence, the number of voters participating in the election is endogenously state dependent.

Myerson (1998b) introduces a Poisson model with population uncertainty in which the expected number of voters may be state dependent. He shows that large electorates aggregate information along some sequence of equilibria. In

[15] Bhattacharya (2013) observes the necessity of preference monotonicity for information aggregation. Bouton and Castanheira (2012) find that voters' imperfect information may help solve certain coordination problems. Mandler (2012) shows that uncertainty about the informativeness of the signals can also lead to failure of information aggregation. A recent generalization is Barelli and Bhattacharya (2013). Gul and Pesendorfer (2009) show that information aggregation fails when there is policy uncertainty.

his model, the ratio of the expected numbers of voters across different states is exogenously fixed along the sequence as the expected number of voters grows. In our model, similar to Myerson's model, the number of voters participating is state dependent. However, the ratio of the number of voters is endogenously determined via the choice of an organizer who incurs a cost for increasing the number of the participating voters. A second difference is that we characterize the limiting outcomes of all symmetric equilibria. We show that there is no equilibrium in which information fully aggregates when the number of voters is endogenous and there also exist equilibria in which the organizer's favorite outcome is implemented regardless of the state.

Methodologically, information aggregation in elections is related to work on large auctions. Among others, this has been studied by Wilson (1977), Milgrom (1979), Pesendorfer and Swinkels (1997, 2000), and Atakan and Ekmekci (2014). The papers study auctions in which the number of bidders becomes large exogenously. Lauermann and Wolinsky (2012) introduce an auction model in which the number of bidders is random and endogenously state dependent.

Related studies of voter (non-)participation in elections include especially Feddersen and Pesendorfer (1996), who identify the swing voters' curse when voters can abstain, and the vast literature on costly voting, especially Palfrey and Rosenthal (1985), Krishna and Morgan (2011) and Krishna and Morgan (2012). In these models, the number of votes cast depends on the state as well. In Feddersen and Pesendorfer (1996), abstention facilitates information aggregation whereas in Krishna and Morgan (2011) the cost of voting helps increasing (utilitarian) welfare by screening according to preference intensities in a model with both common and private values. Those models emphasize choice on the voters' side, showing how this can improve election outcomes, whereas our model emphasizes the organizer's ability to affect participation and how it lowers efficiency.

A related paper that endogenizes the issues that are voted for by a strategic proposer is Bond and Eraslan (2010). Similar to us, they show that the unanimity rule may be superior to other supermajority voting rules. In their model voting behavior under different rules have different implications for the proposals put on the table by a strategic proposer. In particular, unanimity rule disciplines the proposer to make offers preferred by the voters. In our model, unlike in theirs, the alternatives are fixed, but the participation rate is endogenously determined by a strategic organizer. In our model, unanimity rule restricts the organizer's ability to create the asymmetry of participation rates across the states.

Finally, a large literature analyzes a conflicted agent's ability to manipulate one or more decision makers to act in favor of the agent's interests, either through using informational tools or by taking actions that directly affect the decision makers' incentives. An important instance are models of cheap-talk, emanating from Crawford and Sobel (1982), who analyzed a biased sender's ability to transmit information and hence, induce behavior partially beneficial to the sender. Our model shares with these models the feature that the organizer has superior information and has biased preferences. Our model differs from the cheap talk literature, since information transmission mechanism is not through cheap talk messages. The recent literature on Bayesian persuasion, initiated by Kamenica and Gentzkow (2011), and applied to a voting context by Wang (2012) assumes that a sender can commit to an information disclosure rule that generates public signals. Similar to that literature, we are interested in the ability of an agent to induce others to undertake his preferred action, but the tools that the agent can use are different.

8 Conclusion

Understanding the performance of voting mechanisms to pick the best alternatives for the society has always received attention, dating all the way back to the Athenian leader Cleisthenes and later to Condorcet. In this paper, we studied the ability of voting mechanisms to aggregate dispersed information among voters when the election is taking place in the presence of an organizer who has the tools to change the turnout rate and whose interests are not aligned with those of the voters. Our main result is that such an organizer can manage to influence the election outcomes drastically, in his favor, thereby preventing information aggregation completely. This result suggests that although voting mechanisms may be very effective in aggregating information, they may be quite susceptible, and hence not robust, to manipulation activities by outsiders. An interesting feature of our model is that small electorates in which the organizer is not allowed to intervene may perform much better than large electorates in which the organizer can influence the turnout rate.

The organizer's ability to get his desired outcome relies on his being able to recruit many voters and does not rely on the abilities of cherry picking voters who have information supporting his favorite policy, or voters who are *a priori* more inclined to vote for his favorite policy. In practice, however, many of the manipulation schemes involve the use of tools, such as the timing of elections or targeted subsidies. Because in our model the organizer can pick only the turnout rate and cannot distinguish between voters with different characteristics, our sharp result provides a *lower bound* on the ability of an organizer who has more tools than just the ability to pick the turnout rate of the election. We also view our results as having implications for the use of elections as a means to control a common agent by a dispersed group of principals (e.g., shareholders controlling a manager or faculty controlling a chair). In particular, our main result suggests that elections may be an ineffective form of control if the agent can manipulate turnout.

A Appendix

A.1 Miscellaneous Results

In this part, we explore some properties of organizer's best reply correspondence, and the critical likelihood ratio that will be used in proving the theorems.

Organizer's Best Reply:

Let $\tilde{n} := (\tilde{n}_A, \tilde{n}_B)$ be a generic mixed strategy of the organizer, and let the set of all mixed strategies for the organizer in the voting game with recruitment cost c be $\tilde{N}(c)$. The term $\tilde{n}_\omega(i)$ denotes the probability that the strategy $\tilde{n}$ assigns to the integer i in state ω. The organizer's best reply correspondence to a voter cutoff s when the recruitment cost is c is $\eta(s,c) := (\eta_A(s,c), \eta_B(s,c)) \subset \tilde{N}(c)$. Moreover, $\tilde{n}^* \in \eta(s,c)$ if and only if each positive integer that is in the support of $\tilde{n}^*_\omega$ solves

$$\max_{n \in \left\{0,1,\ldots,\frac{1}{2}(N-1)\right\}} \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(s\right)\right)^i \left(1 - q_\omega\left(s\right)\right)^{2n+1-i} - nc.$$

Properties of $\eta(s,c)$:

This part has some repetition from the main text, but we include this to help the reader in the rest of the Appendix. Let

$$\begin{aligned} \Delta\left(n-1, \omega, \hat{s}\right) := & \sum_{i=n+1}^{2n+1} \binom{2n+1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1 - q_\omega\left(\hat{s}\right)\right)^{2n+1-i} \\ & - \sum_{i=n}^{2n-1} \binom{2n-1}{i} \left(q_\omega\left(\hat{s}\right)\right)^i \left(1 - q_\omega\left(\hat{s}\right)\right)^{2n-1-i}. \end{aligned}$$

A simplification of the above expression delivers the following identity.

$$\Delta\left(n-1, \omega, \hat{s}\right) = \binom{2n}{n} \left(q_\omega\right)^n \left(1 - q_\omega\right)^n \frac{1}{2} \left(2q_\omega - 1\right).$$

If $q_\omega(\hat{s}) \leq 1/2$, then $\Delta\left(n-1, \omega, \hat{s}\right) \leq 0$ for every n, so, the organizer recruits no one since recruitment is costly.

If $q_\omega(\hat{s}) > 1/2$, then $\Delta(n-1,\omega,\hat{s}) > 0$ and $\Delta(n-1,\omega,\hat{s}) > \Delta(n,\omega,\hat{s})$. The implication of this is that if $q_\omega(\hat{s}) > 1/2$, then there is a unique n such that $\Delta(n-1,\omega,\hat{s}) \geq c$ and $\Delta(n,\omega,\hat{s}) < c$.

Hence, the support of η_ω contains at most two integers, and if it includes two integers, they have to be adjacent integers.

Critical likelihood ratio, when the organizer uses a mixed strategy

$$\beta(\hat{s}, piv, rec; s, \tilde{n}) = \frac{\pi}{1-\pi}\frac{f(\hat{s}|A)}{f(\hat{s}|B)}\frac{\sum_{i\geq 0}\tilde{n}_A(i)(\frac{2i+1}{N})\binom{2i}{i}q_A(s)^i(1-q_A(s))^i}{\sum_{i\geq 0}\tilde{n}_B(i)(\frac{2i+1}{N})\binom{2i}{i}q_B(s)^i(1-q_B(s))^i}. \tag{7}$$

Lemma 3. *Fix $s \in [0,1]$. For every $\hat{s} \in [0,1]$,*

$$\max_{\tilde{n}\in\eta(s,c)} \beta(\hat{s}, piv, rec; s, \tilde{n})$$

exists, and is attained by some pure strategy $n \in \eta(s,c)$. Moreover, the set of maximizers for each $\hat{s}$ is the same. Similarly,

$$\min_{\tilde{n}\in\eta(s,c)} \beta(\hat{s}, piv, rec; s, \tilde{n})$$

exists, and is attained by some pure strategy $n \in \eta(s,c)$. Moreover, the set of minimizers for each $\hat{s}$ is the same.

Proof. The function β is continuous in $\tilde{n}$, and the the maximum of a continuous function over a compact domain exists. Independence of the maximizers from $\hat{s}$ is seen by inspection of the function β. □

Operator $\tilde{\beta}$:

Definition 1. *Let*

$$\tilde{\beta} : [0,1] \times [0,\infty) \rightrightarrows \mathbb{R}_+$$

be a correspondence that takes a signal and cost of recruitment as arguments, and returns a positive number, that denotes a likelihood ratio. In particular, $x \in \tilde{\beta}(\hat{s}, c)$ if there is a strategy $\tilde{n} \in \eta(\hat{s}, c)$ such that $\beta(\hat{s}, piv, rec; \hat{s}, \tilde{n}) = x$.

The mapping $\tilde{\beta}$ takes the signal $\hat{s}$ as the cutoff strategy of the voters, then calculates the best reply correspondence of the organizer to the cutoff strategy

$\hat{s}$, and then returns the number that is equal to the critical likelihood ratio of type $\hat{s}$ when all other voters follow the cutoff strategy $\hat{s}$ and when the organizer is following a strategy that belongs to the set of best replies to the cutoff strategy with cutoff $\hat{s}$.

Lemma 4. *The correspondence $\tilde{\beta}(\hat{s}, c)$ is convex valued and is upper-hemicontinous in its first argument, $\hat{s}$.*

Proof. The best reply correspondence, $\eta(\hat{s}, c)$ is upper-hemicontinous in $\hat{s}$—which follows from Berge's maximum theorem—and convex valued. The function $\beta(\hat{s}, piv, rec; \hat{s}, \tilde{n})$ is continuous in $\tilde{n}$. Moreover, because the densities $f(\cdot|\omega)$ are continuous for each $\omega \in \{0, 1\}$, the upper hemicontinuity of the organizer's best reply correspondence implies that $\tilde{\beta}$ is upper-hemicontinous. Convex-valuedness follows from the continuity of β in $\tilde{n}$, the fact that β is single-dimensional, and convexity of the best reply correspondence of the organizer. □

Lemma 5. *A signal $s \in (0, 1)$ is an equilibrium cutoff signal of $G(c)$ if and only if $1 \in \tilde{\beta}(s, c)$.*

Proof. By construction, if $1 \in \tilde{\beta}(s, c)$, then there is a strategy $\tilde{n}$ which is a best reply for the organizer to the voter cutoff strategy s such that $\beta(s, rec, piv; s, \tilde{n}) = 1$. Now, observe that $(s, \tilde{n})$ is an equilibrium, because $\tilde{n}$ is a best response of the organizer to voter behavior s, and using s as a cutoff is a best response for any given voter, since he is indifferent between the two alternatives in the events that he has signal s, he is recruited and he is pivotal, when other voters are following the same strategy and when the organizer follows the strategy $\tilde{n}$. The other direction is straightforward, so we skip it. □

A.2 Proof of Theorem 1

Proof:

Recall that s_ω is the median signal in state ω, that is, $q_\omega = F(s_\omega|\omega) = 1/2$.

Our proof strategy is that we will first show that for all small c, there is some $s(c) > s_B + \epsilon$ such that $1 \in \tilde{\beta}(s(c), c)$. This means, there are equilibria in which the voters vote for policy a with probability more than 1/2 in both states. The second part of the proof will show that in such equilibria, as c

disappears, a get selected with probability approaching 1, that the number of voters grows without bound, and that the organizer's payoff approaches 1.

We will start by showing the existence of equilibria with a large cutoff. There are two steps we will show in the following development:

Claim 1:

$$\exists \epsilon > 0 \text{ such that } \lim_{c \to 0} \max \tilde{\beta}(s_B + \epsilon, c) < 1.$$

Claim 2:

$$\exists \epsilon_c > 0, \text{ with } \lim_{c \to 0} \epsilon_c \to 0, \text{ such that } \lim_{c \to 0} \max \tilde{\beta}(1 - \epsilon_c, c) = \infty.$$

These two findings together with the upper-hemicontinuity and convex valuedness of $\tilde{\beta}$ (Lemma 4) imply, via a version of the intermediate value theorem[16], that for all c smaller than a cutoff $\bar{c} > 0$, there is a $s(c) \in (s_B + \epsilon, 1 - \epsilon_c)$ such that $1 \in \tilde{\beta}(s(c), c)$, delivering the desired result.

Claim 1:

$$\exists \epsilon > 0 \text{ such that } \lim_{c \to 0} \max \tilde{\beta}(s_B + \epsilon, c) < 1.$$

Proof of Claim 1:

Remember that $q_\omega(x)$ denotes the probability that a randomly selected voter votes for policy a in state ω, and let $n_\omega(x, c)$ be a pure best reply of the organizer if the voters are using the cutoff strategy x. We will drop the arguments in these objects occasionally to save on notation and for ease of reading.

Using the derivations we have for the organizer's best reply conditions before, i.e., that $\Delta(n_\omega - 1, \omega, x) \geq c$ and $\Delta(n_\omega, \omega, x) < c$, we now posit and prove the following:

[16] We have not been able to find an intermediate value theorem for correspondences in standard textbooks, however it is straightforward to extend the standard intermediate value theorems to convex valued, upper-hemicontinous correspondences, and we skip the proof.

Lemma 6. *Given any $x \in (0,1)$, $c > 0$, and $n_\omega \in \eta_\omega(x,c)$:*

$$\frac{2c}{2q_\omega - 1} \leq \binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1-q_\omega)^{n_\omega} \leq \frac{c}{q_\omega (1-q_\omega)(2q_\omega - 1)} \frac{(n_\omega + 1)}{(2n_\omega + 1)}.$$

Proof: Rewriting the hypothesis,

$$\begin{aligned}
\Delta(n_\omega, \omega, x) &\leq c \implies \\
\binom{2n_\omega + 1}{n_\omega} (q_\omega)^{n_\omega + 1} (1-q_\omega)^{n_\omega+1} (2q_\omega - 1) &\leq c \implies \\
(2n_\omega + 1)\binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1-q_\omega)^{n_\omega} &\leq \frac{c(n_\omega + 1)}{q_\omega (1-q_\omega)(2q_\omega - 1)},
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{2}\binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1-q_\omega)^{n_\omega} (2q_\omega - 1) &\geq c \implies \\
\binom{2n_\omega}{n_\omega} (q_\omega)^{n_\omega} (1-q_\omega)^{n_\omega} &\geq \frac{2c}{2q_\omega - 1}.
\end{aligned}$$

Taken together, the claim follows.

□

For the rest of the Appendix, we will occasionally use Stirling's approximation, which is:

$$\lim_{n \to \infty} \frac{n!}{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n} = 1.$$

Lemma 7. *If $q_\omega(x) \in (0.5, 1)$, then for any selection of pure strategy best replies by the organizer, $\{n_A(x,c), n_B(x,c)\}_{c>0}$*

$$\lim_{c \to 0} \frac{n_B(x,c)}{n_A(x,c)} = \frac{\ln(4(q_A)(1-q_A))}{\ln(4(q_B)(1-q_B))}.$$

Proof: If $q_\omega(x) \in (0.5, 1)$, then $\lim_{c \to 0} n_\omega(x,c) = \infty$.

Rewriting the approximation for state A,

$$
\begin{aligned}
& \binom{2n_A}{n_A} (q_A)^{n_A} (1-q_A)^{n_A} \\
= \; & \frac{(2n_A)!}{(n_A!)^2} (q_A)^{n_A} (1-q_A)^{n_A} \\
\cong \; & \frac{\sqrt{2\pi 2n_A}\left(\frac{2n_A}{e}\right)^{2n_A}}{\left(\sqrt{2\pi n_A}\left(\frac{n_A}{e}\right)^{n_A}\right)^2} (q_A)^{n_A} (1-q_A)^{n_A} \\
= \; & \frac{(4q_A(1-q_A))^{n_A}}{\sqrt{\pi}\sqrt{n_A}}.
\end{aligned}
$$

The approximation for state B is similar, and hence,

$$
\begin{aligned}
& \lim_{c\to 0} \frac{\binom{2n_A}{n_A} (q_A)^{n_A} (1-q_A)^{n_A}}{\binom{2n_B}{n_B} (q_B)^{n_B} (1-q_B)^{n_B}} \qquad (8) \\
= \; & \lim_{c\to 0} \sqrt{\frac{n_B}{n_A}} \left(\frac{(4q_A(1-q_A))}{(4q_B(1-q_B))^{\frac{n_B}{n_A}}} \right)^{n_A},
\end{aligned}
$$

where the equality is from Stirling's approximation and the previous rewriting.

From Lemma 6 and $q_\omega(x) \in (0.5, 1)$, the ratio (8) must be bounded away from zero and infinity, which requires that

$$
\lim_{c\to 0} \frac{n_B(x,c)}{n_A(x,c)} = K \in (0, \infty),
$$

(if $\lim_{c\to 0} \frac{n_B}{n_A} = 0$, then the ratio vanishes, if $\lim_{c\to 0} \frac{n_B}{n_A} = \infty$ it explodes). Using again that the ratio is bounded, this requires

$$
\lim_{c\to 0} \frac{(4q_A(1-q_A))}{(4q_B(1-q_B))^{\frac{n_B}{n_A}}} = 1,
$$

from which the claim of this step follows.

□

Remark 4. *Notice that if there is an equilibrium sequence in which* $\lim_{c\to 0} x(c) = x$ *such that* $q_\omega(x) \in (0.5, 1)$*, the ratio of the number of voters in the two states stays bounded as* c *goes to zero by Lemma 7. A slightly more complicated argument shows that the same conclusion—that the ratio of the number of voters* *stays bounded—holds also if* $\lim_{c\to 0} x(c) = 1$. *The key observation for this argument is that if* $\lim x(c) = 1$, *and if* $\lim n_A(x(c), c)/n_B(x(c), c) \to \infty$ *then* $\beta(s, piv, rec; x(c), n(c)) = 0$ *for every* $s \in [0,1]$, *which contradicts that these* $x(c)$ *are interior equilibrium cutoffs.*

We will now complete the proof for claim 1. From Lemma 6, it follows that,

$$\begin{aligned}
& \max \tilde{\beta}(s,c) \\
= \quad & \max_{(n_A,n_B)\in\eta(s,c)} \frac{\pi f(s|A)}{(1-\pi)f(s|B)} \frac{(2n_A+1)\binom{2n_A}{n_A}(F(s|A)(1-F(s|A)))^{n_A}}{(2n_B+1)\binom{2n_B}{n_B}(F(s|B)(1-F(s|B)))^{n_B}} \\
\leq \quad & \max_{(n_A,n_B)\in\eta(s,c)} \frac{\pi f(s|A)}{(1-\pi)f(s|B)} \frac{(2n_A+1)\frac{c}{q_A(1-q_A)(2q_A-1)}\frac{(n_A+1)}{(2n_A+1)}}{(2n_B+1)\frac{2c}{2q_B-1}} \\
= \quad & \max_{(n_A,n_B)\in\eta(s,c)} \frac{\pi f(s|A)}{(1-\pi)f(s|B)} \frac{2n_A+1}{2n_B+1} \frac{2q_B-1}{2q_A(1-q_A)(2q_A-1)} \frac{n_A+1}{2n_A+1}.
\end{aligned}$$

Note that $\max \tilde{\beta}(s,c)$ denotes the biggest element of the correspondence $\tilde{\beta}$. The term (n_A, n_B) denotes a pure strategy that puts probability 1 to integers n_A and n_B in states A and B respectively. Applying Lemma 7, we obtain that, for any fixed s such that $q_B(s) \in (0.5, 1)$,

$$\lim_{c\to 0} \max \tilde{\beta}(s,c) \leq \frac{\pi f(s|A)}{(1-\pi)f(s|B)} \frac{2q_B-1}{4q_A(1-q_A)(2q_A-1)} \frac{\ln(4(q_B)(1-q_B))}{\ln(4(q_A)(1-q_A))}.$$

Note that the right side vanishes for $s \to s_B$ from above, because $q_B(s) \to 1/2$ (while $q_A(s)$ stays strictly larger than $1/2$). So, there exists some ε and c^* such that for all $c \leq c^*$,

$$\max \tilde{\beta}(s_B + \varepsilon, c) < 1.$$

Note also that the number 1 on the right hand side of the inequality is arbitrary, and the same proof works to show that this inequality holds for any positive number.

<u>Claim 2:</u>

$$\exists \epsilon_c > 0, \text{ with } \lim_{c \to 0} \epsilon_c \to 0, \text{ such that } \lim_{c \to 0} \max \tilde{\beta}(1 - \epsilon_c, c) = \infty.$$

<u>Proof of Claim 2:</u>

Let $f(x) := 2x(1-x)(2x-1)$, and let $\epsilon_c > 0$ be a number that is close to zero that satisfies $f(q_A(1-\epsilon_c)) = 2c$. Existence of such a $\epsilon_c > 0$ is guaranteed when c is small, and $\lim_{c \to 0} \epsilon_c = 0$. We are going to show that $\lim_{c \to 0} \max \tilde{\beta}(1-\epsilon_c, c) = \infty$.

The definition of $1 - \epsilon_c$ is that in state A, the organizer with a marginal cost c is indifferent between recruiting no additional voter and recruiting one pair of voters.

Note that,

$$f'(q) = 12q\,(1-q) - 1.$$

and $f'(q) < 0$ for q sufficiently close to one. Hence, for x close to 1, $q_B(x) < q_A(x)$ implies whenever $f(q_A(x)) = 2q_A(x)(1 - q_A(x))(2q_A(x) - 1) = 2c$, $f(q_B(x)) = 2q_B(x)(1 - q_B(x))(2q_B(x) - 1) > 2c$. Hence, the best reply of the organizer to the cutoff strategy $1 - \epsilon_c$ is that in state B, he recruits at least 1 pair, and actually as $1 - \epsilon_c \to 1$, exactly one pair. This is because, $\Delta(1, B, x) = \binom{4}{2} q_B(x)^2(1 - q_B(x))^2(2q_B(x) - 1) < f(q_A(x)) = 2c$ for all x sufficiently close to one. Writing down the pivotal probability in state B, we get $2q_B(x)(1 - q_B(x))$, and if in state A the organizer recruits no one, then the pivotal probability in state A is 1. Because $\lim_{x \to 1} \frac{q_B(x)}{q_A(x)} = 1$, and $\lim_{x \to 1} \frac{1-q_B(x)}{1-q_A(x)} = \frac{f(1|B)}{f(1|A)}$, and that $\frac{f(1|A)}{f(1|B)} > 0$, we have that

$$\begin{aligned}
\lim_{c \to 0} \max \tilde{\beta}(1 - \epsilon_c, c) &= \lim_{c \to 0} \frac{\pi f(1 - \epsilon_c|A)}{(1 - \pi) f(1 - \epsilon_c|B)} \frac{1}{3} \frac{1}{2q_B(1 - \epsilon_c)(1 - q_B(1 - \epsilon_c))} \\
&= \infty.
\end{aligned}$$

<u>Combining Claims 1 and 2 and Lemma 4:</u>

Because $\tilde{\beta}(s, c)$ is upper-hemicontinous and convex valued (Lemma 4), and combining this with Claims 1 and 2, it follows via a version of the intermediate value theorem that there is a $\bar{c} > 0$ and $\epsilon > 0$ such that for every $c < \bar{c}$, there is an $s(c) > s_B + \epsilon$ such that $1 \in \tilde{\beta}(s(c), c)$. Hence, there is an equilibrium in which the voters vote for policy a with a probability more than 1/2.

Modifying Proof of Claim 2 to ensure large participation:

In this part, we will modify the second part of the above proof (i.e., the proof of claim 2) to show that ϵ_c can be chosen in such a way that the organizer, when faced with voters using a cutoff $1-\epsilon_c$, is indifferent between $m(c)$ and $m(c)-1$ pairs of voters in state A and recruits $m(c)$ pairs of voters in state B, and that $\lim_{c\to 0} m(c) = \infty$.

The alternative mapping that we consider is $x_m(c)$, defined analogously as the solution to

$$\binom{2m}{m} (q_A(x))^m (1-q_A(x))^m (2q_A(x)-1) = 2c.$$

As before, for given c sufficiently small,

$$\begin{aligned} & \binom{2m}{m} (q_B(x))^m (1-q_B(x))^m (2q_B(x)-1) \\ > \quad & 2c > \\ & \binom{2m+2}{m+1} (q_B(x))^{m+1} (1-q_B(x))^{m+1} (2q_B(x)-1). \end{aligned}$$

Thus, we can pick some $\hat{x}_m(c)$ just above $x_m(c)$ such that in state A, the organizer recruits $m-1$ pairs of voters and in state B recruits m pairs of voters. As $c \to 0$, it must be that $\hat{x}_m(c) \to 1$ and similar to before,

$$\lim_{c\to 0} \max \tilde{\beta}(\hat{x}_m(c), c) = \infty.$$

Now consider a sequence of equilibria whose existence has been shown with cutoffs bounded away (above) s_B, i.e., $\lim_{c\to 0} s(c) = s^* > s_B$.

If $s^* < 1$, then $\lim_{c\to 0} n_\omega(s(c), c) \to \infty$. This follows directly from the properties of the best reply correspondence of the organizer.

If $s^* = 1$, then note the following: Consider the function $g(m,x) = \binom{2m}{m} x^m (1-x)^m (2x-1)$, where m is a positive integer and $x \in [0,1]$. There is some $\varepsilon > 0$ such that $g(m,x)$ is decreasing in x and m, in the region where $x > 1-\varepsilon$.[17] This property of the function g together with the property for

[17] To see that $g(m,x)$ is decreasing in m for all $x > 1-\varepsilon$ for some $\varepsilon > 0$, notice that $\frac{g(m+1,x)}{g(m,x)} = \frac{(2m+2)(2m+1)}{(m+1)m} x(1-x) < 1$ for all m, when x is close to 1. To see that $g(m,x)$ is decreasing in x, first notice that if $g(1,x)$ is decreasing in x, then so is $g(m,x)$ for any $m \geq 1$. Showing that $g(1,x)$ is decreasing in x for x close to one is straightforward and we

the equilibrium cutoff $s(c)$ that, $s(c) \leq x_m(c)$, and also the hypothesis that $\lim_{c\to 0} s(c) = 1$ implies that $n_\omega(s(c), c) \geq m(c)$. Since this exercise can be repeated for any arbitrary m, we can pick the sequence $m(c)$ in a way that it grows unboundedly. Therefore, the resulting equilibrium participation grows without bound.

Showing that policy a gets selected:

Let $s\,(c)$ denote the equilibrium cutoff sequence that we showed the existence of in the previous parts of this Proof. By construction, $1 > s\,(c) > s_B + \varepsilon$ for all c smaller than $\bar{c} > 0$. We show that the probability of the majority voting for policy a approaches one as $c \to 0$. Without loss of generality, suppose $s(c)$ converges. The claim is obvious if

$$\lim_{c\to 0} s(c) = 1.$$

If not, then

$$1 > \lim_{c\to 0} s(c) \geq s_B + \varepsilon$$

implies

$$\lim_{c\to 0} q_A\,(s\,(c)) > \lim_{c\to 0} q_B\,(s\,(c)) > 0.5,$$

and $\lim_{c\to 0} n_\omega\,(c) \to \infty$. But this implies the claim also for the second case, because a law of large numbers applies. Thus, we have proven that across the sequence of equilibria we have shown the existence of, the probability that policy a gets implemented approaches 1, as c disappears.

Showing that organizer's payoff is 1 in both states:

Let $U_O^c(s(c), \tilde{n}(c))$ denote the equilibrium payoff of the organizer in the election in which the marginal recruitment cost is c and the equilibrium strategy profile $s(c), \tilde{n}(c)$ is the one identified in the previous parts. Consider the following alternative strategy $\bar{n}(c) := (\lfloor \frac{1}{\sqrt{c}} \rfloor, \lfloor \frac{1}{\sqrt{c}} \rfloor)$, i.e., $\bar{n}(c)$ is the strategy in which the organizer invites $\lfloor \frac{1}{\sqrt{c}} \rfloor$ pairs of voters in both states. Note that, as $c \to 0$, recruitment cost incurred by the strategy $\bar{n}$ disappears. Moreover, because $c \to 0$, the number of recruited voters go to ∞, and because $s(c) \to s^* > s_B + \epsilon$, by a weak version of Law of Large Numbers, the probability that the majority votes for policy a approaches 1 when the organizer employs

skip it.

strategy $\bar{n}(c)$. Hence, $\lim_{c\to 0} U_O^c(s(c), \bar{n}(c)) \to 1$. Because $\tilde{n}(c)$ is a best reply to voter cutoff strategy $s(c)$, it has to be that $U_O^c(s(c), \tilde{n}(c)) \geq U_O^c(s(c), \bar{n}(c))$, for every $c > 0$. Therefore, $\lim_{c\to 0} U_O^c(s(c), \tilde{n}(c)) = 1$, as well. Since, in each state, the organizer's payoff is bounded above by 1, and since each state occurs with positive probability, the organizer's payoff conditional on each state converges to 1, as well.

■

A.3 Proof of Theorem 2

This theorem characterizes all limit points of non-trivial equilibrium cutoffs:
PART 1: Sequences with limit points $s^* > s_B$: Manipulated Equilibria

The second part of Theorem 2, item 2 follows from our main theorem.
PART 2: Sequences with limit points $s^* = s_A$

Step 1: To show s_A is the only possible limit point of non-trivial equilibria, not more than s_B.

Lemma 8. *Let $\{s(c), n_A(c), n_B(c)\}_{c>0}$ be a selection of cutoffs $s(c)$ for the voters, and a pair of integers $(n_A(c), n_B(c))$ that are in the support of the organizer's best reply to voter strategy $s(c)$ with recruitment cost c. If $1 > \lim_{c\to 0} s(c) > s_\omega$, then (we drop the dependence of n_ω on c here)*

$$\lim_{c\to 0}(2n_\omega + 1)\binom{2n_\omega}{n_\omega} q_\omega(s(c))^{n_\omega}(1 - q_\omega(s(c)))^{n_\omega} = 0.$$

Proof. First, if $\lim s(c) > s_\omega$, then $\lim q_\omega(s(c)) = q^* > 1/2$. Therefore, $\lim n_\omega(c) = \infty$. By Stirling's approximation, we get that

$$\lim_{c\to 0} \frac{\binom{2n_\omega}{n_\omega} q_\omega(s(c))^{n_\omega}(1 - q_\omega(s(c)))^{n_\omega}}{(4q^*(1 - q^*))^{n_\omega}} \leq 1.$$

Because $q^* > 1/2$, $4q^*(1 - q^*) < 1$, and hence

$$\lim_{c\to 0}(2n_\omega + 1)(4q^*(1 - q^*))^{n_\omega} = 0.$$

Combining this with the above equality delivers the result. □

Clearly, if $s^* < s_A$, then the probability that a randomly selected voter votes for policy a is strictly less than 1/2 in both states, and the organizer recruits no one. But then this is a trivial equilibrium. So if there is any non-trivial equilibrium sequence, its limit point, $s^* \geq s_A$. If $s^* > s_B$, then these are manipulated equilibria, and we showed their existence and analyzed its properties in Theorem 1. So first suppose that $s_A < s^* < s_B$. In that case, when k is sufficiently large, the organizer recruits no one in state B, and many voters in state A. In fact, because $s^* > s_A$, $q_A(s(c)) \to q_A^* > 1/2$, and therefore, in any sequence of equilibria, for any selection of integers $\{n(A,c)\}$ that are in the support of the equilibrium recruitment strategy of the organizer, $n(A,c) \to \infty$. Therefore, by Lemma 8 above,

$$(2\tilde{n}(A,c)+1)\binom{2\tilde{n}(A,c)}{\tilde{n}(A,c)} q_A(s(c))^{\tilde{n}(A,c)}(1-q_A(s(c)))^{\tilde{n}(A,c)} \to 0.$$

Therefore, $\max \tilde{\beta}(s(c),c) \to 0$, which is a contradiction to $s(c)$ being an equilibrium cutoff.

On the way to a contradiction, suppose that $s^* = s_B$.

We will now argue that this cannot be the case since

$$\lim \max \tilde{\beta}(s(c),c) = 0 \text{ if } s(c) \to s_B.$$

First, note that

$$\lim_{c\to 0} q_A(s(c)) = q_A(s_B) > 1/2,$$

and this implies

$$\lim_{c\to 0}(2n_A+1)\binom{2n_A}{n_A} q_A^{n_A}(1-q_A)^{n_A} = 0.$$

for every sequence of integers $n_A(c)$ in the support of the organizer's best reply, via Lemma 8. So, there cannot be any infinite subsequence of the sequence equilibrium cutoffs in which $q_B(s(c)) \leq 1/2$. This is because, otherwise, along such a subsequence, $n_B(c) = 0$, and hence $\lim \max \tilde{\beta}(s(c),c) = 0$. So, we now look at the infinite subsequence along which $q_B > 1/2$.

Also remember that $\max \tilde{\beta}(s(c),c)$ is attained by some pure strategy that is in $\eta(s(c),c)$, and we will denote such a pure strategy with a pair of integers, n_A and n_B which correspond to the integers in the support of the strategy in states a and b, respectively. Surely these integers depend on c, but for ease of reading, when it does not cause confusion, we will drop the dependence of these integers on c.

In what follows, we will bound $\lim\max\tilde{\beta}(s(c),c)$ from above, by either putting a lower bound on the multiplication of two terms on the denominator, which is

$$(2n_B+1)\binom{2n_B}{n_B}q_B^n(1-q_B)^{n_B},$$

or by directly arguing that

$$\frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}}{(2n_B+1)\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)^{n_B}}\to 0.$$

For any given $q>1/2$, the function $f(q,n):=(2n+1)\binom{2n}{n}q^n(1-q)^n$ can have at most one peak, when viewed as a function of n. This is because,

$$\frac{f(q,n+1)}{f(q,n)}=\frac{2n+3}{2n+1}\frac{(2n+2)(2n+1)}{(n+1)^2}q(1-q)=\frac{4n+6}{n+1}q(1-q).$$

A simple calculation shows that the expression for $\frac{f(q,n+1)}{f(q,n)}$ is a strictly decreasing function of n. When q is sufficiently close to $1/2$, $f(q,n)$ is strictly increasing in n at $n=0$. Therefore, for every nonnegative integer N^*, the minimum of $f(q,n)$ in the domain $n\in\{0,1,...,N^*\}$ is attained at one of the extreme points, i.e., either at $n=0$ or $n=N^*$.

Pick any infinite subsequence along which $n_A(c)\geq n_B(c)$. From the above argument when c is small,

$$f(q_B(s(c)),n_B(c)\geq\min\{f(q_B(s(c)),n_A(c);f(q_B(s(c)),0)\}.$$

Along a subsequence at which the above minimum is attained at $n=0$, $f(q_B(s(c)),0)=1$, and hence our claim follows. This is because,

$$(2n_B+1)\binom{2n_B}{n_B}q_B^n(1-q_B)^{n_B}\geq\min_{n\in\{0,1,\dots,n_B\}}f(q_B,n)\geq\min_{n\in\{0,1,\dots,n_A\}}f(q_B,n)=1.$$

Where the first inequality follows from the definition of $f(q,n)$, and the second one follows from the property of the subsequence that $n_A \geq n_B$, and hence the min is taken over a larger set. This together with

$$\lim(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}=0.$$

delivers that $\lim\max\tilde{\beta}(s(c),c)=0$ along such a sequence.

Along the remaining sequence along which the minimum of the above expression is attained at $f(q_B(s(c)),n_A(c)$,

$$\begin{aligned}\frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}}{(2n_B+1)\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)^{n_B}} &\leq \frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}}{(2n_A+1)\binom{2n_A}{n_A}q_B^{n_A}(1-q_B)^{n_A}}\\ &= \frac{q_A^{n_A}(1-q_A)^{n_A}}{q_B^{n_A}(1-q_B)^{n_A}}\to 0.\end{aligned}$$

The last line follows from the fact that $q_A(1-q_A)<q_B(1-q_B)$, and that $n_A\to\infty$. Here, $q_A(1-q_A)<q_B(1-q_B)$ because $q_A>q_B\geq 1/2$.

Now the only remaining subsequence is the one along which $n_A(c)<n_B(c)$. For such a subsequence, notice that the optimality of the organizer's best reply delivers:

$$\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)^{n_B}\geq\frac{2c}{2q_B-1},$$

and

$$\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}\leq\frac{c}{2q_A(1-q_A)(2q_A-1)}\frac{n+1}{2n+1}\leq\frac{c}{2q_A(1-q_A)(2q_A-1)}.$$

Notice that, $2q_B-1\to 0$, and combining this with $n_A(s(c))<n_B(s(c))$ delivers:

$$\lim_{c\to 0}\frac{(2n_A+1)\binom{2n_A}{n_A}q_A^{n_A}(1-q_A)^{n_A}}{(2n_B+1)\binom{2n_B}{n_B}q_B^{n_B}(1-q_B)^{n_B}}=0.$$

which then implies that $\lim\max\tilde{\beta}(s(c),c)=0$ along such a sequence as well.

Step 2: To show s_A is an attainable limit point.

The proof strategy here will be similar to the proof for the existence of manipulative equilibria in Theorem 1.

Using Lemma 7 (above) as before, it is straightforward to show that there exists some small $\bar{\varepsilon} > 0$ such that for every $0 < \varepsilon < \bar{\varepsilon}$, $\lim \max \tilde{\beta}(s_A + \varepsilon, c) = 0$. We will show that there is an $\varepsilon(c) > 0$ with $\lim \varepsilon(c) = 0$, such that $\lim \max \tilde{\beta}(s_A + \varepsilon(c), c) = \infty$. Clearly, then, via our version of intermediate value theorem, again an equilibrium with a cut point $s(c) \in (s_A + \varepsilon(c), s_A + \varepsilon)$ exists, for every small c. By the previous step, the limit point of $s(c)$ has to be s_A. So all we have to show is that there is a mapping $\varepsilon(c) > 0$ with $\lim \varepsilon(c) = 0$ such that $\lim \max \tilde{\beta}(s_A + \varepsilon(c)) = \infty$.

Note that in state B, the organizer recruits no one, for $\varepsilon(c)$ sufficiently small. So our task is to show that $(2n_A + 1)\binom{2n_A}{n_A} q_A^{n_A}(1 - q_A)^{n_A}$ can be made arbitrarily large, for small c, with a cutoff $s_A + \varepsilon(c)$.

Given any integer a, let $s(a, c) > s_A$ be such that the organizer is indifferent between recruiting a and $a + 1$ pairs of voters when the voters are using a cutoff strategy with a cutoff $s(a, c)$. In particular, $(2q_A(s(a, c)) - 1)\binom{2a}{a} q_A(s(a, c))^a(1 - q_A(s(a, c)))^a = c$. It is evident by now that we can do the selection of $s(a, c)$ such that $\lim_{c \to 0} s_A(a, c) \to s_A$ for every integer a. (Note that such a selection could be made by picking cutoffs that converge to 1 as well)

Let $s(a) > s_A$ be equal to $\min\{\tilde{s}(a), s_A + \bar{\varepsilon}\}$, where $\tilde{s}(a)$ is the largest signal that has the property that, for every $q \in [1/2, q_A(\tilde{s}(a))]$

$$2 - \frac{a(2q - 1)^2}{q(1 - q)} \geq 0.$$

For every a, such a $\tilde{s}(a) > s_A$ exists, by inspection of the inequality. Moreover, for a sufficiently large, $s(a) = \tilde{s}(a) < s_A + \bar{\varepsilon}$.

Note that, $\lim \max \tilde{\beta}(s(a), c) = 0$. Moreover, $\lim \max \tilde{\beta}(s(a, c), c) = O(\sqrt{a})$, which follows from Stirling's approximation,

$$a\binom{2a}{a} q_A(s(a, c))^a(1 - q_A(s(a, c)))^a \cong a\frac{4^a}{\sqrt{\pi}\sqrt{a}}\left(\frac{1}{2}\right)^a\left(\frac{1}{2}\right)^a,$$

so that

$$\lim_{a \to \infty} \lim_{c \to 0} \max \tilde{\beta}(s(a, c), c) = \infty.$$

Therefore, via a version of the intermediate value theorem, for each a sufficiently large, there is a $\bar{c}$ such that for all $c < \bar{c}$, there is $s^*(a, c) \in (s(a, c), s(a))$ such that $1 \in \tilde{\beta}(s^*(a, c), c)$.

Step 3: To show that an equilibrium sequence exists whose limit point is s_A and for which in state A, a majority selects policy a with probability one in the limit.

Note that if $s^* = s_A$, then in state B, no one is recruited, and hence there is only one voter for every c. Therefore, as $c \to 0$, the term $(2n_A + 1)\binom{2n_A}{n_A} q_A^{n_A}(1 - q_A)^{n_A}$ converges to a number $k \in (0, \infty)$. Now consider the cutoff $s^*(a, c)$ defined in step 2, above. Note that the organizer's best reply to this cutoff in state A is to recruit at least a voters, for c small enough. This is because, $s^*(a, c) < \tilde{s}(a)$, where for every $q \in [1/2, q_A(\tilde{s}(a))]$,

$$2 - \frac{a(2q-1)^2}{q(1-q)} \geq 0.$$

The marginal benefit of the organizer,

$$\frac{\partial \binom{2a}{a} (q(1-q))^a (2q-1) 1/2}{\partial q} = \frac{1}{2} \binom{2a}{a} (q(1-q))^a [2 - \frac{a(2q-1)^2}{q(1-q)}] > 0,$$

for every $q \in [1/2, q_A(\tilde{s}(a))]$, and hence the support of the organizer's best replies at state A are at least a pairs of voters, whenever the voters are using a cutoff between $s(a, c)$ and $\tilde{s}(a)$. Because the equilibrium cutoff $s^*(a, c)$ that we identified is in that interval, the organizer indeed recruits at least a pairs of voters in state A.

Since a is arbitrary, we can construct a sequence of equilibria along which $s(c) \to s_A$, and the number of voters recruited in state A grows without bound.

Now we will show that if $s(c) \to s_A$ and if the number of voters in state A grows without bound, then the majority selects policy a with a probability that converges to 1. As we stated in the previous paragraph:

$$\lim_{c \to 0} (2n_A + 1) \binom{2n_A}{n_A} q_A^{n_A} (1 - q_A)^{n_A} = k \in (0, \infty).$$

Where again, n_A, q_A clearly depend on $s(c)$ and c, but we dropped the dependence for ease of reading. Because $n_A \to \infty$, $s(c) > s_A$. The probability that the majority selects policy a in state A is:

$$\sum_{i=n_A+1}^{2n_A+1} \binom{2n_A+1}{i} q_A^i (1-q_A)^{2n_A+1-i}.$$

To show that this probability converges to 1, we use the following lemma.

Lemma 9. *Let* $\{q(c)\}_{c>0}$ *be a selection of probabilities with* $\lim q(c) \to q*$*, and* $\{n(c)\}_{c>0}$ *be a selection of integers such that* $\lim n(c) \to \infty$ *. If*

$$\lim_{c\to 0}(2n(c)+1)\binom{2n(c)}{n(c)} q(c)^{n(c)}(1-q(c))^{n(c)} = k \in (0,\infty),$$

then

$$\lim_{c\to 0}\sum_{i=0}^{n(c)} \binom{2n(c)+1}{i} q(c)^i (1-q(c))^{2n(c)+1-i} = 0.$$

Proof. Pick any pair q, n. Let

$$t(i,n) := \frac{\binom{2n+1}{n+1} q^{n+1}(1-q)^n}{\binom{2n+1}{i} q^i (1-q)^{2n+1-i}} = (\frac{q}{1-q})^{n+1-i}\frac{(2n+1-i)(2n-i)...(n+2)}{n(n-1)...(i+1)}.$$

Note that $t(i,n) > 1$ for $i \leq n$ because $q > 1/2$. Moreover, $t(i,n)$ is decreasing in i.

Pick an arbitrary $\epsilon > 0$. Let $1+\kappa(\epsilon)$ be a lower bound strictly larger than 1 for the term

$$\frac{2n+1-(n(1-\epsilon))}{n(1-\epsilon)+1}.$$

For $i \leq (1-2\epsilon)n$, we have that $t(i,n) \geq (1+\kappa(\epsilon))^{\epsilon n}$. Therefore,

$$\sum_{i=0}^{n} \binom{2n+1}{i} q(n)^i (1-q(n))^{2n+1-i} \leq ((n(1-2\epsilon))(1+\kappa(\epsilon))^{-\epsilon n} + 2\epsilon n)\binom{2n+1}{n} q^{n+1}(1-q)^n.$$

Taking $n \to \infty$, and then using the fact that ϵ was arbitrary, and the fact that

$$(2n(c)+1)\binom{2n(c)}{n(c)} q(c)^{n(c)}(1-q(c))^{n(c)} \to k \in (0,\infty)$$

delivers the result.

$\square$

Step 4: Proof that if inequality (5) holds, there is an equilibrium with limit cutoff s_A and with a bounded number of voters.

Note that $\tilde{\beta}(s_A, c)$ is single valued for every $c > 0$, and that value is equal to $\frac{\pi}{1-\pi}\frac{f(s_A|A)}{f(s_A|B)}$. This is because, $\eta(s_A, c)$ has a single element for every $c > 0$, and this single element is a pure strategy that recruits no one in both states.

Hence, if inequality (5) holds, then $\max \tilde{\beta}(s_A, c) \leq 1$, for every $c > 0$. By the argument in step 2, there is some large a and $\bar{c} > 0$ such that for every $c < \bar{c}$, $\max \tilde{\beta}(s(a, c), c) > 1$. Therefore, by a version of the intermediate value theorem, there is some $s(c) \in [s_A, s(a, c)]$ such that $1 \in \tilde{\beta}(s(c), c)$. Because $s(c) < s(a, c)$, and because for all sufficiently small c, $s(a, c) < \tilde{s}(a)$, and because $s(a, c)$ is the cutoff signal to which the organizer best reply is to recruit at most $a+1$ pairs of voters in state A, the organizer recruits not more than $a+1$ pairs of voters in state A when the voters are using the cutoff $s(c)$. Because this is true for every $c < \bar{c}$, and because $\lim_{c\to 0} s(a, c) = s_A$, we can construct a sequence of equilibrium cutoffs that converge to s_A and along such equilibria, the organizer recruits a bounded number of voters in state A (and no one in state B).

Step 5: Proof that if inequality (5) is not satisfied, the all equilibria with limit cutoff s_A have an unbounded number of voters.

On the way to a contradiction, suppose that there is an equilibrium sequence with limit cutoff s_A, and which has a bounded number of voters in state A, say less than k. Notice that,

$$\liminf_{c\to 0} \sum_{i>0} \tilde{n}(c)(i) \times (2i+1)\binom{2i}{i}(q_A(s(c))(1 - q_A(s(c))))^i \geq 1,$$

where $\tilde{n}(c)$ is the equilibrium strategy of the organizer. This is because, $q_A(s(c)) \to 1/2$, $(2i+1)\binom{2i}{i}(1/4)^i$ is strictly increasing in i, and because $\tilde{n}(c)(i) = 0$ for every $i > k$. Moreover, $\frac{f(s|A)}{f(s|B)}$ is strictly increasing in s, and hence, for every $s > s_A$,

$$\frac{\pi}{1-\pi}\frac{f(s|A)}{f(s|B)} > 1.$$

But this contradicts the equilibrium requirement that $1 = \beta(s(c), piv, rec; s(c), \tilde{n}(c))$.

A.4 Proof of Theorem 4

As we argued in the main text after the statement of the Theorem, items 2 and 3 follow from a slight modification of Feddersen and Pesendorfer (1997). In the rest of the proof, we will show that there is a sequence of equilibria that aggregates information.

For every k, let s_k be an equilibrium cutoff of the voting game in which there are $2m_k + 1$ voters in both states of the worlds. This is the setup of Feddersen and Pesendorfer (1997), and $s_k \to s^* \in (s_A, s_B)$ since $m_k \to \infty$.

We will now construct a sequence of equilibrium cutoffs, $\tilde{s}_k$, such that information aggregates along such equilibria. Consider the mapping $\tilde{\beta}^k(s, c)$ which is a modification of the mapping $\tilde{\beta}(s, c)$ by incorporating that the minimum number of voters is $2m_k + 1$. Clearly for large enough k, $\tilde{n}_B(s_k, c_k) = m_k$, i.e., the organizer recruits no one in state B. If $\tilde{n}_A(s_k, c_k) = m_k$, i.e., if in state A also the organizer recruits no one, then it is an equilibrium that the organizer recruits no one, and the voters use a cutoff strategy with a cutoff s_k, and let $\tilde{s}_k = s_k$ in this case.

If $\tilde{n}_A(s_k, c_k) > m_k$, then because $(2m+1)\binom{2m}{m} q_A(s_k)^m (1 - q_A(s_k))^m)$ is decreasing in m when m is large and s_k is sufficiently close to s^*, $\max \tilde{\beta}^k(s_k, c_k) < 1$. Moreover, $\tilde{\beta}^k(s_A, c_k) > 1$ when k is sufficiently large. Hence, there is a $\tilde{s}_k \geq s_A$ such that $\tilde{s}_k$ is an equilibrium cutoff of the voting game with the organizer. The proof that when $\tilde{s}_k > s_A$, as $k \to \infty$, in state A the probability that policy a is implemented converges to one follows identical reasoning as Lemma 9, so we skip it. Clearly, in state B, the organizer recruits no new voter, and since $m_k \to \infty$, in state B, policy b is implemented with a probability that approaches to 1, and hence information is aggregated in both states.

A.5 Proof of Theorem 5 (Unanimity)

Let $\rho(s) := \frac{f(s|A)}{f(s|B)}$ We start by noting the identity:

$$\frac{F(s_m|A)}{F(s_m|B)} = (1 + \frac{F(s_m|A) - F(s_m|B)}{F(s_m|B)}).$$

The indifference condition of the cutoff signal delivers that:

$$\left(\frac{F(s_m|A)}{F(s_m|B)}\right)^{2m} = \left(1 + \frac{F(s_m|A) - F(s_m|B)}{F(s_m|B)}\right)^{2m} = \frac{(1-\pi)}{\pi\rho(s_m)}.$$

The next calculation follows from the Lemma in the main text which showed that $s_m \to 1$, and hence that $F(s_m|B) \to 1$, and that $F(s_m|A) - F(s_m|B) \to 0$:

$$\lim_{m\to\infty}\left(1 + \frac{(F(s_m|A) - F(s_m|B))}{F(s_m|B))}\right)^{2m} = \frac{(1-\pi)}{\pi\rho(1)}.$$

The probability that a is chosen in state A is $p_0^m := (F(s_m|A))^{2m}$. As $m \to \infty$,

$$\begin{aligned}\lim_{m\to\infty} p_0^m &= \lim_{m\to\infty}\left(1 - (1 - F(s_m|A))\right)^{2m} \\ &= \left(\tfrac{(1-\pi)}{\pi\rho(1)}\right)^{\frac{-f(1|A)}{f(1|A)-f(1|B)}}.\end{aligned}$$

The last line follows because if $\lim_n (1 + x_n)^n = a$ and if $\lim_n \frac{y_n}{x_n} = z$, then $\lim_n (1 + y_n)^n = a^z$. Now if we apply this by defining $x_n = \frac{F(s_m|A)-F(s_m|B)}{F(s_m|B)}$, and $y_n = -(1 - F(s_m|A))$, we get the expression for $\lim p_0^m$. Similarly, the probability that policy b is chosen in state B, $p_1^m = 1 - (F(s_m|B))^{2m}$. As $m \to \infty$, this calculation yields:

$$\begin{aligned}\lim_{m\to\infty} 1 - p_1^m &= \lim_{m\to\infty}\left(1 - (1 - F(s_m|B))\right)^{2m} \\ &= \left(\tfrac{1-\pi}{\pi\rho(1)}\right)^{\frac{-f(1|B)}{f(1|A)-f(1|B)}}.\end{aligned}$$

This completes the first part of the theorem. To show that $\left(\frac{(1-\pi)}{\pi\rho(1)}\right)^{-\frac{f(1|A)}{f(1|A)-f(1|B)}}$ goes to 1 if $\frac{f(1|A)}{f(1|B)}$ goes to 0 is by taking the log of the expression $\left(\frac{(1-\pi)}{\pi\rho(1)}\right)^{-\frac{f(1|A)}{f(1|A)-f(1|B)}}$, and using L'Hopital's rule to show that this ln expression goes to zero. The other part similarly follows.

References

Atakan, A. E. and M. Ekmekci (2014): "Auctions, Actions, and the Failure of Information Aggregation," *American Economic Review*, 104.

Austen-Smith, D. and J. S. Banks (1996): "Information aggregation, rationality, and the Condorcet jury theorem," *American Political Science Review*, 34–45.

Barelli, P. and S. Bhattacharya (2013): "A Possibility Theorem on Information Aggregation in Elections," *Working paper.*

Bhattacharya, S. (2013): "Preference monotonicity and information aggregation in elections," *Econometrica*, 81, 1229–1247.

Bond, P. and H. Eraslan (2010): "Strategic voting over strategic proposals," *The Review of Economic Studies*, 77, 459–490.

Bouton, L. and M. Castanheira (2012): "One person, many votes: Divided majority and information aggregation," *Econometrica*, 80, 43–87.

Crawford, V. P. and J. Sobel (1982): "Strategic information transmission," *Econometrica*, 1431–1451.

Duggan, J. and C. Martinelli (2001): "A Bayesian model of voting in juries," *Games and Economic Behavior*, 37, 259–294.

Feddersen, T. and W. Pesendorfer (1997): "Voting behavior and information aggregation in elections with private information," *Econometrica*, 1029–1058.

——— (1998): "Convicting the innocent: The inferiority of unanimous jury verdicts under strategic voting," *American Political Science Review*, 23–35.

——— (1999a): "Elections, information aggregation, and strategic voting," *Proceedings of the National Academy of Sciences*, 96, 10572–10574.

Feddersen, T. J. and W. Pesendorfer (1996): "The swing voter's curse," *American Economic Review*, 408–424.

——— (1999b): "Abstention in elections with asymmetric information and diverse preferences," *American Political Science Review*, 381–398.

Gul, F. and W. Pesendorfer (2009): "Partisan politics and election failure with ignorant voters," *Journal of Economic Theory*, 144, 146–174.

Kamenica, E. and M. Gentzkow (2011): "Bayesian Persuasion," *American Economic Review*, 101, 2590–2615.

Krishna, V. and J. Morgan (2011): "Overcoming ideological bias in elections," *Journal of Political Economy*, 119, 183–211.

——— (2012): "Voluntary voting: Costs and benefits," *Journal of Economic Theory*, 147, 2083–2123.

Lauermann, S. and A. Wolinsky (2012): "A Common Value Auction with Bid Solicitation," Tech. rep., Working Paper, Northwestern University.

Mandler, M. (2012): "The fragility of information aggregation in large elections," *Games and Economic Behavior*, 74, 257–268.

McLennan, A. (1998): "Consequences of the Condorcet jury theorem for beneficial information aggregation by rational agents," *American Political Science Review*, 413–418.

Milgrom, P. R. (1979): "A convergence theorem for competitive bidding with differential information," *Econometrica*, 679–688.

Myerson, R. B. (1998a): "Extended Poisson games and the Condorcet jury theorem," *Games and Economic Behavior*, 25, 111–131.

——— (1998b): "Population uncertainty and Poisson games," *International Journal of Game Theory*, 27, 375–392.

Palfrey, T. R. and H. Rosenthal (1985): "Voter participation and strategic uncertainty," *The American Political Science Review*, 62–78.

Pesendorfer, W. and J. Swinkels (1997): "The loser's curse and information aggregation in common value auctions," *Econometrica*, 65, 1247–1281.

——— (2000): "Efficiency and information aggregation in auctions," *American Economic Review*, 90, 499–525.

Wang, Y. (2012): "Bayesian persuasion with multiple receivers," *Job Market Paper, Department of Economics, University of Pittsburgh*, 1–45.

Wilson, R. (1977): "A bidding model of perfect competition," *The Review of Economic Studies*, 511–518.