Large tick assets: implicit spread and optimal tick value

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Outline

1. Tick value, tick size and spread
   - Tick value
   - Tick size
   - Large tick asset and spread
   - Spread theory for small tick assets
   - The Wyart *et al.* approach
2. The model with uncertainty zones
   - Simplified version
   - Buy only, sell only and buy/sell areas
   - Some intuitions
3. Implicit spread and volatility per trade
   - Setup
   - Regression design
   - Cost analysis
   - Explanation of microstructure effects
4. Optimal tick value
   - Changing the tick value

Outline

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Definitions

- Exchange rules: ∃ price grid for orders.
- **Tick value**: smallest price increment. Dimension: currency of the asset.
  - Subject to changes by the exchange.
  - In some markets, the spacing of the grid can depend on the price.
  - eg: stocks trading on Euronext Paris have a price dependent tick scheme. Stocks priced 0 to 9.999€ have a tick value of 0.001€ but all stocks above 10€ have a tick of 0.005€.

Outline

Notion of tick size

- In practice: tick value is given little consideration. What is important is the **tick size**.
- **Tick size**: qualifies the traders' aversion to price movements of one tick.

Tick value vs tick size

- The trader's perception of the tick size is qualitative and empirical. It depends on:
  - tick value,
  - price,
  - average daily volumes,
  - volatility,
  - own trading strategy.
- The tick value is not a good measure of the perceived size of the tick.
- eg: ESX futures has a much larger tick size than the DAX index futures, though the tick values are of the same orders.

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What is a large tick asset ?

- Notion of tick size is ambiguous in general. However, we can identify large tick assets.
- From Eisler, Bouchaud and Kockelkoren: *Large tick stocks are such that the bid-ask spread is almost always equal to one tick, while small tick stocks have spreads that are typically a few ticks.*
- This leads to the following questions:

Issues

- Small tick assets: spread is a good proxy for the tick size.
  - If spread $\simeq$ 1 tick $\Rightarrow$ How to quantify the tick size ?
- In the literature: $\exists$ special relationships between the spread and some market quantities. BUT:
  - Not valid for large tick assets: spread bounded by 1.

How to extend these studies in the large tick case?

- Tick value change $\Rightarrow$ What happens to the microstructure?
- Can we define an optimal tick value?

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Madhavan, Richardson, Roomans model

- p_i: ex post efficient price after the i^{th} trade
- ε_i: sign of the i^{th} trade.
- MRR model:

$$p_{i+1} - p_i = \xi_i + \theta\varepsilon_i,$$

- ξ_i: independent centered shock component (new information,...) with variance v^2.
- θ: impact coefficient.

MRR model

- Market makers cannot guess the surprise of the next trade. So, they post (pre trade) bid and ask prices a_i and b_i:

$$a_i = p_i + \theta + \phi, \quad b_i = p_i - \theta - \phi,$$

with ϕ an extra compensation (processing costs and the shock component risk).

- The above rule ensures no ex post regrets for market makers:
- $\phi = 0 \Rightarrow$ the ex post average cost of an ask market order (relative to the efficient price) = $a_i - p_{i+1}$ = 0. (same for bid)

MRR model

We can compute several quantities:

- Spread: $S = a - b = 2(\theta + \phi)$.
- Volatility per trade of the efficient price:

$$\sigma_1^2 = E[(p_{i+1} - p_i)^2] = \theta^2 + v^2 \sim \theta^2$$

(Neglecting the news contribution, see Wyart *et al.*).

- Therefore:

$$S \sim 2\sigma_1 + 2\phi.$$

Outline

Market making strategy

- Market makers: patient traders. Send limit orders ⇒ delayed execution. Pocket the spread. ∃ volatility risk.
- Market takers: impatient traders. Send market orders ⇒ immediate execution. Pay the spread. No volatility risk.
- Wyart *et al.*: consider a simple market making strategy. Its average P&L per trade is

$$P\&L = \frac{S}{2} - \frac{c}{2}\sigma_1,$$

with c depending on the assets but of order $1 \sim 2$.
- P&L = cost of a market order (on average).

Market maker vs market taker

Wyart *et al.*:

- On electronic markets, any agent can choose between market orders and limit orders. ⇒ both types of orders will have the same average (ex post) cost = 0 ⇒ Market makers' P&L = 0 (competition).
- Therefore:

$$S \sim c\sigma_1.$$

- This relationship is very well satisfied on market data.

Outline

Properties

- Model for transaction prices and durations, based on an efficient semi-martingale type price.
- One important scalar parameter: η. Characterizes microstructure.
- Reproduces almost all the stylized facts of (ultra) high frequency and low frequency data.
- Originally built in the purpose of high frequency statistical estimation and hedging.

Thoughts and intuitions

In practice:

- Uncertainty about the efficient price.
- Aversion for price changes.

Thoughts behind the model:

- The price changes only when market participants are convinced that the efficient price is sufficiently far from the last traded price.
- Quantifying the aversion for price changes ⇒ η.

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Notations

- X_t : non observable efficient price. Volatility σ. $X_t = \sigma W_t$, with W a Brownian motion.
- α: tick value. a: ask, b: bid. $m = \frac{a+b}{2}$: midpoint.
- t_i: time of the i^{th} transaction with price change.
- P_t: observable price. P_{t_i}: transaction price at time t_i.
- $U = 2\eta\alpha < \alpha$: uncertainty region around m.
- $U_k = [0, \infty) \times (d_k, u_k)$ with

$$d_k = (k + 1/2 - \eta)\alpha \text{ and } u_k = (k + 1/2 + \eta)\alpha.$$


101.5
101
100.5
100
99.5
99
Price
$2\eta\alpha$
α
0
50
100
150
200
250
300
Time

Notations

- t_i: i^{th} exit time of an uncertainty zone:

$$t_{i+1} = \inf\left\{t > t_i, X_t = X_{t_i}^{(\alpha)} \pm \alpha(\frac{1}{2} + \eta)\right\},$$

  where $X_{t_i}^{(\alpha)}$ the value of X_{t_i} rounded to the nearest multiple of α.

- $P_{t_i} = X_{t_i}^{(\alpha)}$. (∃ transaction on every price change).

Estimation of η

- A continuation is a price variation whose direction is the same as the one of the preceding variation.
- An alternation is a price variation whose direction is opposite to the one of the preceding variation.
- N^c = # continuations. N^a = # alternations.
- Estimator $\widehat{\eta}$:

$$\widehat{\eta} = \frac{N^c}{2N^a}$$

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Bund and DAX, estimation of η

October 2010

Day	η (Bund)	η (FDAX)	Day	η (Bund)	η (FDAX)
1 Oct.	0.18	0.41	18 Oct.	0.16	0.33
5 Oct.	0.15	0.37	19 Oct.	0.13	0.37
6 Oct.	0.15	0.37	20 Oct.	0.13	0.33
7 Oct.	0.15	0.38	21 Oct.	0.15	0.33
8 Oct.	0.15	0.41	22 Oct.	0.11	0.33
11 Oct.	0.14	0.36	25 Oct.	0.12	0.31
12 Oct.	0.14	0.36	26 Oct.	0.14	0.31
13 Oct.	0.14	0.32	27 Oct.	0.14	0.32
14 Oct.	0.16	0.35	28 Oct.	0.14	0.32
15 Oct.	0.16	0.35	29 Oct.	0.14	0.34

Futures	Exchange	Class	Tick Value	Session	# Trades/Day	$\#\eta$	$\#S_=$
BUS5	CBOT	Interest Rate	7.8125 $	7:20-14:00	26914	0.233	94.9
DJ	CBOT	Equity	5.00 $	8:30-15:15	48922	0.246	81.8
EURO	CME	FX	12.50 $	7:20-14:00	46520	0.242	90.6
SP	CME	Equity	12.50 $	8:30-15:15	118530	0.035	99.6
Bobl 1	EUREX	Interest Rate	5.00 €	8:00-17:15	18531	0.268	95.3
Bobl 2	EUREX	Interest Rate	10.00 €	8:00-17:15	11637	0.142	99.2
Bund	EUREX	Interest Rate	10.00 €	8:00-17:15	25182	0.138	98.1
DAX	EUREX	Equity	12.50 €	8:00-17:30	39573	0.275	72.7
ESX	EUREX	Equity	10.00 €	8:00-17:30	35121	0.087	99.5
Schatz	EUREX	Interest Rate	5.00 €	8:00-17:15	9642	0.122	99.4
CL	NYMEX	Energy	10.00 $	8:00-13:30	73080	0.228	75.7

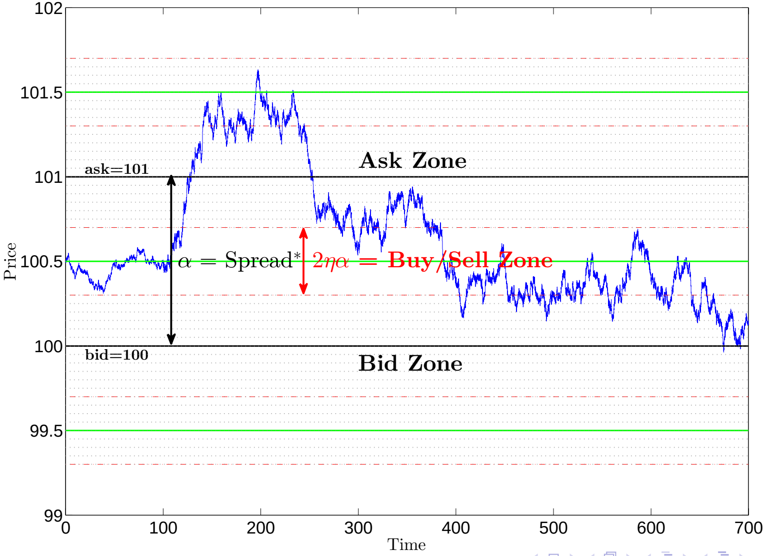
Table 1: Data Statistics. The *Session* column indicates the considered trading hours (local time). The sessions are chosen so that we get enough liquidity and are not the actual sessions.

Outline

The market order areas

- Simplification: $S = \alpha$, constant, 1 tick.
- For given bid-ask quotes, we have:
  - Bid or Buy only zone.
  - Ask or Sell only zone.
  - Uncertainty or Buy/Sell zone.

Ask Zone, Bid Zone and Uncertainty Zone



Outline

Intuitions about η

- $\eta \iff$ Distribution of high frequency tick returns:
- η **small** $\Rightarrow$ Uncertainty zone small $\Rightarrow$ Strong mean reversion in the observed price $\Rightarrow$ Decreasing signature plot, significant ACV of tick returns $\Rightarrow$ Tick size **large**.
- $\eta \sim 1/2 \Rightarrow$ the last traded price can be seen as a sampled Brownian motion $\Rightarrow$ No microstructure effects $\Rightarrow$ Flat signature plot and ACV of tick returns $\Rightarrow$ Uncertainty zone = 1 tick.

Intuitions about η

- Distance between Ask Zone and Bid Zone is $2\eta\alpha$.
- $2\eta\alpha$ represents an **implicit unobservable spread**.
- M: **Total number of trades** (null returns and not).
- Can we extend

$$\frac{S}{2} \sim \frac{\sigma}{\sqrt{M}} \text{ to } \eta\alpha \sim \frac{\sigma}{\sqrt{M}}?$$

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The assets

- We want to investigate the relationship

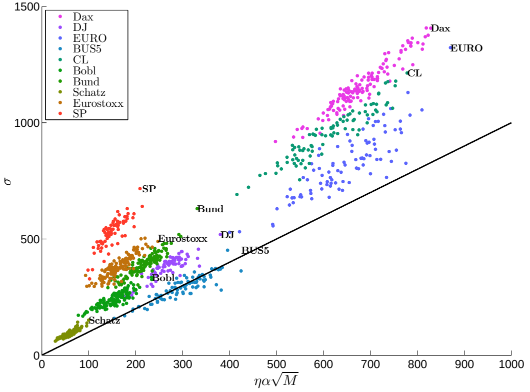
$$\eta\alpha \sim \frac{\sigma}{\sqrt{M}} + \phi$$

  for large tick assets.

- We consider Futures on: the DAX index (DAX), the Euro-Stoxx 50 index (ESX), the Dow Jones index (DJ), SP500 index (SP), 10-years Euro-Bund (Bund), 5-years Euro-Bobl (Bobl), 2-years Euro-Schatz (Schatz), 5-Year U.S. Treasury Note Futures (BUS5), EUR/USD futures (EURO), Light Sweet Crude Oil Futures (CL).

Empirical results

- Cloud $(\eta\alpha\sqrt{M}, \sigma)$, for each day, for each asset.
- Linear relationship, same slope, different intercepts.



Outline

Linear regression

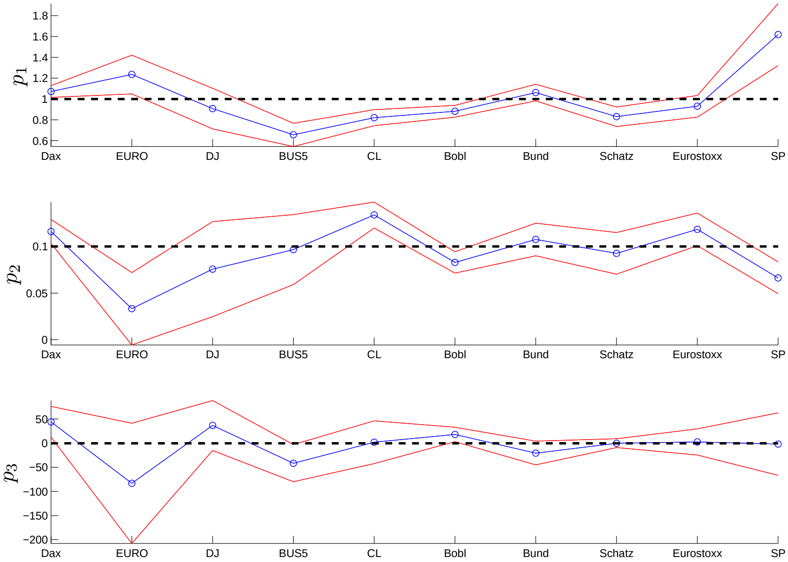
- We consider the relationship

$$\eta\alpha \sim \frac{\sigma}{\sqrt{M}} + \phi$$

  for large tick assets.
- ϕ includes operational costs and **inventory control** $\Rightarrow$ $\phi = k * S$.
- Daily regression:

$$\sigma = p_1 \eta\alpha\sqrt{M} + p_2 S\sqrt{M} + p_3.$$

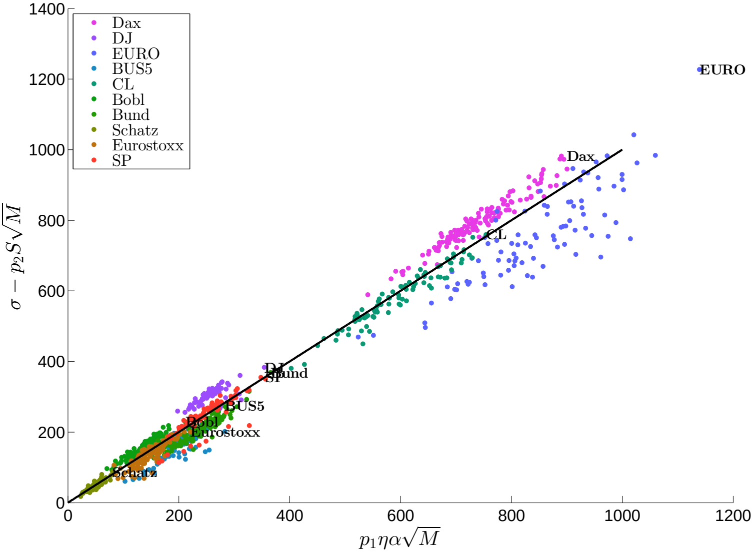
Daily regression



Asset	p_1	p_2	p_3	R^2
BUS5	0.67 [0.55,0.79]	0.10 [0.06,0.14]	-40.21 [-76.28,-4.14]	0.84
DJ	0.93 [0.71,1.15]	0.07 [0.01,0.13]	38.90 [-18.19,96.00]	0.73
EURO	1.31 [1.11,1.51]	0.02 [-0.02,0.07]	-89.23 [-211.08,32.62]	0.75
SP	1.67 [1.37,1.96]	0.07 [0.05,0.08]	-2.84 [-69.90, 64.21]	0.83
Bobl	0.91 [0.84,0.97]	0.08 [0.07,0.09]	19.04 [4.41,33.67]	0.90
Bund	1.11 [1.01,1.20]	0.11 [0.09,0.13]	-29.99 [-54.16,-5.82]	0.92
Dax	1.09 [1.01,1.16]	0.11 [0.10,0.13]	54.94 [23.02,86.86]	0.97
ESX	0.89 [0.78,1.01]	0.13 [0.11,0.15]	-10.15 [-37.71,17.41]	0.90
Schatz	0.80 [0.71,0.90]	0.10 [0.07,0.12]	-0.93 [-9.78,7.92]	0.88
CL	0.97 [0.89,1.05]	0.11 [0.09,0.12]	-11.14 [-51.20,28.92]	0.97

Table 2: Estimation of the linear model with 95% confidence intervals.

The constant is equal to zero



Outline

Market orders cost

- Average ex post cost of a market order (relative to X_t):

$$\alpha/2 - \eta\alpha.$$

- Average P&L per trade of the market makers = average cost of a market order ⇒

$$\eta\alpha = c\frac{\sigma}{\sqrt{M}} + \phi.$$

- $\eta < 1/2$: Limit orders are profitable whereas market orders are costly.

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Signature plot

- Observe P_t at times $t = i/n$, $n \in \mathbb{N}$, $i = 0, \ldots, n$, ($t = 1 \sim 1$ trading day).
- Signature plot:

$$k \to RV_n(k) = \sum_{i=0}^{\lfloor n/k \rfloor - 1} (P_{k(i+1)/n} - P_{ki/n})^2.$$

where $k = 1, \ldots, n$

- If P_t continuous semi-martingale $\Rightarrow RV_n(k)$ converges as $n \to \infty$.
- Empirical data: signature plot has a decreasing behavior.

Example: Bund

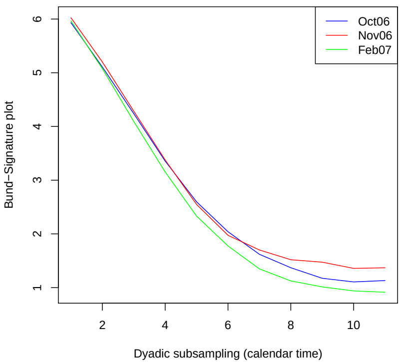

Figure : *Signature plot for Bund contract, one data every second, aggregation of all the trading days in each month.*

Modeling the signature plot

- Many models aim at reproducing this decreasing shape.
- Few agent based explanations for this phenomenon.
- Our approach enables us to provide a very simple one.

Explaining the signature plot

- Recall: ex post expected cost of a market order = $\alpha/2 - \eta\alpha$.
- $\Rightarrow$ for large tick assets with average spread close to one tick, the parameter η is systematically smaller than 1/2.
- Otherwise, the cost of market orders is negative and market makers lose money.
- To avoid that, market makers would naturally increase the spread, which they can always do.
- $\eta < 1/2 \Rightarrow$ Signature plot decreasing. $\Rightarrow$
- Agent based explanation of a phenomenon viewed mostly as a statistical stylized fact.

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Consequences of a change

α too small

- Exchange regulator: faces the question of choosing a tick value α
- α too small encourages free-riding:
    - Market participants jump marginally ahead of market makers.
    - Discourages market makers and tends to suppress liquidity.
    - Sparse order books and might trick people into making absurd judgments about prices.
    - Overloads the platform.
- One has certainly no rational basis for assessing the price of, say, Microsoft, down to the level of fractions of a penny.

Consequences of a change

α too large

- α too large: creates needless frictions or sloppiness in pricing. (strong mean reversions)
    - Favors speed (race to the top of book) $\Rightarrow$ High investments in infrastructure.
    - High entry costs $\Rightarrow$ Reduces competition (at least temporarily).
    - Drives away investors.

Consequences of a change

α too large

- One has a rational basis for pricing it in multiples of a dollar.
- BUT one might think twice if the perceived transaction costs are too high (crossing the spread for eg.)
- It is usually acknowledged that it is not possible to have an a priori idea of what is the "right" tick value. Thus, a market designer could only determine, after the fact, whether his chosen tick value has the desired effect, usually adjudged on the basis of price formation, spread, and liquidity.
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Consequences of a change

- What happens to η if one changes the tick value ?
- How to obtain the following **optimal** situation:
  - η close to 1/2
  - $S \sim 1$
  - Cost of market orders = cost of limit orders = 0.

Assumptions:

- σ, p_1, p_2 and the average traded volume are invariant after a change of the tick value, however, the number of trades M should not.

Changing α

- σ constant $\Rightarrow$

$$p_1\eta_0\alpha_0\sqrt{M_0}+p_2\alpha_0\sqrt{M_0}=p_1\eta\alpha\sqrt{M}+p_2\alpha\sqrt{M}$$

- Volume constant. Assumptions: Average volume per trade $\sim V_{ToB}$.
- Cumulative OB is linear or concave
  1. $V_{ToB}(\alpha)=b\frac{\alpha}{2}\Rightarrow M=M_0\frac{\alpha_0}{\alpha}$

$$\eta=\eta_0\sqrt{\frac{\alpha_0}{\alpha}}+\frac{p_2}{p_1}\sqrt{\frac{\alpha_0}{\alpha}}-\frac{p_2}{p_1}$$

  2. $V_{ToB}(\alpha)=b\sqrt{\frac{\alpha}{2}}\Rightarrow M=M_0\sqrt{\frac{\alpha_0}{\alpha}}$

$$\eta=\eta_0(\frac{\alpha_0}{\alpha})^{3/4}+\frac{p_2}{p_1}(\frac{\alpha_0}{\alpha})^{3/4}-\frac{p_2}{p_1}$$

Changing α

- σ constant $\Rightarrow$

$$p_1\eta_0\alpha_0\sqrt{M_0} + p_2\alpha_0\sqrt{M_0} = p_1\eta\alpha\sqrt{M} + p_2\alpha\sqrt{M}$$

- Volume constant. Assumptions: Average volume per trade $\sim V_{ToB}$.
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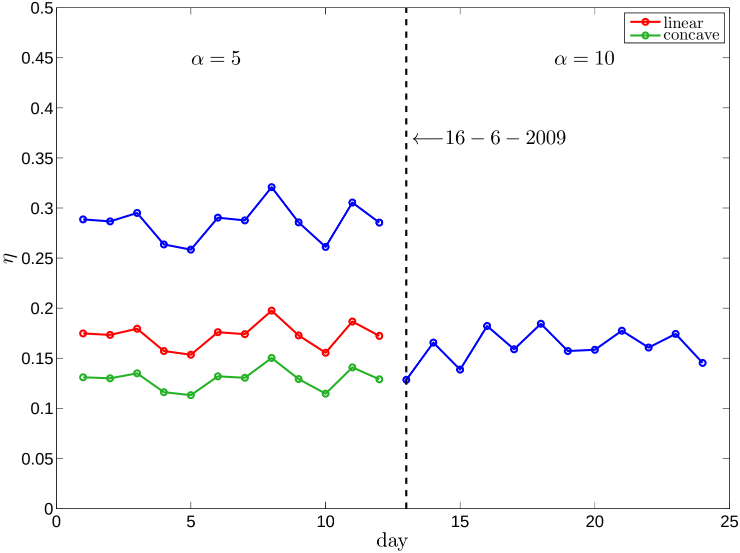
  2. $V_{ToB}(\alpha) = b\sqrt{\frac{\alpha}{2}} \Rightarrow M = M_0\sqrt{\frac{\alpha_0}{\alpha}}$

$$\eta = \eta_0(\frac{\alpha_0}{\alpha})^{3/4} + \frac{p_2}{p_1}(\frac{\alpha_0}{\alpha})^{3/4} - \frac{p_2}{p_1}$$

Verification:Bobl futures



Futures	Tick Value	Optimal tick value $\beta = 1$	Optimal tick value $\beta = 1/2$
BUS5	7.8125 \$	2.7 \$	3.8 \$
DJ	5.00 \$	1.6 \$	2.3 \$
EURO	12.50 \$	3.1 \$	5.0 \$
SP	12.50 \$	0.3 \$	0.9 \$
Bobl 1	5.00 €	1.8 €	2.6 €
Bobl 2	10.00 €	1.6 €	2.8 €
Bund	10.00 €	1.6 €	2.9 €
DAX	12.50 €	4.9 €	6.7 €
ESX	10.00 €	1.3 €	2.6 €
Schatz	5.00 €	0.8 €	1.5 €
CL	10.00 \$	3.1 \$	4.6 \$

Table 3: Optimal tick values for the considered assets, in the linear case and the square root concave case.

Conclusions

- To predict microstructure suffice to predict η after change in α.
- Easy and straightforward formula.
- Spectacular result on the Bobl.
- Inversely, can find α for any choice of η.
- Starting from a large tick asset we can reach the optimal situation: market makers will keep $S = 1$ as long as it is profitable.