Limited Willpower

Yusufcan Masatlioglu
University Michigan

Daisuke Nakajima
Otaru University of Commerce

Emre Ozdenoren
London Business School

Higher School of Economics, 2014

In the standard theory,

$$\textsc{Choices} \quad \Leftrightarrow \quad \textsc{Preferences}$$

Wedge

Choices $\overset{?}{\Leftrightarrow}$ Preferences

A wedge between choices and actual preferences

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A wedge between choices and actual preferences

Wedge

Such a wedge can be created by VISCERAL URGES

Visceral Urges

VISCEREAL URGES

- drives, e.g. hunger and sexual desire,
- emotions, e.g. anger and fear,

VISCERAL URGES

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- emotions, e.g. anger and fear,

Visceral Urges

From the evolutionary perspective, VISCERAL URGES are crucial for human beings:

- hunger protects us against malnutrition;
- anger protects us from exploitation by others;

Visceral Urges

Nowadays, VISCERAL URGES could harm us, e.g.,

- obesity,
- stage fright,

How to suppress and override our visceral urges?

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- not unlimited resource,
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Application

Recent applied papers on willpower depletion

- Ozdenoren, Salant, and Silverman (2011)
- Ali (2011)
- Fudenberg and Levine (2012)

Today's Talk

- Our goal is to provide a choice theoretic foundation for the willpower as a limited cognitive resource model.

  - Provide a simple and tractable model,
  - Temptation modelled as a constraint,
  - Our characterization uses only choices,
  - Identification of one's willpower and visceral urge intensity,

The Model

Three components:

$u(\cdot) \rightarrow$ utility

$v(\cdot) \rightarrow$ visceral urge intensity

$w \rightarrow$ willpower

Choosing an alternative from set A:

$$c(A) = \arg\max_{x \in A} \quad u(x)$$

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subject to

$$\underbrace{\max_{y \in A} v(y)}_{\text{most tempting alternative in } A}$$

The Model

Choosing an alternative from set A:

$$c(A) = \arg\max_{x \in A} \quad u(x)$$

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$$\underbrace{\max_{y \in A} v(y) - v(x)}_{\substack{\text{required amount of} \\ \text{willpower to be able} \\ \text{to choose } x \text{ from } A}}$$

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$$\underbrace{\max_{y \in A} v(y) - v(x)}_{\text{required amount of willpower to be able to choose } x \text{ from } A} \leq w$$

An illustration

$$c(A) = \arg\max_{x \in A} u(x) \text{ s.t. } \quad \max_{y \in A} v(y) - v(x) \leq w$$

Example: Assume willpower stock, $w = 3$,

	u	v
going to gym	10	1
reading book	5	3

An illustration

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⋆ Violation of IIA,

⋆ The middle option is chosen, "Compromise Effect"

Representation

$$c(A) = \underset{x \in A}{\operatorname{argmax}} \quad u(x) \quad \text{subject to} \quad \max_{y \in A} v(y) - v(x) \leq w$$

Two Extreme Cases

- $w = \infty$ (Standard) *NEVER* give in temptation
- $w = 0$ (Strotz) *ALWAYS* give in temptation

Setup

- X: a finite set of alternatives.
- Two pieces of information: $(\succsim, c)$
  - Preferences
  - Choices

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Question: What class of $(\succsim, c)$ can be explained by the Limited Willpower model?

Axioms

Axiom 1: $\succsim$ is complete and transitive.

Axiom 2: If $x \succ c(A \cup x)$ then $c(A) = c(A \cup x)$.

Axiom 3: $c(A) \succsim c(B) \Rightarrow c(A) \succsim c(A \cup B) \succsim c(B)$.

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A Result

Theorem 0

$(\succsim, c)$ satisfies Axioms 1-3 if and only if it admits the following representation:

$$c(A) = \arg\max_{x \in A} u(x) \text{ s.t. } \max_{y \in A} v(y) - v(x) \leq w(x)$$

An Additional Axiom

When $w(x) = w$?

An observation: $y \succ c(y, z) \Rightarrow z$ is more tempting than y ($y \succ z$)

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An Additional Axiom

Axiom 4 Suppose $y \succ c(y, z)$ and $c(t, z) = t$.
If $x \succ c(x, y)$ then $c(x, t) = t$.

- t is more tempting than y,
- x is not choosable over y,
- Then x is also not choosable t.

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Desired Result

Theorem 1

$(\succsim, c)$ satisfies Axioms 1-4 iff $(\succsim, c)$ admits a Limited Willpower representation.

$$c(A) = \arg\max_{x \in A} u(x) \text{ s.t. } \max_{y \in A} v(y) - v(x) \leq w$$

Non-Uniqueness

If preferences and choices coincide ($c(x, y) = x \succ y$), then

- No self-control problem
  - $0 < v(x) - v(y)$
- Self-control problem exists but enough willpower
  - $0 < v(y) - v(x) < w$

v is not even unique in ordinal sense !!!

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A richer structure is needed !!!

LOTTERIES

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LOTTERIES

Willpower with Lotteries

- X: the finite set of potentially available alternatives
- Δ: the set of all lotteries on X
- $\mathcal{X}$: the set of non-empty finite subsets of Δ
- $\succsim$: the preferences on X
- c: choices on $\mathcal{X}$

Linear Limited Willpower

$$c(A) = \underset{p \in A}{\operatorname{argmax}} \quad u(p)$$

subject to

$$\max_{q \in A} v(q) - v(p) \leq w$$

where

- u, v are linear functions
- w is a positive scalar.

New Axioms

Axiom A (Temptation Independence) Let $p \succ q$ and $\alpha \in [0, 1]$.
i) If $c(p, q) = p$, $c(p', q') = p'$ and $p' \succsim q'$, then $c(p\alpha p', q\alpha q') = p\alpha p'$
ii) If $c(p, q) = q$, $c(p', q') = q'$ and $p' \succ q'$ then $c(p\alpha p', q\alpha q') = q\alpha q'$

Axiom B (Invariance to Replacement) If $c(p\alpha r, q\alpha r) = p\alpha r$ then $c(p\alpha r', q\alpha r') = p\alpha r'$ for any r'.

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New Axioms

Axiom C: (Conflict) There exist p and q such that $p \succ c(p, q)$.

Axiom D: (Consonance) For all $p \succ q$, there exists $\alpha > 0$ such that $p\alpha q = c(p\alpha q, q)$.

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Characterization

Main Result

$(\succsim, c)$ satisfies Axiom 1-3 and "*some technical axioms*" iff $(\succsim, c)$ admits a linear Limited Willpower representation with $w > 0$.

Uniqueness: If (u, v, w) and (u', v', w') represent $(\succsim, c)$ then there exist scalars $\alpha > 0, \alpha' > 0, \beta, \beta'$ such that

$$u' = \alpha u + \beta, \quad v' = \alpha' v + \beta', \quad w' = \alpha' w$$

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Preferences from Choices

Can we reveal preferences from choices?

In the standard approach, preferences are revealed by choices.

$$x \succ y \quad \text{if} \quad x = c(x, y)$$

In the limited willpower, this is no longer true. It is possible that

$$x \succ y \quad \text{and} \quad y = c(x, y)$$

because of limited willpower ($v(y) - v(x) > w$)

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Preferences from Choices

Take two points x and y, and consider a mixture of them,

- If $u(x) > u(y)$ then $u(\alpha x + (1-\alpha)y) > u(y)$,
  - Order of utility does not change
- $v(y) - v(\alpha x + (1-\alpha)y) = \alpha (v(y) - v(x))$,
  - Self-control problem gets smaller

Preferences from Choices

Given c, we define revealed preference, $\succ^c$,

$x \succ^c y$ if one of the following is true

- $x = c(x, y)$ and no mixture can reverse the choice,
- $y = c(x, y)$ and some mixture can reverse the choice,

Preferences from Choices

Given c, define $\succ^c$

$x \succ^c y$ if one of the following is true

- $x = c(x, y)$ and $\nexists \alpha \in (0, 1)$ such that $y \in c(x\alpha y, y)$,
- $y = c(x, y)$ and $\exists \alpha \in (0, 1)$ such that $x\alpha y = c(x\alpha y, y)$.

Proposition

If $(\succsim, c)$ admits a linear willpower representation, then $\succsim = \succsim^c$.

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Modeling Options

Cold State	Hot State
Menu Preferences	Choices
Choose among menus	Choose from the menu
Existing Literature on Self-Control	Our Model

Preference for Commitment

Costly Self-control (GP)

- In Cold State, preference for commitment,

Limited Willpower (MNO)

- In Hot State, preference for commitment,

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Conclusion

- Provide a limited willpower model,
- Our characterization uses only choices,
- Temptation modeled as a constraint,
- Model is simple and tractable,
  - A monopolist facing a consumer with limited willpower
  - More complicated contracts
  - Qualitatively different results (Strotz or Costly Self-control)
  - "Compromise Effect" as a market outcome

THANK YOU