***SERIES:*** *ECONOMICS*

*Dmitry I. Malakhov[[1]](#footnote-1), Nikolay P. Pilnik [[2]](#footnote-2), Igor G. Pospelov [[3]](#footnote-3)*

**Money emissions, interbank transactions, and structure of banking industry[[4]](#footnote-4)**

Banking sphere is rather different from real sector industries. We show, that exogenous demand for specific financial products and necessity of developing peculiar economic mechanisms determine structure of banking industry.

In the first part of our paper we propose new model of banking system of an open economy. This model shows how money is distributed across banks. We get that distribution of shares of assets of individual banks is stable within certain time intervals, characterized by a small change in the number of banks.

In the second part we test this result using data from Russian banking system. We show that, using generalized versions of well-known distribution functions, distributions of shares of assets, deposits and credits can be approximated with high accuracy and, moreover, distributions of shares of these key aggregate variables of Russian banks are stable over time.

Such mechanism decreases structural risks of banking system, improving adaptability of the industry.

Key words: bank, industry structure, market design, economic mechanism, money emission, interbank transaction.

JEL classification: D47, L11, G21

**Introduction**

This work is devoted to the issue of industry structure and the evolution of the banking system. Today there are many works (Lotti, Santarelli, Vivarelli (2003), Sutton (1991)), which discussed evolution of real sector firms, but principles and mechanisms of evolution of banking industry are far from real sector ones. For example, evolution of real sector firms are highly connected with innovations and technology development, but growth of banking sphere is connected in fact mainly with growth of real sector firms. Moreover, bank money multiplier models, such as Johannes, Rasche (1979), Bernanke, Blinder (1988), Carpenter, Demiralp (2012) did not pay much attention to interbank transactions and industry structure.

The last financial crisis of 2007-2009 years revealed, that architecture of the financial system is crucial for stability. So attentions from famous 4 «L», leverage, liquidity, losses, and linkages, now shifted to last «L», because risk measures for the first 3 «L» are rather investigated, but not for the last one. Modern studies, which analyzed structure of financial industry (Acemoglu, Ozdaglar, Tahbaz-Salehi (2015), Billio, Getmansky, Pelizzon (2012), Iori, De Masi, Precup, Gabbi, Caldarelli (2008)) show, that number of linkages and their characteristics are very important for good risk resistance, but these models in fact don’t pay a lot of attention to specific products or mechanisms of banking industry.

In modern economic system loans, interbank credits, money transactions (such as money transfer between two individuals or companies, etc.) greatly affect principals of functioning of banks. In the following chapter we develop dynamic stochastic approach to money creation and transmission mechanisms in large group of interacted banks. We show how mechanisms of money distribution affect structure of banking industry.

In our opinion the dynamics of the banking system in terms of the summary indicators in the modern economy is clearly insufficient. The existing crisis doesn’t influence only on the values of summary indicators, but also radically changes the distribution of shares in these rates between different banks, which is directly in the summary measures are not visible. For example, the fall of the banking system assets by 10% can mean a decrease of assets of each bank on 10%, and the withdrawal from the market of banks whose assets account for 10% of the assets of the entire banking system. Obviously, these cases are fundamentally different in terms of the threats to the banking system, and that it is desirable to predict at the level of individual banks and at the level of the monetary authorities. In this sense, this article is the beginning of the research tool to describe and predict the crisis on a new level.

На наш взгляд, описание динамики банковской системы в терминах суммарных показателей в современной экономике является явно недостаточным. Имеющие место кризисные явления затрагивают не только значения суммарных показателей, но и радикально меняют распределение долей в этих показателях между разными банками, что напрямую в суммарных показателях не видно. Так, например, падение активов банковской системы на 10% может означать как снижение активов каждого банка на 10%, так и уход с рынка банков, чьи активы составляют 10% от активов всей банковской системы. Естественно, что эти случаи принципиально отличаются с точки зрения угроз банковской системы, которые желательно прогнозировать и на уровне отдельных банков, и на уровне монетарных властей. В этом смысле данная статья является началом исследования инструмента, позволяющего описывать и прогнозировать кризисные явления на новом уровне.

Plan of this work is following. In the first part we discuss theoretical model of banking system. In the second part we will provide empirical testing of our model using data from Russian banks. And finally we will make conclusions.

**1.** **Interbank money transactions**

**1.1. General description of the model**

We analyze the distribution of money among banks and dynamics of value of assets of banking system. In this section large scale closed economy with great amount of perfectly competitive banks will be discussed.

**1.1.1. Money emissions**

New money appearing in the economy due to two reasons:

1. Loans from outside of banking system. Residents and nonresidents can put their money into national banks. National banks can give credits to residents and nonresidents or can use other financial instruments, such as debt emissions.
2. Credit emission. Bank can give credit to client with corresponding creation/changing of his/her current account. We do not consider in this paper the impact of the monetary policy pursued by the Central Bank. In this sense, we can assume that the banking system operates in a constant monetary policy.

In our model these two ways of money accumulation are not differenced. Moreover, we could consider interbank credit as a special case of transactions listed above. Value of accumulated money, , depends only on number of clients and is independent to conjuncture.

We propose, that all clients are identical to each other (if number of clients is great (which is true for developed banking system), than this assumption is realistic or we can divide transaction of one individual into several ones).

**1.1.2. Withdrawals and repayments**

Withdrawals from bank occur only when bank repays for its’ debts or client’s debt relief. We assume that emission generates interest income (which can be potentially negative). Also we propose that all losses are covered and all profits are derived.

**1.1.3. Value of emissions**

Bank can potentially transfer some amount of liabilities to other banks. Bank, which initiates the transaction, transfers money from client’s account to correspondent account of receiving bank. So only the structure of liabilities of banks will change (value will be the same). Also transaction between clients does not affect value of assets of banking system.

**1.1.4. Duration**

Assume that all credits are repaid with frequency, which is proportional to duration of credits,. All assets are identical to each other in the sense of duration (only moments of creation of bank’s assets are changing). Also we consider, that all emissions are equal in size and value of assets of banks depend only on the number of bank’s clients (we propose, that one client during one moment of time can induce only one single emission). Potentially, we can divide all transactions into tranches to hold this assumption.

**1.1.5. Validity of assumptions**

If we analyze developed banking system with great amount of highly competitive banks and identical clients during rather long period of time, then assumptions about durations and deposits sizes are relevant, because we can average all transactions. So if we discuss real banking sectors of Russian Federation these assumptions will be reliable.

**1.2. Induced emission**

Propose, that there are set of banks ****. Share of individual bank’s assets in the overall amount is , . We assume that set **** is stable over time.

Bank  induced initial emission , transaction is needed with probability. We can assume, that each emission induces chain of emissions (like in banking multiplier models), if amount of banks is great and  is small, than corresponding series converge. Moreover, this assumption does not decrease an explanatory power of model. So let’s assume, that with probability  transaction  is needed. Average assets change after transaction will be

.  

**1.3. Stochastic process of assets change**

Let’s assume, that at moment  bank  has assets 

,    
 During period  one client of randomly chosen bank , independently on the others initiates the emission with probability  with size , which is much smaller, than . Note, that  is “real” demand and  is proxy for inflation and society welfare.

We propose that :

1. Does not depend on bank’s index
2. Depends only on share of individual bank’s assets
3. Homogeneous of zero degree
4. Does not change if assets of some banks are merged

So for simplicity we will further consider, that  is constant.

With probability  induced emissions are independently covered. Furthermore, we propose, that  is big enough, so all loans are short-term (long-term loans can be divided into parts and analyzed as series of short-term loans) and we can assume, that  can change during repayment time.

**1.4. Dynamic of generalized moments**

Now discuss averaged value of some function  over realization of stochastic process :

,   
where – mathematical expectation of  over . Calculate ,  using chain rule for mathematical expectation:

.   
 During the period :

1. with probability  assets are not changed
2. with probability  assets of bank  decrease by 
3. with probability  initial emission  occurs at bank  and with probability bank  continues this transaction. Here we assume, that probability of continuation of emission is proportional to amount of assets.

Now we derive conditional mathematical expectation using the probabilities, which were mentioned above:







.

Than we use (4):





. (5)

**1.5. Diffusion approximation**

Diffusion approximation helps to find the solution to equation, equal to asymptotic one, when we are far away from borders of researched area. We assume with probability equal to 1, that , but  is finite. Then:

, (6)  
 , (7)  
 , (8)  
Substitute the expressions (6), (7), (8) into (5) and using (2):

   
 . (9)

**1.6. Kinetic equation**

**1.6.1. Dirac δ-function**

For each smooth function :

,

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**1.6.2. Theorem of averaging of δ-function**

If , where  - Dirac function,  - constant parameters, and distribution of random variable  near  has smooth density , then:

 - density of joint distribution.

Let’s return to our model.  is parameter, so:

, (10)  
****,

Using expression given above:



.

Simplifying expression above, we get:





  . (11)  
 Substitute the expressions (10) and (11) into (9):



. (12)  
 Simplify the derivatives in (12) and divide both sides of equation by :







Than



**,



. (13)  
 If we integrate second and third summands over the whole space and in each element integrate over , then this integral will be equal to 0, because  turns into 0 at the edges.

Because number of banks , so we can write down:



 . (14)

**1.7. Stationary solution**

We can make in (14) following substitution:

. (15)  
 Then we substitute (15) into (14):



.

We make the following change of variables:

 . (16)  
 After simplification we get:

. (17)  
 Functions, which depend on  are homogenous of zero degree, so:

.

Notice, that this equation is independent on time, so we can find the stationary solution:

,

.

**1.8. Partial separation of variables**

We can write the preceding relation in the following form:

. (18)  
 General solution of homogeneous equation:

. (19)  
 Free term of (18) can be expressed as series:

, . (20)  
 We can find particular solution as series:

,

,

.

The series, which were mentioned above, can be solution if:

.

Homogeneous equation  ⇔  has solution . So we can use variation of parameters:

,

 ⇒ .

We need only particular solution, so assume, that , then  and so partial solution will be:

. (21)  
 Then general solution of equation is following:

.

Now return to original variables :

.

We notice again, that  does not depend on time, so, substituting 

we get:

,

and

. (22)

Notice, that density function  can be represented as a quotient of two typically different factors. Numerator  depends only on share of assets, but does not depend on time or absolute value of assets. Denominator depends on absolute values of assets of individual banks are only in the expression . If this expression has constant value over time, then it makes sense only to pay attention to aggregate value of assets in the denominator.

Since the function  is a function of density, of course, that the integration of all the values of  at any time  gives 1. After the change of variables in the integral, we can go to the  indexes of shares and the total amount of assets, as is easily seen, the possibility of integrating separate the numerator and denominator. Therefore, the function  can be regarded as up to a constant a function of density, depending upon only from a fraction of banks' assets. Moreover, since it is not directly dependent on time, this function within our assumptions must be constant. This function depends only on the shares, we will explore further the real data. Moreover, when it will continue to go on the description of the general population, we will have in mind precisely this function.

**2. Empirical testing**

**2.1. Model validation**

To validate the model we provide empirical tests. We decide to use financial statement of banks as source of information, because financial statement is really informative for bank analysis[[5]](#footnote-5). Moreover, we decide to use information from Russian banks, because Russian banking system is rather developed and competitive, especially for the last 10 years.

**2.1.1. Data**

We use information from 101 turnover balance sheet of individual credit organizations. 101 turnover balance sheet is trial balance with debit and credit subtotals per account, we can get information about assets, deposits, credits and other financial indexes from this report. We collect information only from credit organizations, both bank and nonbank organizations, which can provide banking services and are registered in Russia and report balance sheets publicly. Share of nonbank credit organizations is very small if we consider both number of firms or volume of assets. For simplicity we will name all credit organizations as banks.

Information about 101 turnover balance sheet is collected from the official website of the Central Bank of Russian Federation <http://www.cbr.ru/>. In our sample in average for period 2009-2015 we have about 99% of overall number of banks and 99% of overall banking system assets for all time periods. So our sample is approximately equal to the amount of Russian banks (for details see Figure 3).

Subaccounts are rather minor, so they are noisy and are not very representative indicators of financial health of individual banks. We use aggregate variables, because they are very informative, are not so noisy and number of these variables is not very high, so they are informative, reliable and useful subjects of analysis. All values are gave in thousands of rubles.

We decided to use following variables:

1. Total amount of assets.
2. Fixed date deposits of banks and other credit organizations, including overdraft (further we will use abbreviations for financial variables, so this variable is Db)
3. Fixed date deposits of non-residents (Df),
4. Fixed date deposits of natural persons – residents (Dh),
5. Fixed date deposits of nonfinancial organizations (Da),
6. Fixed date credits to commercial nonbank organizations-residents, including overdraft (La),
7. Fixed date credits to natural persons – residents (Lh),
8. Fixed date credits to foreign organizations (Lf).

All our variables are calculated by summing corresponding subaccounts of 101 turnover balance sheet. We choose these variables, because they are significant shares of total amount of assets (liabilities). Final data are tables, where columns indicate time period and rows indicate bank id. We have separate tables for each financial variable. Period of observation begins at January 2004 and ends by Febrary 2015 (monthly data). We have actual data for each time period.

We will use in our analysis share of individual banks in total amount of particular variable. So, for example, share of assets of Bank A is total amount of assets of Banks A at the end of month i, divided by total amount of assets of all banks in the sample at the end of month i. We use shares of assets instead of assets because distribution of shares is investigated in the first part of our work. Moreover, we need not to deflate them and they give relevant picture of banking system structure.

Number of banks in Russia changed through time, also portion of banks which gave information to the Central Bank changed too, so we have different number of observations each month. Generally, number of banks didn’t vary greatly. Number of banks with nonzero values at the beginning of time period is approximately 700 and 1100 by the end. It is important to mention, that although we work only with banks, which provide information to the Central Bank.

We don’t drop any banks from our sample, so we estimate distributions including very big banks, such as Sberbank and VTB. Moreover, we don’t ignore very small banks, which form left tail of distribution.

**2.1.1. The possibility of aggregation of the banking system**

Let's go back to the equation (22). We plot values of  for each month (vertical axes is value of corresponding parameter, horizontal axe is time (January 2004-Febrary 2015)).

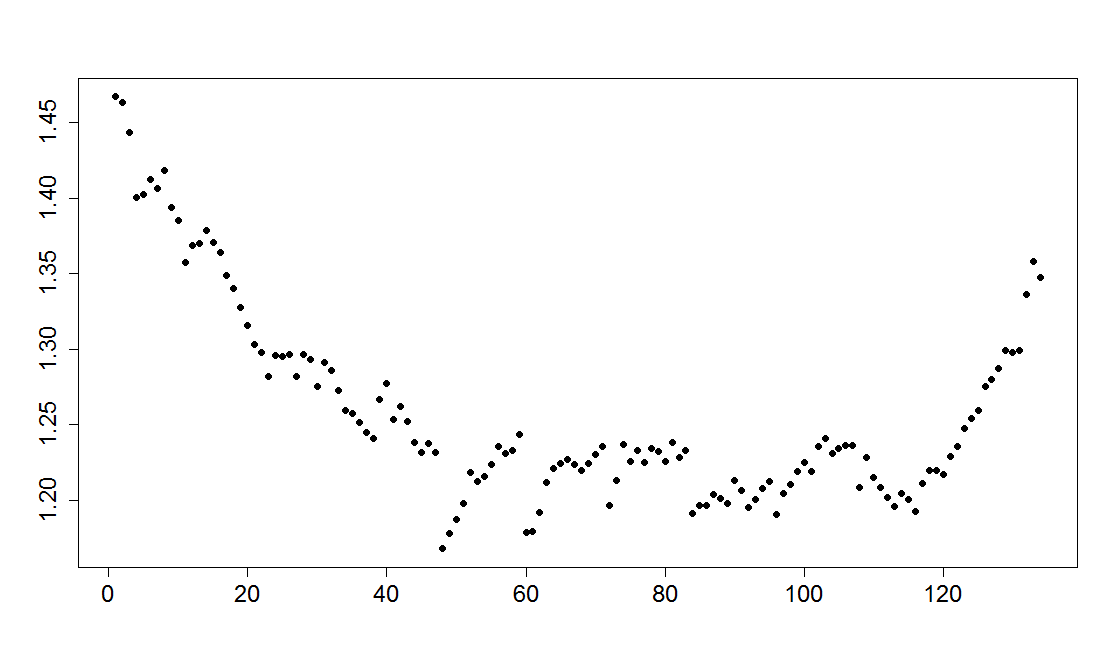


Figure 1. Values of the main factor of (22).

As we can see, for a fairly long period of time from 2006 to 2013 value of  changes significantly less than in other time periods. Consequently, during this period, it can be expected that the distribution of assets banks do not change as much as in the remaining periods. Therefore, we can continue to talk about the distribution of shares of the assets of individual banks introduced above terms. In addition, special mention is the period from 2004 to 2005 and 2014-2015. As we shall see, these time periods were characterized by the displacement distribution of the shares of assets of banks.

Also we can investigate dynamic of . The chart (similar axes) clearly shows two periods: before and after the crisis of 2008, the ratio considered really grows linearly, although with a different pace. Note that in Figure 1 no fracture in 2008-2009 are observed. In this regard, we can assume that the same crisis of 2008-2009 affected the banks of varying sizes. This fact is one more argument in favor of the above model.

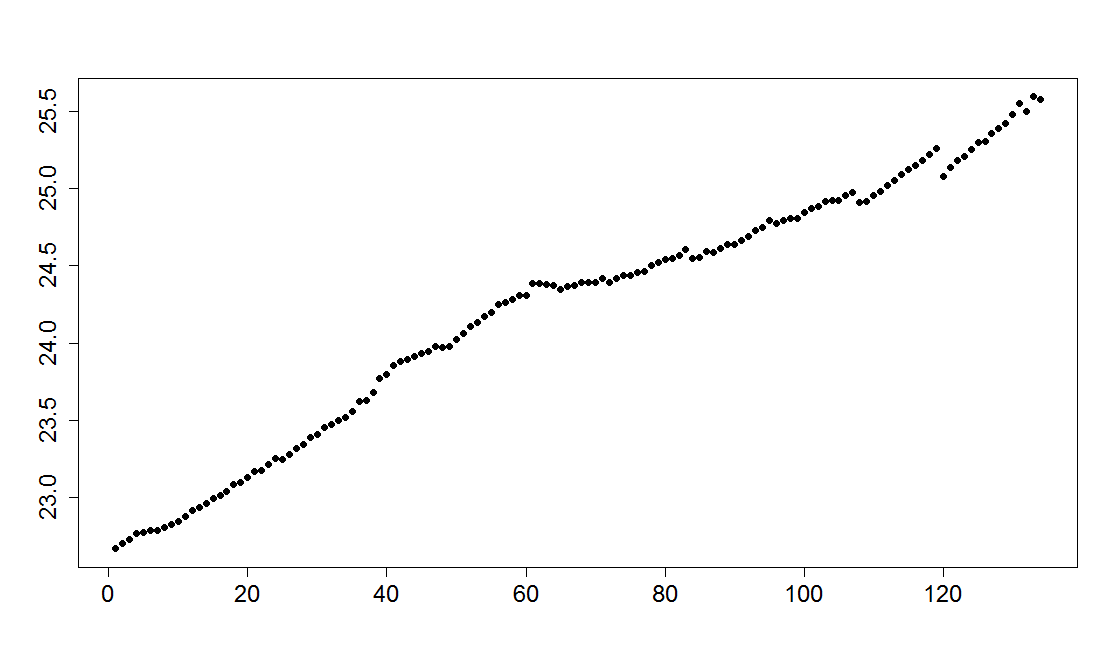


Figure 2. Logarithm of the total assets of the Russian banking system.

**2.2. Approaches to modeling distributions of firms’ sizes**

Theoretical model showed, that distribution of shares of assets of banks is stable over time, so these results is highly connected with industry evolution and development. Today there are many works connected with evolution of particular industries. Classical work is Gibrat (1931), in which the following hypothesis is formulated: firm size and its growth rate are independed. If this hypothesis is true, than firm growth rate is independent of its size, so big and small firms have approximately equal growth rates. Researchers tested this hypothesis for many different industries. Generally, it is difficult to say if this this Law correct for real economy or not: firms in some industries grow independently of their size, but firms in other industries don’t (Javonovich (1982)).

Lotti, Santarelli, Vivarelli (2003) provide empirical test of Gibrat Law for young firms. Authors postulate that there are three approaches to Gibrat Law testing. First approach is the most general: Gibrat Law is correct for all firms, independently of their bankruptcy fact during period of observation. Second approach: Gibrat Law is correct only for firms, which are functioning during period of observation. Third approach: Gibrat Law is correct only for firms which size is bigger than minimum efficient scale. Authors provide great literature survey (actual for 2003 year), they show that for some industries these Law is correct, for some - not.

If Gibrat Law is correct, than firms sizes can be approximated by lognormal distribution (for example, Gibrat(1931), Axtell (2001)). But fat tails of distribution of firm sizes may occur, because positive feedback can exist. So Pareto distribution may be also helpful.

Prescott, Janicki (2006) investigate data for American banks over 1960-2005 time period. Authors postulate that lognormal and Pareto distributions are good approximations for banks’ sizes, but right tail of empirical distribution is much fatter, than lognormal one, so they use lognormal distribution as main distribution for central part and left tail of data, but right tail is approximated by Pareto distribution. Also Prescott, Janicki (2006) show, that Gibrat Law is correct for American banks.

In paper Cont, Moussa (2010) a quantitative methodology for analyzing the potential for contagion and systemic risk in a network of interlinked financial institutions is presented. This methodology is applied to a data set of mutual exposures and capital levels of financial institutions in Brazil in 2007 and 2008, and the role of balance sheet size and network structure in each institution's contribution to systemic risk is analyzed. Results emphasize the contribution of heterogeneity in network structure and concentration of counterparty exposures to a given institution in explaining its systemic importance. In this paper, which analyzes interbank sector, it is shown, that right tail of distribution can be approximated by Pareto distribution. For testing this hypothesis Cont, Moussa (2010) shows, that linear regression models are good approximations of the logarithmic data.

In Andreev, Pilnik, Pospelov (2009) authors analyze rang distribution of Russian banks and come to a conclusion, that it can be approximated by Pareto distribution with high quality. Moreover, this distribution is stable over time.

**2.3. Preliminary analysis of data**

In this part we will focus only on shares of assets[[6]](#footnote-6). We calculate descriptive statistics for all time periods, but for better visualizing we print only time means:

Table 1. Descriptive statistics.

|  |  |
| --- | --- |
|  | Value |
| Stand. deviation | 0.01345341 |
| Min | 6.414726e-08 |
| Max | 0.3835583 |

So we can see, that difference between the mean smallest bank and the mean biggest bank[[7]](#footnote-7) is rather significant. So we have some clusters of big banks, such as Sberbank, VTB, and cluster of very small banks. It is important to mention, that mean of shares is , n – number of banks. Moreover, we plot dynamic of number banks on our sample, standard error for shares of assets, skewness for shares of assets and kurtosis for shares of assets (vertical axes are values of corresponding parameters, horizontal axes are time periods (January 2004 - Febrary 2015)).

|  |  |
| --- | --- |
|  |  |
| Fig. 3. Number of banks in our sample (black points) and the amount of Russian banks (grey points). | Fig. 4. Dynamic of standard error of shares of assets. |
|  |  |
| Fig.5. Dynamic of skewness of shares of assets. | Fig.6. Dynamic of kurtosis of shares of assets. |

It can be easily seen, that since October 2009 (70th point) our sample is approximately equal to the amount of Russian banks. Inequality (measured as standard error) has nontrivial dynamic, but it has rising dynamics since 70th point. Skewness and kurtosis are from normal ones, so we expect, that distribution functions will be nontrivial.

**2.4. Used distribution**

Random variable x has power distribution, if probability density function is, where  - parameter. Often Pareto distribution is used as the basic power distribution:



Much attention is paid to parameter value . If , than big values have significant effect on the average value. If , than observation with small values have significant effect on the average. Pareto distribution is rather popular in modeling distribution of firm sizes (Axtell (2001), Crosato, Ganugi (2007)).

If Gibrat Law is correct for the particular sample of firms, than we can use lognormal distribution for firms’ sizes, as it is shown in the original paper of Gibrat. So we pay a lot of attention to lognormal and Pareto distributions and their quality of approximation.

Today there are a lot of generalizations of these distributions. Generalized distributions are very important tools, when data set is rather heterogeneous and it is difficult to use basic distributions, because of their small numbers of parameters. Larger number of parameters and much more flexible functional form help to get precise results. But when we estimate parameters of these distributions we face a lot of problems. Numerical procedures very often can’t converge and estimation results are not very precise and robust. In our particular case we successfully avoid many estimation problems, because data set is homogenous and rather big. We use maximum likelihood and method of L-moments estimation procedures as the most reliable methods with good properties of estimators.

Further we will discuss Pareto and lognormal distribution and their generalizations. It is important to notice, that many generalizations are connected with these basic distributions, but sometimes this connection is rather nontrivial and not very strong.

### 2.4.1 Family of Pareto Distributions

**Generalized Pareto distribution (Gen. Pareto)** has three parameters: location, scale and shape. Including of shape parameter allows generalizing standard Pareto distribution:

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where ,- location parameter, –scale parameter,  - shape parameter.

**Wakeby distribution** is generalization of generalized Pareto distribution. This distribution is equivalent Pareto, exponential and uniform distributions as special cases. Cumulative distribution function is very complicated and it is easier to use quantile function:

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- uniform random variable with support [0,1], -location parameter, -scale parameters,-shape parameters.

So, Wakeby is a very general distribution with 5 parameters, it is widely used in financial application (for example, Negrea (2014)).

Another way of generalization of Pareto distribution is **Pareto IV type**. Pareto IV is a generalization of many different Pareto type distributions and has following cumulative distribution function:

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where  location parameter, scale parameter,  shape (inequality) parameter, tail parameter.

Pareto IV type is one of the most popular generalized Pareto distributions. Location parameter affects mathematical expectation and tail fatness, scale parameter stretches cumulative distribution and probability density functions along the OX axes. Gini coefficient is greatly affected by shape parameter. And tail parameter has significant impact on tail fatness. Due to its flexibility Pareto IV type is very often used in firm size modeling Crosato, Ganugi (2007).

**Generalized Beta of the second kind (Beta prime distribution)** is generalization of many different distributions, including Pareto IV type distribution. Probability density function for Generalized Beta of the second kind has following functional form:

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where - shape parameters and -scale parameter.

Beta prime distribution is very often used in income distribution analyses and financial modelling, but there are many problems with parameters’ estimation of this distribution, because its probability density function is rather complicated.

### 2.4.2 Family of Normal-related distributions

It is important to mention, that some of these distributions are not strictly connected with normal distribution, but for simplicity (and only for it) we form family of normal-related distribution.

**Generalized normal distribution (Gen. normal distribution)** has probability density function: ,where



where  - parameters of location, scale and shape, respectively. According to three parameters, this distribution is much more flexible than typical normal distribution. Moreover, this distribution is skewed and it can be very important in modelling of income or firm sizes distribution, because this data is typically asymmetrically distributed.

**Skew normal distribution** has following probability density function:

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where  - parameters of location, scale and shape, respectively. This distribution has very important feature – its distribution function is asymmetric, which is very relevant for our dataset. During parameter estimation problems may occur, for example, probability density functions are calculated using simulation methods, so accuracy and robustness of parameters’ estimates are not very high.

We also used **asymmetric exponential power distribution (asymmetric generalized error distribution or simply AEP)** with cumulative probability function:

 ,

where  incomplete gamma function, -location parameter, - scale parameter, ,- shape parameters.

Asymmetric exponential power distribution was developed as an asymmetric generalization of exponential power distribution (also known as generalized error distribution), which in turn is a generalization of normal distribution with kurtosis parameter. It is important to notice that asymmetric exponential power distribution has maximum entropy in the very wide class of distributions with support  (Zhu, Zinde-Walsh (2009)). Moreover, tails of this distribution can potentially have different fatness and they are much fatter, than normal ones. In paper Buldyrev, Growiec, Pammolli, Riccaboni, Stanley (2007) asymmetric exponential power distribution is used for modelling firms sizes.

**Generalized lambda distribution (Gen. lambda distribution)** is also used for modelling data. Quantile function is following:

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- uniform distribution with support [0,1], -location parameter, -scale parameter,-shape parameters.

Generalized lambda distribution is asymmetric distribution with power law tails, so it is often used for financial modelling. Generalized lambda distribution was developed as an approximation of many standard distributions, because it was originally used in Monte Carlo modelling, so flexibility of this distribution should be very high. In Beena, Kumara (2010) generalized lambda distribution is used for modelling inequality of income distribution.

**2.5. Modelling of bank distribution**

Our data set is homogenous and big, so we will use empirical distribution function as a benchmark for our analysis. Empirical function is consistent estimate of true cumulative distribution function. We use R software for our analysis.

**2.5.1 Lognormal and Pareto distribution**

We use maximum likelihood and L-moments approach, because these methods give estimators with “good” properties and they don’t depend on chosen metrics (Asquith (2015), Hosking (2015), Yee, Wild (1996)).

Quality of approximation is rather stable through time and financial variables. So we will show graph of cumulative distribution function as an example (only for one month (May 2012) and for shares of assets). We use shares of assets, as we mentioned above. For better visualization top 3-4 banks with very big assets for family of Pareto distributions were not shown (this will not affect analysis results, because pattern will be the same).

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| Figure 7. Assets[[8]](#footnote-8), Pareto distribution. | Figure 8. Rang distribution of assets in both logarithmic axes. |

Pareto approximation is not very good, it can be easily explained by low number of parameters and inflexible functional form. Moreover, rang distribution graph shows us, that this data can’t be correctly approximated by Pareto distribution, because it is not a strict line in log axes: we can see the curve in the right side of graph (cluster of small banks). Similar situation will be for other time periods and financial variables. Typically researchers ignore the right part of the graph and use Pareto distribution only for modelling of the middle part of the graph. But we try to model overall set of banks by entire distribution.

Now let’s estimate lognormal distribution for the same data. We use log of shares of individual banks as quantiles. So in fact we estimate normal distribution for the log data. As we can see quality of approximation is medium, because the left tail is approximated (quantiles less than -12) not very good and there is noticeable bias of the middle part (quantiles is around -8), which is connected with calibration of the right tail. So we move to analysis of the rest distributions.

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| Figure 9. Logarithm of assets, normal distribution. |  |

**2.5.2 Generalized distributions**

We will discuss graphs for 4 variables: Assets, La, Dh, Db. We show graphs only for May 2012, because structure of industry did not change significantly during time. We decided not to show results for generalized beta of the second kind, because its results is equal to Pareto IV ones and graphs become more comlicated. Wakeby distribution is turned into Pareto IV for our data, so we exlude this distribution too. Also we assume, that location parameter for Pareto IV type is 0. This assumption is recociled with data and does not lead to any limitations (Brazauskas (2003)).

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| Figure 10. Assets, family of Pareto distribution (Pareto, Gen. Pareto) | Figure 11. Assets, family of Pareto distribution (Pareto IV) |
|  |  |
| Figure 12. Logarithm of assets, family of normal-related distributions (Normal, Gen. Normal) | Figure 13. Logarithm of assets, family of normal distributions (Skew Normal, Gen. Lambda) |
|  |  |
| Figure 14. Logarithm of assets, family of normal-related distributions (AEP) | Figure 15. Credits to firms (La), family of Pareto distributions (Pareto, Gen. Pareto) |
|  |  |
| Figure 16. Credits to firms (La), family of Pareto distributions (Pareto IV) | Figure 17. Logarithm of credits to firms (La), family of normal-related distributions (Normal, Gen. Normal) |
|  |  |
| Figure 18. Logarithm of credits to firms (La), family of normal distributions (Skew Normal, Gen. Lamda) | Figure 19. Logarithm of credits to firms (La), family of normal distributions (AEP) |
|  |  |
| Figure 20. Interbank deposits (Db), family of Pareto distributions (Pareto, Gen. Pareto) | Figure 21. Interbank deposits (Db), family of Pareto distributions (Pareto IV) |
|  |  |
| Figure 22. Logarithm of interbank deposits (Db), family of normal-related distributions (Normal, Gen. Normal) | Figure 23. Logarithm of interbank deposits (Db), family of normal-related distributions (Skew Normal, Gen. Lambda) |
|  |  |
| Figure 24. Logarithm of interbank deposits (Db), family of normal-related distributions (AEP) |  |

Quality of approximation is rather high for all financial variables. We eliminate skew normal distribution from our futher analysis, because estimation results are unstable and their quality is rather low. Pareto and Generalized Pareto distribution can not approximate left tail of empirical distribution due to specific values of parameters. Typicall normal distribution is not good approximation, because empirical distribution in fact rather asymmetric and fat tailed. But we would use Pareto distribution and lognormal distribution as benchmarks for further analysis.

According to graphical analysis results it is difficult to conclude which distribution is better, so we calculte formal distancies between empirical and theorethical distributions for each month:

1. 
2. , where n – number of observations in k-th month.

We show maximum, minimum, average and standard error of distances. Following two tables are calculated for shares of assets.

Table 2. Extreme metric for assets distribution

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Pareto | Gen. Pareto | | Pareto IV |
| Maximum | 0.48313 | 0.18795 | | 0.02455 |
| Minimum | 0.34992 | 0.10712 | | 0.01191 |
| Average | 0.41238 | 0.14349 | | 0.01709 |
| Stand. err | 0.03508 | 0.02008 | | 0.00254 |
|  | | | | | | |
|  | Normal | Gen. Normal | AEP | | | Gen. Lambda |
| Maximum | 0.06809 | 0.04451 | 0.02064 | | | 0.04773 |
| Minimum | 0.03485 | 0.02327 | 0.00972 | | | 0.01185 |
| Average | 0.05275 | 0.03178 | 0.01805 | | | 0.01924 |
| Stand. err | 0.00803 | 0.00470 | 0.00314 | | | 0.00468 |

Table 3. Average metric for assets distribution

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Pareto | | Gen. Pareto | | Pareto IV |
| Maximum | 0.21907 | | 0.07107 | | 0.00720 |
| Minimum | 0.17427 | | 0.03248 | | 0.00323 |
| Average | 0.19391 | | 0.05107 | | 0.00506 |
| Stand. err | 0.01084 | | 0.01077 | | 0.00093 |
|  | | | | | | | |
|  | Normal | Gen. Normal | | AEP | | | Gen. Lambda |
| Maximum | 0.03006 | 0.01635 | | 0.00734 | | | 0.02029 |
| Minimum | 0.01363 | 0.00929 | | 0.00268 | | | 0.00334 |
| Average | 0.02378 | 0.01267 | | 0.00498 | | | 0.00575 |
| Stand. err | 0.00523 | 0.00149 | | 0.0008 | | | 0.00198 |

Pareto IV is the best distribution among Pareto family of distributions and asymmetric exponential power distribution is the best among normal-related distributions. We can easily notice, that difference of approximation between asymmetric exponential power and Pareto IV distributions is rather insignificant (Pareto IV has bigger distances in maximum metrics). For better visualization, we show graphs for these distributions for November 2004 and November 2014 (assets). November is one of months with small seasonality effect. Also it is interesting to investigate differences in distributions which occurred during last 10 years.

|  |  |
| --- | --- |
|  |  |
| Figure 25. Assets, Pareto IV distribution (November 2004) | Figure 26. Logarithm of assets, asymmetric exponential power distribution (November 2004) |
|  |  |
| Figure 27. Assets, Pareto IV distribution (November 2014) | Figure 28. Logarithm of assets, asymmetric exponential power distribution (November 2014) |

We can see, that distribution of banks has not changed significantly during last 10 years. Both distributions become more sloping, so in general banking system become more homogenous. Tail formed by little banks is appeared. Recent crisis did not affect banking system greatly, because there are no significant changes in the form of distribution. Quality of approximation is very high. It is important to mention, that results will not change, if we eliminate random part of our sample.

Analysis of quality of approximation of other financial variables shows that AEP and Pareto IV distributions are the best choices for modeling. In table 3 maximum, minimum, average and standard error of extreme (max) and average (mean) metric is given.

Table 4. Maximum and mean metrics for other financial variables

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | | Maximum | Minimum | Average | Stand.Err. |
| Db | Max, Pareto IV | 0.0561 | 0.0217 | 0.0381 | 0.0068 |
| Max, AEP | 0.0478 | 0.0163 | 0.0293 | 0.0068 |
| Mean, Pareto IV | 0.0211 | 0.0073 | 0.0135 | 0.0029 |
| Mean, AEP | 0.0182 | 0.0043 | 0.0090 | 0.0028 |
|  |  |  |  |  |  |
| Df | Max, Pareto IV | 0.0942 | 0.0318 | 0.0546 | 0.0165 |
| Max, AEP | 0.0574 | 0.0173 | 0.0296 | 0.0078 |
| Mean, Pareto IV | 0.0377 | 0.0093 | 0.0196 | 0.0078 |
| Mean, AEP | 0.0213 | 0.0048 | 0.0099 | 0.0039 |
|  |  |  |  |  |  |
| Dh | Max, Pareto IV | 0.0601 | 0.0171 | 0.0325 | 0.0100 |
| Max, AEP | 0.0472 | 0.0168 | 0.0312 | 0.0081 |
| Mean, Pareto IV | 0.0200 | 0.0065 | 0.0121 | 0.0038 |
| Mean, AEP | 0.0209 | 0.0056 | 0.0117 | 0.0041 |
|  |  |  |  |  |  |
| Da | Max, Pareto IV | 0.0401 | 0.0131 | 0.0260 | 0.0059 |
| Max, AEP | 0.0429 | 0.0145 | 0.0269 | 0.0062 |
| Mean, Pareto IV | 0.0149 | 0.0040 | 0.0087 | 0.0025 |
| Mean, AEP | 0.0162 | 0.0044 | 0.0089 | 0.0026 |
|  |  |  |  |  |  |
| Lf | Max, Pareto IV | 0.0531 | 0.0163 | 0.0318 | 0.0065 |
| Max, AEP | 0.0441 | 0.0142 | 0.0252 | 0.0053 |
| Mean, Pareto IV | 0.0172 | 0.0054 | 0.0102 | 0.0027 |
| Mean, AEP | 0.0127 | 0.0042 | 0.0072 | 0.0017 |
|  |  |  |  |  |  |
| Lh | Max, Pareto IV | 0.0355 | 0.0128 | 0.0222 | 0.0043 |
| Max, AEP | 0.0326 | 0.0093 | 0.0158 | 0.0048 |
| Mean, Pareto IV | 0.0113 | 0.0041 | 0.0073 | 0.0019 |
| Mean, AEP | 0.0097 | 0.0029 | 0.0047 | 0.0014 |
|  |  |  |  |  |  |
| La | Max, Pareto IV | 0.0335 | 0.0151 | 0.0240 | 0.0042 |
| Max, AEP | 0.0387 | 0.0162 | 0.0262 | 0.0051 |
| Mean, Pareto IV | 0.0116 | 0.0044 | 0.0080 | 0.0019 |
| Mean, AEP | 0.0118 | 0.0052 | 0.0079 | 0.0016 |

It is difficult to make a choice which distribution is better. Different financial variables are approximated better by different distributions (generally, AEP distribution is better for most cases). Distances between empirical and theoretical distributions did not change during our time period (for all our financial variables), so our results are stable and robust. We decide to use both, AEP and Pareto IV, distributions for data modeling.

Also we find out, that individual banks can change their position on the sorted list, but structure of the overall distribution is stable over time. So in fact we estimate distributions not of banks names, but of banks of Russian banking system. In fact, we can interpret it as institutional restriction of Russian banking system.

We decided to provide Kolmogorov-Smirnov test for compering two nearest distributions. But properties of estimates of AEP and Pareto IV distributions can be potentially far from “good”, because likelihood functions is rather complex and we can’t guarantee, that we find global maximum. Thus to due to multiplicity of “best” distributions and properties of estimates, we decided to compare empirical distributions, using standard two-sided Kolmogorov-Smirnov test. Mean p-value for all 133 pairs is 0.94, which means that hypothesis “both empirical distribution come from the same continuous distribution” can’ be rejected.

**2.6. Dynamic of Pareto IV parameters**

We made sure, that Pareto IV type and AEP are good approximations for our data. Also according to formal and graphical analysis functional form of data is table over time, so now it is important to define stability of parameters of Pareto IV type distribution. Figures 25-27 shows dynamic of values of parameters of Pareto IV distribution (the first point is January 2004 and the last point is January 2015).

|  |  |
| --- | --- |
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| Figure 29. Dynamic of parameter of scale of Pareto IV distribution | Figure 30. Dynamic of parameter of inequality of Pareto IV distribution |
|  |  |
| Figure 31. Dynamic of parameter of shape of Pareto IV distribution |  |

After the 60-th point values of all parameters become stable (the 60-th point is December 2008). We can see only a slight downward movement for shape and scale parameter and a slight upward trend for inequality parameter. So recent crisis events don’t affect greatly parameters of distribution.

Analysis of dynamic of parameters of AEP distribution (Figures 32-35) gives other results. Location and first shape parameters are rather stable since 70th point and have a slight downward trend possibly connected with recent crisis. But scale and second shape parameters are not stable since October 2009 and have very nontrivial dynamic for the last year.

|  |  |
| --- | --- |
|  |  |
| Figure 32. Dynamic of parameter of location of AEP distribution | Figure 33. Dynamic of parameter of scale of AEP distribution |
|  |  |
| Figure 34. Dynamic of first parameter of shape of AEP distribution | Figure 35. Dynamic of second parameter of shape of AEP distribution |

We can see that parameters changed during period of observation, but in fact distributions for 2004 and 2014 years are identical, so parameters changes are minor in absolute value and neglect each other changes. Also it is important to notice again, that our sample was changing even since October 2009, because number of banks in Russia was decreasing, so some part of dynamic of parameters can be explained not only by institutional changes of Russian banking industry, but also by sample changing (but of course these processes are connected). Moreover, as it was mentioned before, likelihood functions for AEP and Pareto IV distributions were complicated, so some instability of estimates can potentially presence.

**Conclusion**

We show, that specificity of economic activity of bank imply restrictions on the structure of industry: distribution of shares of assets of individual banks is stable over time. Data from Russian banking system is used to validate theoretical model. We showed that Pareto IV and asymmetric exponential power distributions were very good approximation of empirical data.

Quality of approximation is stable over time, so we can say, that functional form of empirical distributions is stable too. Parameters values for Pareto IV distributions are stable, location and first shape parameter for AEP distribution are stable, but scale and second shape parameters have rather nontrivial dynamic. Moreover, absolute changes in parameters values are not very significant. Question about strength of connection of parameters dynamic with sample size and stability of results of maximization routine is also open.

We found out, that individual banks can change their position in the distribution, but overall distribution is stable over time. So maybe it is an institutional feature of Russian banking system. It is very important to investigate causes of this phenomenon in future.

Thus the observed effect is a natural self-regulation of the industry, when structure of the whole industry is stable, but individual banks can move in this distribution. Existence of such rigid adaptive mechanism decreases a risk of unexpected and, we can say, ineffective structural changes of banking system, consequently, industry much more successfully passes crises. This feature should be taken into account for effective economic policy providing and banking sphere reforming.

In future research project we potentially analyze dynamic of inequality of banks’ asset distributions (for example, using Gini index). Also it will be useful to develop forecasting technique for predicting evolution of banking system.

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Dmitry I. Malakhov

National Research University Higher School of Economics. Laboratory for Macrostructure Modeling of Russian Economy, Research Assistant; Department of Applied Economics, Assistant Professor; E-mail: [dmalakhov@hse.ru](mailto:dmalakhov@hse.ru), Tel.: +79032206238

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1. National Research University Higher School of Economics. Laboratory for Macrostructure Modeling of Russian Economy; Department of Applied Economics, Assistant Professor; E-mail: dmalakhov@hse.ru [↑](#footnote-ref-1)
2. National Research University Higher School of Economics. Laboratory for Macrostructure Modeling of Russian Economy, Senior Research Fellow; Department of Applied Economics, Associate Professor; E-mail:u4d@ya.ru [↑](#footnote-ref-2)
3. Federal Research Center “Informatics and Control” of Russian Academy of Science, professor, corresponding member, E-mail: [pospeli@yandex.ru](mailto:pospeli@yandex.ru) [↑](#footnote-ref-3)
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5. It is not correct for real sector firms (physical indicators (output, etc.) for firms sometimes are very useful, but rather subjective). [↑](#footnote-ref-5)
6. Due to space restrictions. [↑](#footnote-ref-6)
7. Values are stable over time. [↑](#footnote-ref-7)
8. For simplicity we omit “shares of” from figure names. [↑](#footnote-ref-8)