Manipulated News Model: Electoral Competition and Mass Media *

Shintaro Miura †

March 1, 2015

Abstract

This paper concerns the distortions in electoral outcomes when mass media strategically distorts the interactions between candidates and voters. We develop an election model where a voter cannot directly observe the policies proposed by two office-motivated candidates. The voter learns this information through media reports before voting takes place, while the media outlet strategically conceals some part of this information. Because incorrect decision-making by the voter is unavoidable (*direct distortion*), the candidates have an incentive to influence the media outlet's behavior through policy settings that are indirectly appealing to the voter (*indirect distortion*). As a result, policy convergence never occurs if and only if the outlet is sufficiently biased. We then measure the degree of distortion in the equilibrium outcomes by the voter's ex ante expected utility, and characterize the least and most distorted scenarios. This characterization shows that the distortion becomes severer as the outlet becomes more biased. By decomposing total distortion into its components, we also illustrate the tension arising between the direct and the indirect distortion.

Journal of Economic Literature Classification Numbers: C72, D72, D82.

Key Words: Downsian voting model, media manipulation, policy convergence/divergence, direct/indirect distortion, persuasion games

*This paper is based on the fourth chapter of my Ph.D. dissertation at Washington University in St. Louis. I am particularly indebted to Marcus Berliant, Haluk Ergin, John Nachbar and John Patty for their continuous supports and encouragements. I am also grateful to Yoshimasa Aoki, Pierre Boyer, Levent Celik, Jimmy Chan, Wei-Cheng Chen, Junko Doi, Philip Dybvig, Kazumi Hori, Ikuo Ishibashi, Akifumi Ishihara, Hideshi Itoh, Shinsuke Kambe, Navin Kartik, Kohei Kawamura, Kazuya Kikuchi, Hideo Konishi, Taro Kumano, Takashi Kunimoto, Wen-Chieh Lee, Noriaki Matushima, Sho Miyamoto, Fumitoshi Moriya, Daisuke Nakajima, Ryo Ogawa, Marco Ottaviani, Filippo Pavesi, Maher Said, Tadashi Sekiguchi, Takashi Shimizu, Yasuhiro Shirata, Takuo Sugaya, Koichi Tadenuma, Gustavo Torrens, Yasuharu Ukai, Tsz-Nga Wong, Haibo Xu, Takuro Yamashita, Yosuke Yasuda and all participants in 5th Annual Economic Graduate Student Conference, 17th KMSG, 2011, 2013 NASM, CTW Summer Conference 2010, GAMES 2012, Japanese Economic Association Fall Meeting, PET 11, SWET 2011 and the seminars in Hitotsubashi University, Kansai University, Otaru University of Commerce, and Washington University in St. Louis for helpful comments and discussion. All remaining errors are my own.

†Department of Economics, Kanagawa University, 3-27-1, Rokkakubashi, Kanagawa-ku, Yokohama, Kanagawa 221-8686, JAPAN. E-mail: smiura@kanagawa-u.ac.jp

1 Introduction

It is well accepted that mass media has a substantial influence on political outcomes. In modern elections, the interactions between the candidates and the voters are indirect in the sense that mass media exists between candidates and voters, and thus provides essential information for their decision-making. For instance, most voters use the news as an information source for voting instead of directly acquiring relevant information. Likewise, candidates decide the content of their electoral campaigns after taking account of polls. That is, mass media can influence electoral outcomes by acting as an intermediary in the transmission of information between candidates and voters.

Because of their informational advantage, media outlets potentially have an incentive to manipulate the content of released news to influence electoral outcomes. For example, in the Iowa Presidential Straw Poll of Republicans in August 2011, Michele Bachmann defeated Ron Paul by just 152 votes. Despite this tiny margin (152 votes of some 16,892 total votes cast), Bachmann's media exposure after the poll completely dominated that of Paul, such that according to Eddlem (2011), "Michele Bachmann managed to book herself on all five major Sunday national television political talk shows. But Ron Paul, who finished in a virtual statistical tie with Bachmann–just 152 votes and less than a one-percent difference–was booked on none of them. Zero." Carney (2011) argues that the reason why Ron Paul was ignored was that "[t]he mainstream media and the Republican establishment wish he would just go away." That is, media outlets strategically ignored Ron Paul to make him appear little supported by voters. This strategic suppression of some election-relevant information is just one of several common forms of media manipulation.[1]

The main concern of this paper is to investigate how and to what extent media manipulation affects electoral outcomes. In other words, we evaluate the "severeness" of media manipulation when media outlets strategically distort the transmission of information between candidates and voters. Media manipulation in the above sense is commonplace in the literature, but most of the existing models omit the competition among candidates or the strategic aspect of mass media. Further, although it is a useful simplification, its cost is not small. Media manipulation tends to be strategic, and strategic manipulation could affect both candidate and voter behaviors. Hence, to evaluate correctly the severeness of media manipulation, we require a model where all the behaviors of candidates, media outlets, and voters are endogenously determined; otherwise, we may over- or underestimate the severeness. Thus, the objective of this paper is twofold. First, we develop

[1] Bagdikian (1997) argues that "Every basic step in the journalistic process involves a value-laden decision: Which of the infinite number of events in the environment will be assigned for coverage and which ignored? Which of the infinite observations confronting the reporter will be noted? Which of the facts noted will be included in the story? ...None of these is a truly objective decision."

a model of electoral competition including both strategic media manipulation and competition among candidates. Second, we make clear the mechanism governing the distortion of electoral outcomes, and evaluate its severeness using the model developed.

We consider the following Downsian voting model including media outlets. There are two office-motivated candidates, a single media outlet, and a single voter, all of whom are rational. Unlike standard models, we assume that the voter cannot directly observe the policies proposed by the two candidates. Instead, the voter learns this information through reports from the media outlet. In other words, we consider the following two-stage game. In the first stage, the candidates simultaneously propose policies that only the media outlet observes. In the second stage, the media outlet whose preference differs from that of the voter decides the release of information about the proposed policies, after which the voter chooses one of the candidates.

The results are as follows. First, we demonstrate distortion in the equilibrium outcomes compared with the no manipulation scenario because of the following mechanism. When the media outlet suppresses information, the voter attempts to infer the reason why. However, this inference is imperfect in the sense that the voter cannot uniquely pin down the reason, despite being fully rational. Hence, the voter has to choose the ex post unfavored candidate with some positive probability. In other words, the voter's decision-making is unavoidably incorrect ex post, which represents the distortion in the voter's behavior (*direct distortion*). Because of this ex post incorrect decision-making, appealing to the voter by proposing the voter's ideal policy becomes less attractive to the candidates. The candidates then have an incentive to influence the media outlet's behavior through policy settings that are indirectly appealing to the voter. That is, policies that are not ideal for the voter are in the equilibrium, which represents the distortion in the candidates' behavior (*indirect distortion*). As a result, there exist multiple equilibria, but convergence to the voter's ideal policy cannot be supported in an equilibrium when the bias of the media outlet is not small, which contrasts with the results of the standard model.

Second, we measure and characterize the degree of distortion with the voter's ex ante expected utility. In particular, we specify the least and the most distorted scenarios in its terms. The characterization shows that the distortion becomes severer in the following sense as the outlet becomes more biased. To start with, both the lower and upper bounds of the distortion are increasing in the outlet's bias. In addition, the difference between the lower and upper bounds of the distortion is also increasing in the bias. That is, the variance of electoral outcomes increases. We then decompose the degree of distortion into its parts, namely, the direct and the indirect distortion. This decomposition suggests a tension between the two distortion channels. When the bias of the outlet is not small, minimizing the direct distortion is incompatible with minimizing the indirect distortion. Especially, when the bias is moderate, it is possible to maximize the total degree of distortion while minimizing the direct distortion. Thus, we conclude that the cost of omitting strategic manipulation and candidate competitions is not negligible.

The paper is organized as follows. In the following subsection, we briefly review the related literature. Section 2 defines and discusses the formal model. Section 3 analyzes a benchmark model without media manipulation, and Sections 4 and 5 analyze a model with media manipulation. We make clear the distortion mechanism in Section 4, and characterize the degree of distortion in Section 5. We conclude the paper in Section 6. All proofs are in Appendix A.

1.1 Related literature

This paper mostly relates to the literature of political ecnomics of mass media.[2] We can divide this literature into the following two strands depending on the role of mass media. In the first strand, media outlets are modeled as "outside observers" that provide additional election-relevant information instead of distorting information transmission between candidates and voters. In other words, the voters update their beliefs about payoff-relevant uncertainty by observing both candidate behavior and reports by the media outlets. For example, Chan and Suen (2008, 2009), and Gul and Pesendorfer (2012), consider a two-candidate election model where the outlets endorse one of the candidates by sending cheap-talk messages, and investigate the relationship between political polarization and media competition. Elsewhere, Ashworth and Shotts (2010) and Warren (2012) examine a retrospective voting model where the incumbent politician has reputational concern and the media outlets again provide cheap-talk endorsements.[3] Overall, these studies show that media outlets improve voter welfare, even though the outlets are biased. However, Chakraborty and Ghosh (2013) show the opposite implication by considering a model where outlets send cheap-talk endorsement about the character of candidates that is unobservable to voters.

In the second strand of this literature, media outlets are modeled as "intermediaries" in the information transmission process. That is, media manipulation could distort voters' observations. This strand further divides into the subgroups of strategic and nonstrategic mass media. With strategic mass media, Bernhardt et al. (2008) consider a political campaigning model where the media outlets conceal negative news about the candidates, and show that censorship could exag-

[2]According to a recent survey in Prat and Strömberg (2013), there are several key theoretical area of research, including (i) media capture by government (Besley and Prat 2006), (ii) how mass media affect government public policy (Strömberg 2004), and (iii) how media bias is generated (Mullainathan and Shleifer 2005; Baron 2006; Gentzkow and Shapiro 2006).

[3]While the outlets' information is exogenous in Ashworth and Shotts (2010), it is endogenous in Warren (2012).

gerate political polarization. In other work, Duggan and Martinelli (2011) develop a retrospective voting model where the media outlets reduce two-dimensional information about the challenger's policy into a one-dimensional "story" through media manipulation, and show that biased outlets could improve social welfare over neutral outlets. For nonstrategic mass media, Adachi and Hizen (2014) analyze a retrospective voting model where media outlets systematically add noise to voters' observations, and show that media bias, even anti-incumbent bias, never improves social welfare. Recently, Pan (2014) develops a two-candidate Wittman model with a noise structure similar to that of Adachi and Hizen (2014), and shows that more noise exaggerates political polarization.

There are two remarks concerning the relation of this body of work to the current analysis. First, we can neatly classify our analysis into the second strand of inquiry.[4] However, in contrast with the other studies, our model includes both competitions between candidates and strategic media manipulation. Because of the difference in the scope of the paper, Bernhardt et al. (2008) and Duggan and Martinelli (2011) exogenously fix the proposed policies of candidates. Likewise, Adachi and Hizen (2014) and Pan (2014) omit the strategic aspect of media manipulation. Thus, the literature does not fully respond to the question of how rational candidates react to strategic media manipulation distorting the interaction between candidates and voters. Therefore, we present a model that assumes that the candidates, the media outlets, and the voters are fully rational. Second, this paper is a complement of Chakraborty and Ghosh (2013) in the sense that their model is similar to ours except for the role of mass media; that is, media outlets are outside observers in their setup, but intermediaries in our setup. Although the role of mass media is completely different, we observe a tension between the loss of information and agenda distortion in both setups.

To describe the suppression of information by the media outlets, we adopt a persuasion game framework from the strategic communication literature. Persuasion games are sender–receiver games with hard private information, as first formalized by Milgrom (1981), for which there is now a voluminous literature. See, for example, Milgrom and Roberts (1986), Seidmann and Winter (1997), Giovannoni and Seidmann (2007), and Hagenbach et al. (2014). In contrast to cheap-talk games à la Crawford and Sobel (1982), the sender's private information in this framework is certifiable, so the sender is unable to misrepresent information, but can conceal unfavorable information. Based on Miura (2014a, 2014b), we analyze a hierarchical persuasion game where the sender's private information is affected by the strategies of others.[5]

[4] This paper is also regarded as a paper demonstrating policy divergence by introducing voter uncertainty like Kartik and McAfee (2007) and Kikuchi (2009).

[5] Other types of hierarchical communication are also studied in the literature. See, for example, Ivanov (2010), Li (2010) and Ambrus et al. (2013).

2 The Model

We define the baseline model in Section 2.1, and discuss its plausibility in Section 2.2.

2.1 Setup

There are four players in our model: candidates 1 and 2, a single media outlet and a single voter.[6] The players play the following two-stage game. In the first stage, called the *policy-setting stage*, each candidate simultaneously proposes a policy, and only the outlet observes the proposed policies. In the second stage, called the *news-reporting stage*, the outlet sends a message about the proposed policies to the voter. After observing the message, the voter casts the ballot for one of the candidates. The winning candidate then implements his proposed policy.

Let $X \equiv [x^-, x^+] \subset \mathbb{R}$ be the set of available policies for the candidates with $x^- < 0 < x^+$. Let $x_i \in X$ be the policy proposed by candidate $i \in \{1, 2\}$, and $z \equiv (x_1, x_2) \in Z \equiv X^2 \subset \mathbb{R}^2$ describe a policy pair proposed by the candidates. We assume that the information regarding policy pair z is hard information. In addition, we assume that the media outlet, but not the voter, correctly observes policy pair z. Hence, the information about policy pair z is the media outlet's private information in the news-reporting stage. The message space given policy pair z is defined by $M(z) \equiv \{m \in 2^Z | z \in m\}$. That is, the available messages under policy pair z are subsets of policy pair space Z containing the truth z.[7] Let $m \in \bigcup_{z \in Z} M(z)$ be a message from the outlet. Let $y \in Y \equiv \{y_1, y_2\}$ be the action of the voter, where y_i represents that the voter certainly casts a ballot for candidate i.

We assume that there are two types of candidates: an *opportunistic-type* candidate and an *ideological-type* candidate. The opportunistic type is the standard office-motivated strategic type of candidate. Alternatively, the ideological type is a nonstrategic type of candidate that always proposes his preferred policy. We assume that if candidate 1 (resp. 2) is the ideological type, then he always proposes policy $r \in (0, x^+)$ (resp. $l \in (x^-, 0)$), and $|r| < |l|$. That is, we assume an asymmetry between the candidates. Let $\Theta \equiv \{O, I\}$ be the candidates' type space, and O (resp. I) represents the opportunistic (resp. ideological) type. We assume that candidate i's type $\theta_i \in \Theta$ is candidate i's private information, and θ_1 and θ_2 are independently determined. Let $p \in (0, 1)$ be the probability that each candidate is the opportunistic type, and assume this is common knowledge.

We define the players' preferences as follows. Define the opportunistic-type candidate i's von

[6] Throughout the paper, we treat the candidates and the voter as male and the outlet as female.

[7] It is worthwhile to remark that for any subset $P \subseteq Z$, message $m = P$ has the property that $M^{-1}(P) = P$ where $M^{-1}(P)$ represents the set of policy pairs under which message $m = P$ is available. That is, information about policy pair is *fully certifiable* in the sense of persuasion games.

Neumann–Morgenstern utility function $u_i : Y \to \mathbb{R}$ by:

$$u_i(y) \equiv \begin{cases} 1 & \text{if } y = y_i, \\ 0 & \text{Otherwise.} \end{cases} \tag{1}$$

We assume that the voter and the outlet have single-peaked preferences over the implemented policies.[8] Define the voter's von Neumann–Morgenstern utility function $v : Z \times Y \to \mathbb{R}$ by:

$$v(z, y) \equiv \begin{cases} -|x_1| & \text{if } y = y_1, \\ -|x_2| & \text{if } y = y_2. \end{cases} \tag{2}$$

Similarly, define the outlet's von Neumann–Morgenstern utility function $w : Z \times Y \to \mathbb{R}$ by:

$$w(z, y) \equiv \begin{cases} -|x_1 - b| & \text{if } y = y_1, \\ -|x_2 - b| & \text{if } y = y_2. \end{cases} \tag{3}$$

The voter's ideal policy is 0, but that of the outlet is $b > 0$. Hence, parameter b represents the difference between the preferences of the voter and the outlet. We refer to this parameter throughout the paper as *preference bias*. We assume that the exact value of b is common knowledge.

We formalize the timing of the game as follows. At the policy-setting stage, nature chooses candidate i's type $\theta_i \in \Theta$ according to the prior distribution p, and only candidate i correctly learns his own type θ_i. Then, given θ_i, each candidate simultaneously proposes a policy pair $x_i \in X$. Only the outlet correctly observes the policy pair $z = (x_1, x_2) \in Z$. At the news-reporting stage, given the observed pair z, the outlet sends a message $m \in M(z)$. After observing the message, the voter undertakes an action $y \in Y$. The policy announced by the winning candidate is then implemented.

We define the players' strategies and the belief as follows. The opportunistic-type candidate i's strategy is represented by $\alpha_i \in \Delta(X)^*$, where $\Delta(X)^*$ is the set of finite-support probability distributions over the policy space.[9] Let $\alpha_i(x_i)$ represent the probability that candidate i proposes policy x_i. The outlet's strategy $\beta : Z \to \Delta(M)$ is a function from an observed policy pair to a probability distribution over the entire message space. The voter's strategy $\gamma : M \to \Delta(Y)$ is a function from an observed message to a probability distribution over the voter's action set Y. The voter's strategy is represented by $\gamma(m) = (\gamma_1(m), 1-\gamma_1(m))$, where $\gamma_1(m)$ represents the probability that the voter chooses candidate 1 when he observes message m. With some abuse of notation,

[8] We assume the policy-motivated outlet through the paper. This assumption is discussed in section 2.2.6.

[9] If distributional strategies are allowed, we face serious multiplicity of equilibrium. To avoid this problem, we exclude distributional strategies of the candidates.

the pure strategies of the players are simply represented by $\alpha_i = x_i$, $\beta(z) = m$ and $\gamma(m) = y$, respectively. Let $\mathcal{P}^* : M \to \Delta(Z)$ represent the voter's posterior belief, which is a function from an observed message to a probability distribution over the set of proposed policy pairs Z.

We use the perfect Bayesian equilibrium (hereafter, PBE) as a solution concept. Because of the certifiability of policy pair z, the message must contain the true policy pair. Hence, we insert the following requirement as a restriction to off-the-equilibrium-path beliefs. Let $S(f(\cdot))$ be the support of probability distribution $f(\cdot)$.

Requirement 1 *For any message $m \in M$, $S(\mathcal{P}(\cdot|m)) \subseteq m$ holds.*

Definition 1 ***PBE***

A quintuple $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE if it satisfies the following conditions:

(i) for any $i, j \in \{1, 2\}$ with $i \neq j$ and any $x_i \in S(\alpha_i^*)$:

$$x_i \in \arg\max_{x_i' \in X} \sum_{x_j \in X} \left(\sum_{m \in M} \left(\sum_{y \in Y} u_i(y) \Pr(y|\gamma^*(m)) \right) \Pr(m|\beta^*(x_i', x_j)) \right) \Pr(x_j|\alpha_j^*); \tag{4}$$

(ii) for any $z \in Z$ and any $m \in S(\beta^*(z))$:

$$m \in \arg \max_{m' \in M(z)} \sum_{y \in Y} w(z, y) \Pr(y|\gamma^*(m')); \tag{5}$$

(iii) for any $m \in M$ and any $y \in S(\gamma^*(m))$:

$$y \in \arg\max_{y' \in Y} \sum_{z \in Z} v(z, y') \mathcal{P}^*(z|m); \tag{6}$$

(iv) the posterior $\mathcal{P}^*$ is derived by α_1^*, α_2^* and β^* consistently with Bayes' rule whenever it is possible. Otherwise, $\mathcal{P}^*$ is some probability distribution over Z satisfying Requirement 1.

We assume the following tie-breaking rules, one for the voter and the other for the outlet.

Requirement 2 ***Tie-breaking rules***

(i) If the voter is indifferent between y_1 and y_2 under belief $\mathcal{P}^*(\cdot|m)$, then $\gamma^*(m) = (1/2, 1/2)$.

(ii) If the outlet observes policy pair z such that $x_1 = x_2$, then $\beta^*(z) = z$.[10]

[10] To economize on notation, $\beta^*(z) = \{z\}$ is simply represented by $\beta^*(z) = z$.

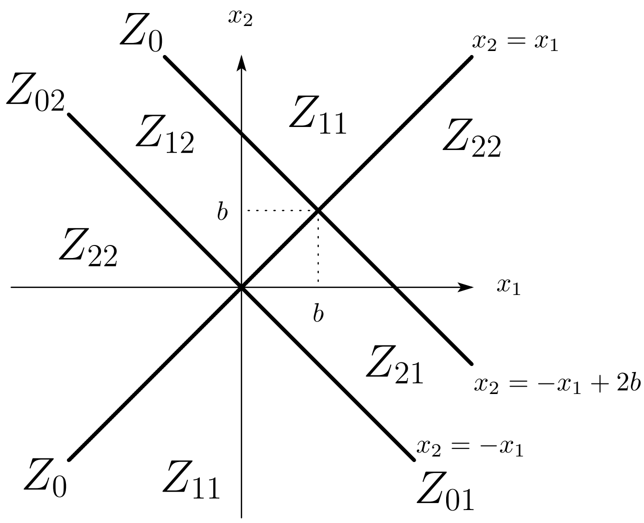

Figure 1: Distribution of preferences

In the subsequent analysis, we focus on PBEs where (i) the tie-breaking rules are satisfied, and (ii) the voter adopts undominated strategies. To simplify the referencing, an equilibrium in which the opportunistic-type candidate proposes x_1 and x_2 for certain is called (x_1, x_2) *equilibrium*.

Define the following notation and terminologies frequently used hereafter. Let $Z(\alpha_1, \alpha_2) \equiv \{z \in Z | \Pr(z|\alpha_1, \alpha_2) > 0\}$ denote the set of possible policy pairs given strategies α_1 and α_2. Depending on the preferences defined by (2) and (3), the space of policy pairs Z is divided into the following regions, as shown in Figure 1. For $i, i', j, j' \in \{1, 2\}$ with $i \neq i'$ and $j \neq j'$:

$$\begin{aligned} Z_{ij} &\equiv \{z \in Z | v(z, y_i) > v(z, y_{i'}) \text{ and } w(z, y_j) > w(z, y_{j'})\}, \\ Z_{0j} &\equiv \{z \in Z | v(z, y_1) = v(z, y_2) \text{ and } w(z, y_j) > w(z, y_{j'})\}, \\ Z_0 &\equiv \{z \in Z | w(z, y_1) = w(z, y_2)\}. \end{aligned} \tag{7}$$

We refer to regions Z_{11}, Z_{22} and Z_0 as *agreement regions*, and regions Z_{01}, Z_{02}, Z_{12} and Z_{21} as *disagreement regions*. If a proposed policy pair lies in an agreement region, then the voter's and the outlet's preferences agree. In region Z_{11} (resp. Z_{22}), both the voter and the outlet strictly prefer y_1 (resp. y_2). In region Z_0, the outlet is indifferent between y_1 and y_2 while the voter could have a strict preference. On the other hand, if a proposed policy pair lies in a disagreement region, then the voter's and the outlet's preferences disagree. In regions Z_{12} and Z_{02} (resp. Z_{21} and Z_{01}), the voter has a weak preference for y_1 (resp. y_2), but the outlet strictly prefers y_2 (resp. y_1). Define $Z_j(\alpha_1, \alpha_2) \equiv Z_j \cap Z(\alpha_1, \alpha_2)$ for $j \in \{11, 22, 0, 01, 02, 12, 21\}$. For easy reference, define $\bar{Z}_{12} \equiv Z_{12} \cup Z_{02}$ (resp. $\bar{Z}_{21} \equiv Z_{21} \cup Z_{01}$) and $\bar{Z}_{12}(\alpha_1, \alpha_2) \equiv \bar{Z}_{12} \cap Z(\alpha_1, \alpha_2)$ (resp. $\bar{Z}_{21}(\alpha_1, \alpha_2) =$

$\bar{Z}_{21} \cap Z(\alpha_1, \alpha_2))$. Let $y^v(z)$ be the voter's ex post correct decision-making defined by:

$$y^v(z) \equiv \begin{cases} (1,0) & \text{if } |x_1| < |x_2|, \\ (1/2,1/2) & \text{if } |x_1| = |x_2|, \\ (0,1) & \text{if } |x_1| > |x_2|. \end{cases} \tag{8}$$

2.2 Discussion of the model

2.2.1 Certifiability of information about the policies

We assume that information about the policies is certifiable. In reality, this information is explicitly stated in the manifesto of each candidate, which anyone can check if he wishes. That is, there is objective evidence that proves whether the media outlet's news is true.

2.2.2 Impossibility of fabrication

As Groseclose and Milyo (2005) argue, media manipulation through fabrication of information is less likely than manipulation by omission.[11] Hence, this assumption seems reasonable to capture the standard behavior of mass media. Moreover, even if the outlet were able to fabricate information, she would not use this option because of its certifiability. As such, because of certifiability, fabrication is easy to disclose, and once disclosed to the public, the outlet's reputation would be severely tarnished. In other words, fabrication is too costly in our setup.

2.2.3 Unique voter

The unique voter in the model can be regarded as a representative swing voter. It is well accepted that swing voters have a major impact in determining the outcome of an election. Because swing voters change their minds depending on the information they face, it seems reasonable to interpret that swing voters are not partisan with respect to particular parties and policies.[12] The voter in the model having ideal policy 0 is consistent with this interpretation.

2.2.4 No direct messages from candidates

The assumption that the candidates cannot send a direct message to the voter reflects the fact that active information acquisition is too costly for voters, as Downs (1957) argues. In reality, candidates

[11] Groseclose and Milyo (2005) argue that "Instead, for every sin of commission, such as Glass or Blair, we believe that there are hundreds, and maybe thousands, of sins of omission–cases where a journalist chooses facts or stories that only one side of political spectrum is likely to mention."

[12] Campbell (2008) argues that "[t]hey (swing voters) are either moderates or people who are unable or unwilling to characterize their ideology."

have opportunities to send their own messages directly to voters (e.g., stump speeches), but this seems too costly for most voters, especially swing voters, to actively acquire such information (e.g., attending stump speeches and asking questions by themselves). Thus, to save costs, instead of actively acquiring information, the voters mainly rely on news released by media outlets.

2.2.5 Perfect commitment of the winning candidate

There are two justifications for the assumption that the winning candidate implements the proposed policy. First, a theoretical reason is that we wish to separate the effects of media manipulation from the effects of imperfect commitment. For example, it is well known that imperfect commitment to policy implementation could induce policy divergence (e.g., Banks (1990) and Harrington (1992)). However, the main purpose of this paper is to examine the effects of media manipulation on electoral competitions. To highlight the manipulation effects, we include the extreme perfect commitment assumption.

Second, a rather more practical reason is that the long-run-perfect suppression of information appears impossible. In other words, information about the proposed policies could leak after the election via several channels, even if not well reported during the election. Once we imagine repeated interactions between the candidates and the voter, the candidates' reputations should be severely tarnished as a punishment if they breach their promises and it is disclosed. Hence, implementing the proposed policies seems rational if we interpret the model as a reduced form of repeated interaction and if the candidates are sufficiently farsighted.

2.2.6 Policy-motivated outlets

It is popular for the principal media outlets in democratic countries to separate marketing and editorial services in order to protect the neutrality of the press from commercialism. In other words, the content of the news might not be derived from the outlets' profit maximization. Furthermore, editorial services tend to have sufficient discretion over the contents of the news in such a situation. That is, the preference of editors/directors could affect the contents. The *Tsubaki Scandal* in Japan is an example. According to Krauss (2000):

> [c]onservative newspapers leaked the transcript of a private talk in late September 1993 by the head of TV Asahi's News Division, Tsubaki Sadayoshi, in which he candidly revealed he had encouraged (but not directed) the news staff to "conduct our news reporting with a view to assisting the non-LDP forces in establishing a coalition government."

The setup with the policy-motivated outlet seems consistent with this scenario.

2.2.7 Single media outlet

We can regard the model with a single media outlet as a reduced form of a model with multiple media outlets whose influence is unbalanced. In democratic countries, ideologically different media outlets coexist, but media coverage tends to be biased in one direction.[13] Hence, if the news from liberal outlets has weaker influence than that from conservative outlets because of biased media coverage, then the aggregate tone of mass media in that country could have a conservative bias. We interpret the single outlet in this paper analysis as a representative media outlet whose preference bias reflects the aggregate tone of media coverage in the country. We revisit this point in Appendix B.5.1, and demonstrate that the results in a multiple-outlet model are similar to those in the single-outlet scenario if the outlets' influence is unbalanced.

2.2.8 Asymmetry between the candidates

We assume that the candidates are asymmetric in the sense that the preferred policies of the ideological-type candidates are different and $|r| < |l|$. This assumption is essential to the result: if the candidates are completely symmetric, then policy convergence is more persistent. However, we do not require large asymmetry. It is sufficient to exclude completely symmetric scenarios for obtaining the results. The detailed discussion is in Appendix B.5.2.

2.2.9 Tie-breaking rule for the media outlet

We assume that the outlet surely discloses all information, i.e., $\beta(x) = x$, if the proposed policies are convergent. This is an assumption to avoid the serious multiplicity of equilibrium; that is, if $\beta^*(z) = Z$ holds for any $z \in Z$, then any policy pair can be supported in equilibrium. However, if we require that the outlet fully discloses the true information with positive probability when the proposed policies are convergent, then such serious multiplicity disappears. Furthermore, we can show that the set of policy pairs that can be supported under the restriction is identical to that under the tie-breaking rule. That is, the tie-breaking rule is not crucial to the results. The detailed discussion is in Appendix B.5.3.

[13] For example, Groseclose and Milyo (2005) find a strong liberal bias in the US.

probability	(θ_1, θ_2)	proposed policy pair	winner	equilibrium policy
p^2	(O, O)	$(0, 0)$	1 or 2	0
$p(1-p)$	(O, I)	$(0, l)$	1	0
$(1-p)p$	(I, O)	$(r, 0)$	2	0
$(1-p)^2$	(I, I)	(r, l)	1	r

Table 1: Equilibrium outcomes in the benchmark model

2.2.10 Nonstrategicness of the ideological type

While the nonstrategicness of the ideological type seems essential to the results, it is merely for simplification, and thus irrelevant to the results. That is, we can obtain similar results in the model where any type of candidate is fully rational, as shown in Appendix B.5.4. The most important factor of this model is the voter's uncertainty about how the candidates behave.

3 Benchmark: No Manipulation

In this section, we analyze the model without media manipulation as a benchmark model. Because the voter always learns the true proposed policies, the voter can certainly cast the ballot for the candidate whose policy is closer to his ideal policy 0. Thus, as in standard Downsian models, the $(0, 0)$ equilibrium is the unique equilibrium in the benchmark model. The equilibrium outcomes are summarized in Table 1, which shows that we can support the voter's ideal policy as the equilibrium policy unless both candidates are of the ideological type. The following proposition summarizes the result in the benchmark model.

Proposition 1 *Consider the benchmark model. Then:*

(i) the $(0, 0)$ *equilibrium is the unique equilibrium; and*

(ii) the voter's ideal policy is supported as the equilibrium outcome unless both candidates are of the ideological type.

Proof. Omit.[14] ■

4 Manipulated News Model

Now, we return to the model involving media manipulation. We refer to this as the *manipulated news model*. In this section, we make clear the mechanism for the distortion by media manipulation.

[14]The details are in Appendix B.1.

The equilibrium outcome is distorted compared with that in the benchmark model through the following two channels. The first is the distortion in the voter's behavior; that is, the voter's decision-making could be incorrect ex post because of the remaining uncertainty about the proposed policies. The second is the distortion in the candidates' behaviors; that is, the candidates have an incentive to influence the outlet's behavior through policy settings indirectly appealing to the voter. That is, media manipulation does not only distort the information that the voter received, but also distorts the alternatives that the voter can choose. We refer to these as *direct distortion* and *indirect distortion*, respectively. As a result, there exist multiple equilibria including policy divergence, and the $(0, 0)$ equilibrium could not exist, which are the main contrasts with the benchmark model.

4.1 News-reporting stage

First, we analyze a persuasion game between the outlet and the voter given the candidates' proposed policies. It is worthwhile making clear the voter's uncertainty at the beginning of the news-reporting stage. The voter faces uncertainty about the proposed policy pair because of the uncertainty about the candidates' types. For example, suppose that $\alpha_1^* = \alpha_2^* = 0$ are the opportunistic-type candidates' equilibrium strategies. The voter then knows that either one of the pairs in $Z(\alpha_1^*, \alpha_2^*) = \{(0, 0), (0, l), (r, 0), (r, l)\}$ is proposed in the equilibrium, but he cannot specify which policy pair is actually proposed. In other words, given equilibrium strategies α_1^* and α_2^*, the voter, in the equilibrium, could face uncertainty represented by a distribution over Z whose support is $Z(\alpha_1^*, \alpha_2^*)$ at the beginning of the news-reporting stage.[15] Therefore, the news from the outlet is crucial for the voter to choose the correct candidate in this model.

The following proposition states that the existence of a strategic media outlet unavoidably forces the voter's decision to be incorrect ex post even though the voter is fully rational.

Proposition 2 *Consider the manipulated news model.*

(i) There exists an equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *where* $\gamma^*(\beta^*(z)) = y^v(z)$ *for any* $z \in Z(\alpha_1^*, \alpha_2^*)$ *if and only if either (1)* $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$ *or* $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$ *or (2)* $Z_{02}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, $Z_{01}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, *and* $Z_{12}(\alpha_1^*, \alpha_2^*) = Z_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$ *holds.*

(ii) In any equilibrium, there exists at least one policy pair $z \in Z$ *such that* $\gamma^*(\beta^*(z)) \neq y^v(z)$.

[15] As long as we use the Nash concept, players correctly expect the strategies of others in equilibrium. In the manipulated news model, the policies proposed are the strategies of the candidates and so the voter correctly expects the candidates' strategies in equilibrium. However, because the voter does not know the type of candidates, he faces uncertainty about the proposed policy pair. This is the reason why $p = 1$ is excluded.

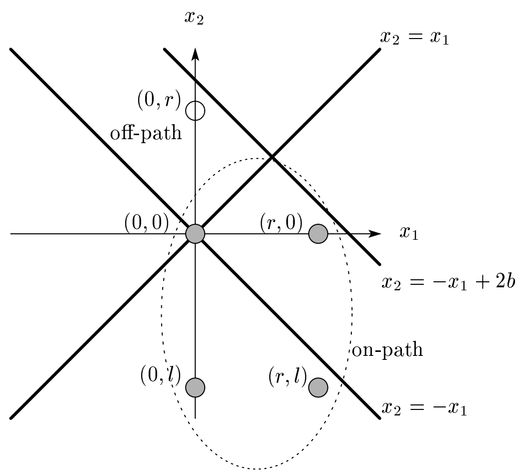

Figure 2: Incorrect decision-making for off-the-equilibrium-path policy.

Proposition 2-(i) is a corollary of the well-known result in the literature of persuasion games, that is, this is the necessary and sufficient condition for the existence of the *worst-case inference* for any message $m \in M$.[16] Intuitively, whether the voter's ex post correct decision-making is guaranteed on the equilibrium path depends on whether the voter can correctly infer the outlet's motivation behind suppression. For example, if $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, then the voter's decision-making is correct ex post because he can correctly infer that the outlet concealed the policy pair in disagreement region $\bar{Z}_{12}$ after observing ambiguous messages. However, if $Z_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then the voter cannot specify the outlet's motivation behind the suppression. Because of this indeterminacy, the voter's decision-making should be incorrect with positive probability on the equilibrium path, which is the direct distortion.

Even if the voter's decision-making is ex post correct on the equilibrium path, it must be incorrect at some off-the-equilibrium-path policy pair. Suppose, for example, that $b > r/2$ and $\alpha_1^* = \alpha_2^* = 0$. Because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$ as shown in Figure 2, there exists an equilibrium in which the voter's ex post correct decision-making is guaranteed on the equilibrium path by Proposition 2-(i). To support this equilibrium, the voter's response to any message $m' \in M(r, 0)$ should be $\gamma^*(m') = (0, 1)$.[17] In this equilibrium, policy pair $z = (0, r)$ is off the equilibrium path, and the voter prefers candidate 1 but the outlet prefers candidate 2 at this policy pair. Thus, given the voter's response $\gamma^*(m') = (0, 1)$ where $(r, 0) \in m'$, the outlet observing policy pair $z = (0, r)$ sends such a message m', e.g., $m' = \{(0, r), (r, 0)\}$, and so candidate 2 wins for certain. That is, the voter's decision-making at policy pair $z = (0, r)$ is incorrect ex post.

[16] See Giovannoni and Seidmann (2007), Hagenbach et al. (2014) and Miura (2014a).

[17] To economize on the notation, $M((x_1, x_2))$ is simply represented as $M(x_1, x_2)$.

In summary, the outlet successfully conceals part of the unfavorable information in any equilibrium, that is, the voter's decision-making is unavoidably incorrect ex post. While the incorrect decision-making on the equilibrium path definitely distorts equilibrium outcomes, it might seem that incorrect decision-making off the equilibrium path is irrelevant. However, even incorrect decision-making off the equilibrium path significantly affects the candidates' incentives.

4.2 Policy-setting stage

Now, we analyze how the opportunistic-type candidates behave. There are the following two main contrasts between the manipulated news and the benchmark models. First, there exist multiple equilibria including policy-divergence equilibria. Second, a $(0, 0)$ equilibrium does not exist unless the preference bias is small. While opportunistic-type candidates appeal to the voter, the effective way of appealing is altered in the manipulated news model because of the voter's incorrect decision-making, which is the indirect distortion. This is the origin of these contrasts.

The multiplicity of equilibria arises because proposing the voter's ideal policy, which we refer to as *direct appealing*, becomes less attractive to the candidates because of the incorrect decision-making by the voter. In the benchmark model, directly appealing is the effective way to maximize the winning probability. That is, because the voter correctly recognizes the proposed policy pair, only the candidate who proposes a policy that is closer to the voter's ideal policy wins with positive probability. However, in the manipulated news model, the voter could not correctly recognize the attractiveness of the candidate who directly appeals because of the media manipulation. As a result, the candidate who proposes an ex post less attractive policy for the voter could win with positive probability. Therefore, from the perspective of the candidates, proposing other than the voter's ideal policy is not a bad idea. An example of an equilibrium other than $(0, 0)$ is as follows.[18]

Claim 1 *Consider the manipulated news model, and suppose that* $b > r$*. Then, there exists an equilibrium where for any* $q \in (0, p)$*, candidate* 1 *(resp. candidate* 2*) randomizes policies* 0 *and* r *with probabilities* q/p *(resp.* q*) and* $1 - q/p$ *(resp.* $1 - q$*), respectively.*

As demonstrated in this example, directly appealing may not be dominant for the candidates. In this equilibrium, candidate 2's winning probability is $1/2$. Now, we consider candidate 2's deviation to strategy $\alpha_2 = 0$. In the benchmark model, this deviation strictly improves his winning probability: candidate 2 wins with probability $1 - q/2 > 1/2$ after this deviation because he wins for certain if policy pair $z = (r, 0)$ occurs. Hence, such a mixed strategy equilibrium never

[18]The proof of Claim 1 is in Appendix B.3.

exists in the benchmark model, as shown in Proposition 1. However, in the manipulated news model, this deviation does not strictly improve his winning probability. In this equilibrium, the outlet observing policy pair $z = (r, 0)$ successfully suppresses the information, and then the voter chooses candidate 1 with positive probability: that is, $\gamma^*(\beta^*(r, 0)) = (1/2, 1/2)$. Hence, candidate 2's winning probability does not change after this deviation. That is, because the voter's ex post incorrect decision-making at policy pair $z = (r, 0)$ makes direct appealing less attractive to candidate 2, an equilibrium other than $(0, 0)$ exists in the manipulated news model. It is worthwhile to remark that policy divergence occurs on the equilibrium path.

The second contrast with the benchmark model is the fragility of the $(0, 0)$ equilibrium. The necessary and sufficient condition for the existence of a $(0, 0)$ equilibrium is as follows.

Theorem 1 *Consider the manipulated news model. Then, there exists a* $(0, 0)$ *equilibrium if and only if either (i)* $b \leq r/2$ *or (ii)* $b = r$ *and* $p \leq 1/2$ *holds.*

As shown in Theorem 1, a $(0, 0)$ equilibrium generically does not exist when the preference bias is not small. This fragility of the $(0, 0)$ equilibrium arises from the candidates' incentive to appeal to the voter by exploiting the media manipulation. When the information is suppressed, the voter could strictly prefer one candidate to the other. We refer to the more preferred candidate as the *front-runner* and the less preferred candidate as the *underdog*. The front-runner then has an incentive to propose a policy likely suppressed to maintain the advantage under manipulation. On the other hand, the underdog has an incentive to propose a policy likely disclosed to mitigate the disadvantage under manipulation. That is, the candidates appeal to the voter by taking account of the outlet's response, which is *indirect appealing*. The incentives for the indirect appealing are the main force in breaking down the $(0, 0)$ equilibrium.

Suppose, for example, that there exists a $(0, 0)$ equilibrium with $\gamma^*(\beta^*(r, 0)) = (0, 1)$ when $b > r$. It is worthwhile noticing that to support this equilibrium, $\gamma^*(m) = (0, 1)$ should hold for any message $m \in M(r, 0)$; otherwise, the outlet observing policy pair $z = (r, 0)$ deviates. Because this message is also available to the outlet observing a policy pair in disagreement region $\bar{Z}_{12}$, $\gamma^*(\beta^*(z)) = (0, 1)$ should hold for any $z \in \bar{Z}_{12}$; otherwise, the outlet sends a message including policy pair $z = (r, 0)$. In this scenario, candidate 2 is the front-runner, and then has a strong incentive for the indirect appealing to lead the media manipulation to exploit the advantage. In other words, candidate 2 proposes a policy that is more likely to induce a policy pair that lies in disagreement region $\bar{Z}_{12}$ because he wins for certain if such a policy pair is realized. Hence, candidate 2 deviates to strategy $\alpha_2 = b$ because the realized policy pair under this strategy certainly

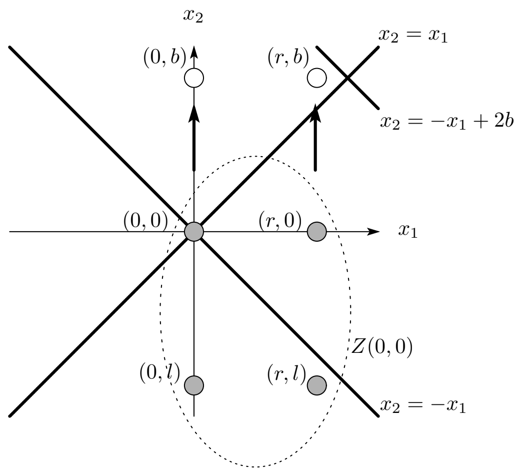

Figure 3: Corruption of the $(0, 0)$ equilibrium by the incentive for indirect appealing

lies in disagreement region $\bar{Z}_{12}$, as shown in Figure 3. That is, the $(0, 0)$ equilibrium collapses because of candidate 2's incentive for indirect appealing. This collapse demonstrates how the voter's ex post incorrect decision off the equilibrium path affects the candidates' incentives.

In summary, the voter's unavoidably incorrect decision-making alters the candidates' effective way of appealing. Instead of direct appealing, the candidates indirectly appeal to the voter by exploiting the media manipulation. Strategic media manipulation induces the direct and the indirect distortion, and both distort equilibrium outcomes. This is the distortion mechanism in the manipulated news model.

5 Degree of Distortion

In this section, we specify the extent to which the distortion mechanism specified in the previous section distorts the equilibrium outcomes. First, we focus on a particular class of equilibria as a refinement, and then show that candidate 2 is never the front-runner in equilibrium. Second, by using this fact, we specify the least and the most distorted level as measured by the voter's ex ante expected utility. Third, we show a tension between the direct and the indirect distortion by decomposing the total degree of distortion into its parts.

5.1 Undominated simple equilibria

5.1.1 Definition

To obtain clear results, we focus on the following equilibria. Define $\Gamma \equiv \{\gamma \in \Delta(Y)^M | S(\gamma(m)) \subseteq \bigcup_{z \in m} S(y^v(z))$ for any message $m \in M\}$. Note that the voter never chooses an action that is not supported by any policy pairs in the observed message if $\gamma \in \Gamma$. We say that the outlet's strategy $\bar{\beta}$ is *simple* if it satisfies the following properties: (i) $S(\bar{\beta}(z)) \subseteq \{\{z\}, \bar{Z}_{12} \cup \bar{Z}_{21}\}$ for any $z \in Z$; and (ii) $\bar{\beta}(z) = z$ for any $z \in Z_0 \cup Z_{11} \cup Z_{22}$ and $\bar{Z}_{12} \cup \bar{Z}_{21}$ for any $z \in Z_{12} \cup Z_{21}$.[19] That is, the outlet fully discloses the information over the agreement regions $Z_0 \cup Z_{11} \cup Z_{22}$, and suppresses it over the disagreement regions $Z_{12} \cup Z_{21}$. Let B be the set of the simple strategies of the outlet. For the outlet's simple strategy $\bar{\beta} \in B$ and the voter's strategy $\gamma \in \Gamma$ where $\gamma(\bar{Z}_{12} \cup \bar{Z}_{21}) = (c, 1-c)$ with $c \in \{0, 1/2, 1\}$, define $\bar{\gamma}_c : Z \to \Delta(Y)$ by $\bar{\gamma}_c(z) \equiv \gamma(\bar{\beta}(z))$ for any $z \in Z$, which is called the *induced outcome of the news-reporting stage*. Define $\bar{\Gamma} \equiv \{\bar{\gamma}_c \in \Delta(Y)^Z |$ there exist $\bar{\beta} \in B$ and $\gamma \in \Gamma$ such that (i) $\bar{\gamma}_c = \gamma \circ \bar{\beta}$ and (ii) $\bar{\beta}$ is a best response to $\gamma\}$, which is the set of induced outcomes of the news-reporting stage that can be supported in some PBE. Hereafter, we restrict our attention to the *undominated simple equilibria* defined below.

Definition 2 ***Undominated simple equilibrium***
An undominated simple equilibrium (hereafter, USE) $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is a PBE satisfying the following conditions: (i) $\gamma^* \in \Gamma$; (ii) $\beta^* \in B$; and (iii) $(\alpha_1^*, \alpha_2^*) \in \Delta([0, b])^{*2}$.

Intuitively, the USE is a PBE that passes the dominance criterion under the restriction. Each condition can be justified as follows. Condition (i) is consistent with the certifiability of the policy pairs. Because the true policy pair is certainly included in the observed message, it seems reasonable to assume that the rational voter only chooses actions that can be optimal under some policy pair contained in the message. Although condition (ii) seems demanding, focusing on simple strategies is not loss of generality. That is, we can show that if we focus on a strong version of PBE where the outlet's equilibrium strategy is undominated with respect to Γ, then the set of equilibrium outcomes of the news-reporting stage should be $\bar{\Gamma}$. Finally, condition (iii) also seems an ad hoc restriction, but we can show that the set of undominated strategies under some restriction is $\Delta([0, b])^*$. It is worthwhile noticing that the dominance criterion has no bite in refinements without restrictions. Hence, when applying the dominance criterion to the candidates' strategies, we restrict our attention to set $\bar{\Gamma}$ of the induced outcomes of the news-reporting stage. As a result, the set of candidates' undominated strategies with this restriction is identical to $\Delta([0, b])^*$. All of the formal proofs for this justification are in Appendix B.4.

[19]There is a degree of freedom for behaviors when the observed policy pair is in regions $Z_{01} \cup Z_{02}$.

5.1.2 Characterization of the front-runner

Once we focus on the USEs, we can characterize the front-runner as the following proposition. This is a key observation for representing the degree of distortion.

Proposition 3 *Consider the manipulated news model.*

(i) Suppose that $b \leq r/2$. Then, there exists no USE with the front-runner.

(ii) Suppose that $b > r/2$. If there exists a USE with the front-runner, then it is candidate 1.

The main message of this proposition is that candidate 1 can be the front-runner but candidate 2 cannot. This implies that the voter is likely to choose the candidate whose ideological policy is more preferred by the media outlet. In other words, voting behavior is biased toward the direction that the outlet prefers.[20] It is worthwhile emphasizing that this phenomenon occurs even if the voter is sophisticated and completely recognizes the bias. Because the voter is fully rational, he chooses a candidate whose expected policy is less biased when the information is suppressed, that is, either candidate 1 is less biased or candidate 2 is less biased. The former scenario can be sustained in the USE, but the latter scenario cannot. Because information could be suppressed when candidates 1 and 2 are ideological and opportunistic types, respectively, the optimal behavior of the underdog is different in each scenario. As a result, this asymmetry appears.

On the one hand, the scenario where candidate 2 is the front-runner cannot be supported in equilibrium. Suppose, for example, that $b < |l|$. Note that if candidate 2 is the front-runner, then both candidates generally win for certain if the opponent is the ideological type.[21] That is, each candidate's decision is mainly based on the outcome when both are the opportunistic type. Because candidate 2 is the front-runner, he attempts to avoid policies chosen by the opportunistic-type candidate 1 for inducing policy divergence. Conversely, candidate 1, as the underdog, has an incentive to propose the policy chosen by the opportunistic-type candidate 2 for inducing policy convergence. That is, the structure of the policy-setting stage is like a zero-sum game in the sense that the front-runner's and the underdog's incentives are incompatible. As a result, we cannot construct a USE. Because a zero-sum game structure is unavoidable in this scenario, a USE where candidate 2 is the front-runner never exists.

On the other hand, a USE does exists where candidate 1 is the front-runner and when the preference bias is not small. Suppose, for example, that $r < b < |l|$. Like the previous scenario,

[20]This phenomenon is empirically supported. See, for example, DellaVigna and Kaplan (2007).

[21]Consider the scenario where $\theta_1 = I$ and $\theta_2 = O$. The winning probability for candidate 2 is less than 1 if $r \in S(\alpha_2)$; otherwise, he wins for certain in this scenario.

candidate 1 generically wins for certain if candidate 2 is the ideological type. However, the converse is not true; that is, candidate 2 could not win even though candidate 1 is the ideological type because the proposed policy pair is suppressed when the preference bias is not small. As a result, proposing a policy chosen by the opportunistic-type candidate 1 might not maximize candidate 2's winning probability. For instance, if candidate 1 adopts a mixed strategy that chooses many policies with equally likely, then proposing policy $x_2 = r$ is candidate 2's best response. Because candidate 2 should take account of the scenario where candidate 1 is the ideological type in his decision-making, the candidates' incentives can be compatible. In other words, the zero-sum game structure might not appear in this scenario.

5.2 Degree of distortion

Hereafter, we focus on the USEs, and characterize the extent to which the equilibrium outcomes are distorted. We measure the distortion in equilibrium outcomes by the voter's ex ante expected utility. The degree of distortion $D(e)$ in equilibrium $e = (\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is defined as follows:[22]

$$D(e) = \left| \sum_{z \in Z} \left(\sum_{m \in M} \left(\sum_{y \in Y} v(y, x) \Pr(y | \gamma^*(m)) \right) \Pr(m | \beta^*(z)) \right) \Pr(z | \alpha_1^*, \alpha_2^*) \right| - (1-p)^2 r. \quad (9)$$

Note that $D(e) \geq 0$ for any USE e, and it is normalized to be 0 under the $(0, 0)$ equilibrium.

First, we characterize the degree of distortion $D(e)$. Because of Proposition 3, we can assume that candidate 1 is the winner when the proposed policy pair is in the disagreement regions $\bar{Z}_{11} \cup \bar{Z}_{21}$ without loss of generality. Hence, the representation of $D(e)$ is given by the following theorem.

Theorem 2 *Consider the manipulated news model, and let $e = (\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ be a USE. Then:*

$$D(e) = \begin{cases} p \sum_{x_1 \in [0,b]} \alpha_1^*(x_1) x_1 + p(1-p) \sum_{x_2 \in [0,b]} \alpha_2^*(x_2) x_2 & \text{if } 0 < b \leq r/2, \\ p \sum_{x_1 \in [0,b]} \alpha_1^*(x_1) x_1 + p(1-p) \sum_{x_2 \in [-r+2b,b]} \alpha_2^*(x_2) x_2 & \text{if } r/2 < b \leq r, \\ p \sum_{x_1 \in [0,b]} \alpha_1^*(x_1) x_1 + p(1-p) r & \text{if } b > r. \end{cases} \quad (10)$$

As a corollary of this theorem, we can determine the lower and upper bounds of the degree of

[22]For simplification, we assume that $S(\beta^*(z))$ is countable for any $z \in Z$ in this representation.

distortion $D(e)$ as follows. There are the following subcases depending on the magnitude of the preference bias: (i) $0 < b \leq r/2$, (ii) $r/2 < b \leq r$, (iii) $r < b < |l|$, and (iv) $b \geq |l|$.[23]

Corollary 1 *Consider the manipulated news model. Then:*

$$\inf D(e) = \begin{cases} 0 & \text{if } 0 < b \leq r/2, \\ p(1-p)(-r+2b) & \text{if } r/2 < b \leq r, \\ p(1-p)r & \text{if } b > r. \end{cases} \tag{11}$$

$$\sup D(e) = \begin{cases} p(2-p)b & \text{if } 0 < b \leq r, \\ pb + p(1-p)r & \text{if } r < b < |l|, \\ p|l| + p(1-p)r & \text{if } b \geq |l|. \end{cases} \tag{12}$$

In Case (i), the upper and lower bounds of $D(e)$ can be supported by $(0,0)$ and (b,b) equilibria, respectively. Note that $\bar{Z}_{12}(\alpha_1,\alpha_2) \cup \bar{Z}_{21}(\alpha_1,\alpha_2) = \emptyset$ for $(\alpha_1,\alpha_2) = (0,0)$ and (b,b). Hence, these strategy profiles can be supported in equilibrium. Furthermore, by Theorem 2, it is straightforward that these equilibria are the least and the most distorted, respectively.

In Case (ii), the upper bound of $D(e)$ is identical to that in Case (i), that is, a (b,b) equilibrium exists, which maximizes the degree of distortion. On the other hand, the lower bound increases because a $(0,0)$ equilibrium never exists, as shown in Theorem 1. In this scenario, $\alpha_1 = 0$ still can be supported, but α_2 such that $S(\alpha_2) \cap [0, -r+2b.) \neq \emptyset$ is never supported in equilibrium. Intuitively, proposing policy $x_2 \in [0, -r+2b)$ is dominated by proposing policy $x_2 \in [-r+2b, b]$ because candidate 2 cannot be the front-runner as mentioned in Proposition 3. As a result, the lower bound is given by $\alpha_1 = 0$ and $\alpha_2 = -r+2b$.

In Cases (iii) and (iv), the lower bound is constant, such that the $(0,(1-p)r)$ equilibrium attains the minimum distortion. Once again, a $(0,0)$ equilibrium does not exist because candidate 2 becomes the front-runner in this scenario, which cannot be supported as shown in Proposition 3. To be an equilibrium, candidate 2's expected policy should be weakly more biased than that of candidate 1. The least bound of such biased policies is $x_2 = (1-p)r$. For the upper bound, a (b,b) equilibrium never exists in these cases, which contrasts with the previous cases. If $\alpha_1 = \alpha_2 = b$ is supported in equilibrium, then candidate 1 becomes the front-runner because $(r,b) \in Z_{12}$ and $Z_{21}(b,b) = \emptyset$. However, the front-runner proposes a policy chosen by the underdog, so candidate 1 has an incentive to deviate to strategy $\alpha_1' \in \Delta([0,b))^*$ to win for certain, which is a contradiction.

[23]See also Appendix C for figures in each case.

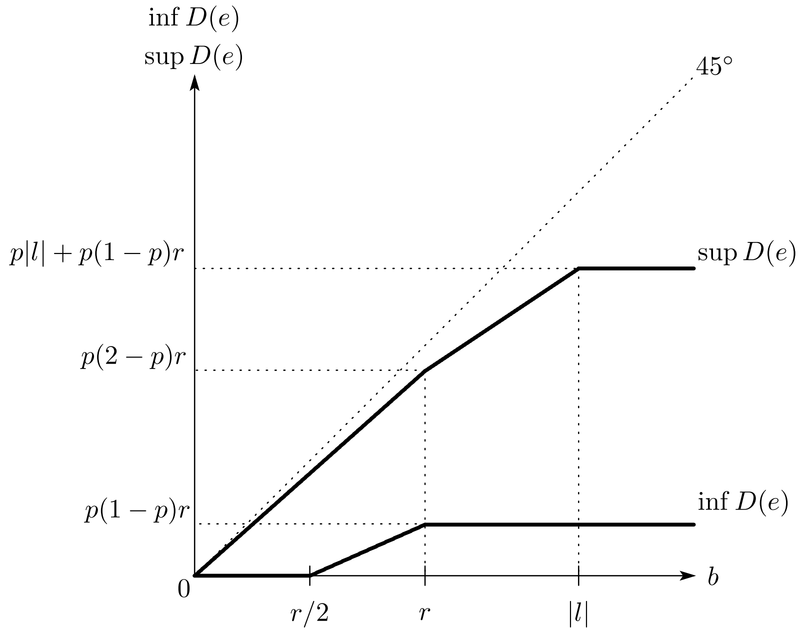

Figure 4: Corollary 1

As a result, the most distortion in Case (iii) is achieved by the $(b, pb + (1-p)r)$ equilibrium where there is no front-runner. In Case (iv), the expectation of the equilibrium policies is bound above by $|l|$. Therefore, the supremum of $D(e)$ is also bounded above by $p|l| + p(1-p)r$.

Corollary 1 is summarized in Figure 4, and there are two remarks to be mentioned. First, the distortion of the equilibrium outcomes becomes severer as the outlet becomes more biased. Note that both the lower and the upper bounds are weakly increasing in preference bias b, but these behaviors are different. The lower bound is constant in the bias except for $r/2 < b \leq r$. That is, the best scenario for the voter is being insensitive to the bias when it is either sufficiently small or sufficiently large. On the other hand, the worst scenario for the voter is being sensitive to the bias, that is, the upper bound is strictly increasing in b up to $|l|$. As a result, the difference between the upper and the lower bounds is also strictly increasing in the bias up to $b \leq |l|$. Thus, we can conclude that the equilibrium outcomes become more distorted in the sense that they not only increase in magnitude but also in variance.

Second, the impact of the competition between the candidates seems not to be negligible. As long as we focus on the USEs, the pooling equilibria are played in the news-reporting stage. However, the equilibrium outcomes are not constant. There is variance in the outcomes, which is a consequence of the policy-setting stage. In particular, by comparing the upper bound with the lower bound, we observe that competition between the candidates makes the voter worse off than the scenario where only the relevant information is lacking. A new question arises from

probability	proposed policy pair	media	winner	equilibrium policy
pq^2	$(0,0)$	discloses	1 or 2	0
$pq(1-q)$	$(0,r)$	suppresses	1 or 2	0 or r
$q(1-p)$	$(0,l)$	disclose	1	0
$pq(1-q)$	$(r,0)$	suppresses	1 or 2	0 or r
$p(1-q)^2$	(r,r)	discloses	1 or 2	r
$(1-q)(1-p)$	(r,l)	discloses	1	r

Table 2: Equilibrium outcomes in the mixed strategy equilibrium

this observation: to what extent does each distortion channel affect the total distortion, and how do these distortion channels interact with each other? In the next subsection, we discuss these questions by decomposing $D(e)$.

5.3 Decomposition of $D(e)$

We measure the direct and the indirect distortion as follows. Let $e = (\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ be a USE where the induced outcome of the news-reporting stage is $\bar{\gamma}_c$. Define the degree of direct distortion $d(e)$ of USE e by:

$$d(e) \equiv c \sum_{(x_1,x_2)\in Z_{21}} (x_1 - x_2)\Pr(x_1, x_2|\alpha_1^*, \alpha_2^*) + (1-c) \sum_{(x_1,x_2)\in Z_{12}} (x_2 - x_1)\Pr(x_1, x_2|\alpha_1^*, \alpha_2^*). \tag{13}$$

That is, the degree of direct distortion is measured by the loss of utility from the incorrect decision-making of the voter. Let $i(e)$ be the degree of indirect distortion of USE e defined by:

$$i(e) \equiv D(e) - d(e). \tag{14}$$

The indirect distortion is measured by the residual loss of utility, which comes from the candidates proposing policy other than 0.

For example, we revisit USE e^M characterized in Claim 1. The outcome of the mixed strategy equilibrium is summarized in Table 2. Then, the degree of distortion is $D(e^M) = (-p^2+2p-q)r > 0$. The direct distortion appears in the second and fourth rows of Table 2, that is, the voter chooses the unfavored candidate with positive probability. Hence, the degree of direct distortion is:

$$d(e^M) = \frac{1}{2}pq(1-q)(r-0) + \frac{1}{2}pq(1-q)(r-0) = pq(1-q)r > 0. \tag{15}$$

On the other hand, the indirect distortion appears in the other rows of Table 2. Because of the randomization by the candidates, policy r is more likely to be proposed. Hence, the degree of indirect distortion is:

$$i(e^M) = D(e^M) - d(e^M) = (-p^2 + 2p - q - pq + pq^2)r > 0. \tag{16}$$

The following theorem characterizes the degree of direct distortion $d(e)$. By combining with Theorem 2, we can decompose the degree of total distortion $D(e)$.

Theorem 3 *Consider the manipulated news model, and let* $e = (\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ *be a USE where the induced outcome of the news-reporting stage is* $\bar{\gamma}_c$.

(i) If $c = 1/2$*, then:*

$$d(e) = \begin{cases} p^2 \displaystyle\sum_{(x_1,x_2)\in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2 - x_1) & \text{if } b \le r; \\ p^2 \displaystyle\sum_{(x_1,x_2)\in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2 - x_1) + p(1-p) \sum_{x_2 \in (r,b]} \alpha_2^*(x_2)(x_2 - r) & \text{if } r > b. \end{cases} \tag{17}$$

(ii) If $c = 1$*, then:*

$$d(e) = \begin{cases} 0 & \text{if } r/2 < b \le r; \\ p^2 \displaystyle\sum_{x_1 \in (r,b]} \alpha_1^*(x_1)(x_1 - r) & \text{if } r < b < |l|; \\ p^2 \displaystyle\sum_{x_1 \in (r,b]} \alpha_1^*(x_1)(x_1 - r) + p(1-p) \sum_{x_1 \in (|l|,b]} \alpha_1^*(x_1)(x_1 - |l|) & \text{if } b \ge |l|. \end{cases} \tag{18}$$

Although a natural question arising from this decomposition is which distortion is primary, the answer is ambiguous. As demonstrated in the above example, the larger distortion is altered depending on the structure of the equilibrium. Notice that the indirect distortion is larger than the direct distortion in USE e^M if and only if either (i) $p \le 1/2$ or (ii) $p > 1/2$ and $q \le (2p + 1 - \sqrt{8p^3 - 12p^2 + 4p + 1})/4p$ holds.[24] Because there still exist multiple equilibria, even though we focus on the USEs, finding a general answer to the question is hopeless. Instead of comparing the distortion, we hereafter investigate the compatibility of the two distortion channels. That is, the

[24] The detail is available from the author upon the request.

question is, "Is minimizing the direct distortion always compatible with minimizing the indirect distortion?" The answer is negative, as shown in the following corollary. Define $e^- \in \arg\min d(e)$.

Corollary 2 *Consider the manipulated news model.*

(i) Suppose that $b > r/2$. Then, for any e^-, there exists $\varepsilon > 0$ such that $D(e^-) - \inf D(e) > \varepsilon$.

(ii) Suppose that $b \leq r$. Then, there exists e^- such that $D(e^-) = \sup D(e)$.

Corollary 2 means that if the outlet is sufficiently biased, then we have to give up minimizing the direct distortion in minimizing the total distortion. In particular, minimizing the direct distortion could maximize the total distortion when the bias is moderate, i.e., $r/2 < b \leq r$. That is, we can identify a tension between the direct and the indirect distortion. Intuitively, this incompatibility comes from the fact that the direct distortion becomes smaller as the proposed policies are more convergent, as shown in Theorem 3. In particular, if the candidates adopt a symmetric pure strategy, then the direct distortion disappears because the voter does not need information from the outlet. When the outlet is sufficiently biased, the candidates should propose a biased policy to minimize the direct distortion. However, choosing a less biased policy is necessary for minimizing the indirect distortion. This discrepancy is the origin of the incompatibility.

There are two remarks for Corollary 2. First, the cost of ignoring candidate competition is large in evaluating media manipulation. As mentioned in the literature review, most of the existing work omits competition. Hence, their evaluations are only based on the direct distortion. However, as demonstrated in Corollary 2, ignoring the indirect effect could mislead the evaluation. That is, the total distortion could be mitigated by allowing more direct distortion. Therefore, we emphasize the importance of including candidate competition when we analyze strategic media manipulation. Second, the tension between the direct and the indirect distortion seems prevalent in electoral competition with mass media. Chakraborty and Ghosh (2013) analyze a model where media outlets provide additional information instead of distorting information transmission, and show the tension between the loss of information (*information-aggregation effect*) and the agenda distortion (*policy drift effect*). That is, to mitigate the information-aggregation effect, the policy drift effect must be exaggerated. As a result, eliminating the information-aggregation effect makes the voter worse off when the outlet's bias is not small.[25] This result corresponds to Corollary 2. Because the two different models demonstrate the tension between the loss of information and the agenda distortion, this phenomenon seems prevalent in the context of electoral competition with mass media.

[25] See Proposition 7 of Chakraborty and Ghosh (2013).

6 Conclusion

This paper has investigated how and to what extent strategic mass media affects electoral outcomes by analyzing the manipulated news model where the candidates, the outlet, and the voter are rational, and their behaviors are endogenously determined. First, we have specified a mechanism that distorts equilibrium outcomes, that is, direct and indirect distortion. In the manipulated news model, the voter's incorrect decision-making is unavoidable, even though the voter is fully rational (direct distortion). Hence, direct appealing by the candidates is less attractive. Instead, the candidates have an incentive to indirectly appeal to the voter by taking account of media manipulation (indirect distortion). As a result, there exist multiple equilibria, but policy convergence to the voter's ideal policy cannot be supported in equilibrium when the outlet is sufficiently biased. Second, we have characterized the degree of distortion as measured by the voter's ex ante expected utility. We then have shown that the distortion in equilibrium outcomes becomes severer as the outlet becomes more biased. The decomposition of the total degree of distortion into direct and indirect distortion has demonstrated that there is a tension between the two distortion channels. That is, we have to give up minimizing the direct distortion in minimizing the total distortion. Hence, we emphasize the importance of the explicit modeling of competitions between the candidates because the evaluation of media manipulation could otherwise be misleading.

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Appendix A: Proofs

A.1 Preliminaries

Notice that the voter's equilibrium strategy γ^* and posterior belief $\mathcal{P}^*(\cdot|m)$ for on-the-equilibrium-path message m are the same structure in any equilibrium. Given equilibrium strategies α_1^*, α_2^* and β^*, the posterior belief for on-the-equilibrium-path message m is determined by Bayes' rule as follows:

$$\mathcal{P}^*(z|m) = \begin{cases} \dfrac{\Pr(z|\alpha_1^*, \alpha_2^*)}{\displaystyle\sum_{z' \in Z(\alpha_1^*, \alpha_2^*) \cap \beta^{-1}(m)} \Pr(z'|\alpha_1^*, \alpha_2^*)} & \text{if } z \in Z(\alpha_1^*, \alpha_2^*), \\ 0 & \text{otherwise}, \end{cases} \tag{A.1}$$

where $\beta^{-1}(m) \equiv \{z \in Z | m \in S(\beta^*(z))\}$. Given posterior belief $\mathcal{P}^*$, the voter's undominated equilibrium strategy is uniquely determined as follows:

$$\gamma^*(m) = \begin{cases} (1,0) & \text{if } \mathbb{E}[|x_1||m] < \mathbb{E}[|x_2||m], \\ (1/2, 1/2) & \text{if } \mathbb{E}[|x_1||m] = \mathbb{E}[|x_2||m], \\ (0,1) & \text{if } \mathbb{E}[|x_1||m] > \mathbb{E}[|x_2||m]. \end{cases} \tag{A.2}$$

Hereafter, to avoid trivial repetition, we omit the description of the voter' equilibrium strategy and on-the-equilibrium-path beliefs. Furthermore, off-the-equilibrium-path beliefs are either conditional probability or density functions depending on the cardinality of observed messages. Hence, to simplify the representation, we characterize off-the-equilibrium-path beliefs by their supports. Any probability distribution with specified support is compatible with the equilibrium strategies.

A.2 Proof of Proposition 2

(i) (Necessity) Suppose, in contrast, that there exists equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $\gamma^*(\beta^*(z)) = y^v(z)$ for any $z \in Z(\alpha_1^*, \alpha_2^*)$ when either (1) $Z_{02}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ or (2) $Z_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ holds.[26] First, consider Case (1). Let $z \in Z_{02}(\alpha_1^*, \alpha_2^*)$ and $z' \in Z_{21}(\alpha_1^*, \alpha_2^*)$, and then $\gamma^*(\beta^*(z)) = (1/2, 1/2)$ and $\gamma^*(\beta^*(z')) = (0,1)$ holds. To be incentive compatible, the following conditions must be satisfied: (a) $\gamma_1^*(m) \geq 1/2$ for any message $m \in M(z)$, and (b) $\gamma_1^*(m') \leq 0$ for any message $m' \in M(z')$; otherwise, the outlet that observes either policy pair z or z' has an incentive to deviate from the equilibrium strategy. However, there is no incentive

[26]To simplify the notation, we, hereafter, use this representation even when the outlet adopt mixed strategies.

compatible reaction to message $m = \{z, z'\} \in M(z) \cap M(z')$, which is a contradiction. We can derive a contradiction in Case (2) by the similar argument.

(Sufficiency) Consider Case (1). Without loss of generality, we assume that $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$. We then show that the following is a desired PBE in the news-reporting stage:

$$\beta^*(z) = \begin{cases} \bar{Z}_{12} \cup Z_{21} & \text{if } z \in \bar{Z}_{12} \cup Z_{21}, \\ z & \text{otherwise.} \end{cases} \tag{A.3}$$

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} m \cap Z_{21} & \text{if } m \cap Z_{21} \neq \emptyset, \\ m \cap Z_{01} & \text{if } m \cap Z_{21} = \emptyset \text{ and } m \cap Z_{01} \neq \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.4}$$

Note that $\gamma^*(\bar{Z}_{12} \cup Z_{21}) = (0, 1)$ and $\gamma^*(\{z\}) = y^v(z)$ given posterior $\mathcal{P}^*$.[27] We then check the optimality of the outlet's equilibrium strategy β^* given the voter's equilibrium strategy γ^*. If $z \notin \bar{Z}_{21}$, then the outlet has no incentive to deviate from $\beta^*(z)$ because it induces her preferred policy for certain. If $z \in \bar{Z}_{21}$, then the outlet also has no incentive to deviate because she cannot induce her preferred policy with probability more than $1/2$. Thus, β^* is the outlet's best response. Finally, note that if $z \in Z(\alpha_1^*, \alpha_2^*) \backslash Z_{21}(\alpha_1^*, \alpha_2^*)$, then $\gamma^*(\beta^*(z)) = y^v(z)$ holds because the outlet fully discloses the information. If $z \in Z_{21}(\alpha_1^*, \alpha_2^*)$, then $\gamma^*(\beta^*(z)) = (0, 1) = y^v(z)$ holds. Therefore, this is a desired PBE.

Consider Case (2). We show that the following is a desired PBE in the news-reporting stage:

$$\beta^*(z) = \begin{cases} \bar{Z}_{12} \cup \bar{Z}_{21} & \text{if } z \in \bar{Z}_{12} \cup \bar{Z}_{21}, \\ z & \text{otherwise.} \end{cases} \tag{A.5}$$

For off-the-equilibrium-path message m such that $m \cap \bar{Z}_{12} \neq \emptyset$ and $m \cap \bar{Z}_{21} \neq \emptyset$:

$$\mathcal{P}^*(\tilde{z}|m) = \begin{cases} a & \text{if } \tilde{z} = z, \\ 1 - a & \text{if } \tilde{z} = z', \\ 0 & \text{otherwise,} \end{cases} \tag{A.6}$$

where $z = (x_1, x_2) \in m \cap \bar{Z}_{12}$, $z' = (x'_1, x'_2) \in m \cap \bar{Z}_{21}$, and $a \equiv (|x'_1| - |x'_2|)/(|x'_1| - |x'_2| + |x_2| - |x_1|)$ whenever it is well-defined; otherwise, a is any value in $[0, 1]$.[28] For other off-the-equilibrium-path

[27] Message $m = \bar{Z}_{12} \cup Z_{21}$ is sent on the equilibrium path if $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$. Note that, in this scenario, $S(\mathcal{P}^*(\cdot|\bar{Z}_{12} \cup Z_{21})) \subset Z_{21}$ holds on the equilibrium path because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \emptyset$.

[28] Because $z \in \bar{Z}_{12}$ and $z' \in \bar{Z}_{21}$, then $|x_1| \leq |x_2|$ and $|x'_1| \geq |x'_2|$ hold. That is, $a \in [0, 1]$, and it is well-defined unless $|x_1| = |x_2|$ and $|x'_1| = |x'_2|$.

messages:

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} m \cap \bar{Z}_{12} & \text{if } m \cap \bar{Z}_{12} \neq \emptyset \text{ and } m \cap \bar{Z}_{21} = \emptyset, \\ m \cap \bar{Z}_{21} & \text{if } m \cap \bar{Z}_{21} \neq \emptyset \text{ and } m \cap \bar{Z}_{12} = \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.7}$$

Note that $\gamma^*(m) = (1/2, 1/2)$ for any message m such that $m \cap \bar{Z}_{12} \neq \emptyset$ and $m \cap \bar{Z}_{21} \neq \emptyset$.[29] We then check the optimality of the outlet's equilibrium strategy β^* given the voter's equilibrium strategy γ^*. If $z \notin \bar{Z}_{12} \cup \bar{Z}_{21}$, then the outlet has no incentive to deviate from $\beta^*(z)$ because it induces her preferred policy for certain. If $z \in \bar{Z}_{12} \cup \bar{Z}_{21}$, then the outlet also has no incentive to deviate because any available message induces her preferred policy with probability at most 1/2. Finally, note that if $z \in Z(\alpha_1^*, \alpha_2^*) \backslash (Z_{02}(\alpha_1^*, \alpha_2^*) \cup Z_{01}(\alpha_1^*, \alpha_2^*))$, then $\gamma^*(\beta^*(z)) = y^v(z)$ holds because the outlet fully discloses the information. If $z \in Z_{02}(\alpha_1^*, \alpha_2^*) \cup Z_{01}(\alpha_1^*, \alpha_2^*)$, then $\gamma^*(\beta^*(z)) = (1/2, 1/2) = y^v(z)$ holds. Therefore, it is a desired PBE.

(ii) It is a corollary of Proposition 2-(i). Hence, the proof is omitted.[30] ■

A.3 Proof of Theorem 1

A.3.1 Preliminaries

Let $\mu_i(z)$ be candidate i's winning probability under policy policy pair z, and $\sigma_i(\alpha_1, \alpha_2)$ be candidate i's winning probability under strategies α_1 and α_2.[31] First, we show the following lemma.

Lemma 1 *In any equilibrium,* $\gamma^*(\beta^*(z)) = y^v(z)$ *for any* $z \in Z_{11} \cup Z_{22}$.

Proof of Lemma 1. Suppose, in contrast, that there exists an equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $\gamma^*(\beta^*(z)) \neq y^v(z)$ for some policy pair $z \in Z_{11} \cup Z_{22}$. However, the outlet observing policy pair z also prefers action $y = y^v(z)$ to any other actions, and that preferred action can be induced by sending message $m = z$ because of Requirement 1, That is, the outlet has an incentive to deviate, which is a contradiction. ■

[29]Because $Z_{02}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{01}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is sent on the equilibrium path. Notice that any policy pair in regions $Z_{02} \cup Z_{01}$ is indifferent for the voter. Thus, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$ holds.

[30]The detail is available on Appendix B.

[31]Although the winning probabilities depends on the outlet's and the voter's strategies, we omit the explicit description of these factors for saving notation if it is not confusing.

A.3.2 Proof of Theorem 1

(Necessity) Suppose, in contrast, that there exists a $(0,0)$ equilibrium when either (i) $b > r/2$ with $b \neq r$, or (ii) $b = r$ and $p > 1/2$ holds. Note that, in both scenarios, $Z_0(0,0) = \{(0,0)\}$, $Z_{11}(0,0) = \{(0,l),(r,l)\}$, $Z_{21}(0,0) = \{(r,0)\}$, and then $Z(0,0) = Z_0(0,0) \cup Z_{11}(0,0) \cup Z_{21}(0,0)$. First, suppose that $b > r/2$ with $b \neq r$ (Case (i)). By Requirement 2, there are the following three cases to be considered.

Case (i)-1: Suppose that $\gamma^*(\beta^*(r,0)) = (1,0)$. Note that $\sigma_1(0,0) = 1 - p/2$. If candidate 1 deviates to strategy $\alpha_1 = r$, then his winning probability is $\sigma_1(r,0) = 1$ by Lemma 1. That is, candidate 1 has an incentive to deviate to $\alpha_1 = r$, which is a contradiction.

Case (i)-2: Suppose that $\gamma^*(\beta^*(r,0)) = (0,1)$. To support this equilibrium, $\gamma^*(m) = (0,1)$ should hold for any message $m \in M(r,0)$; otherwise, the outlet observing policy pair $z = (r,0)$ deviates to such a message. Hence, $\gamma^*(\beta^*(z)) = (0,1)$ must hold for any policy pair $z \in Z_{12}$; otherwise, the outlet observing policy pairs lying in region Z_{12} has an incentive to deviate to a message containing policy pair $z = (r,0)$. Note that $\sigma_2(0,0) = 1 - p/2$. There are further two cases to be considered depending on the preference bias.

Case (i)-2-a: Suppose that $r/2 < b < r$. If candidate 2 deviates to $\alpha_2 = b$, then the realized policy pair under this deviation is either $z = (0,b) \in Z_{12}$ or $(r,b) \in Z_{22}$. Hence, by Lemma 1, candidate 2's winning probability from strategy $\alpha_2 = b$ is $\sigma_2(0,b) = 1$. That is, candidate 2 has an incentive to deviate, which is a contradiction.

Case (i)-2-b: Suppose that $b > r$. If candidate 2 deviates to $\alpha_2 = x'_2 \in (r,b)$, then the realized policy pair under this deviation is either $z = (0,x'_2) \in Z_{12}$ or $(r,x'_2) \in Z_{12}$. Hence, candidate 2's winning probability from strategy $\alpha_2 = x'_2$ is $\sigma_2(0,x'_2) = 1$. That is, candidate 2 has an incentive to deviate, which is a contradiction.

Case (i)-3: Suppose that $\gamma^*(\beta^*(r,0)) = (1/2,1/2)$. To support this equilibrium, the outlet observing policy pair $z = (r,0)$ must be pooling with the outlet observing policy pair either $z = (0,l)$ or (r,l); otherwise, $\gamma^*(\beta^*(r,0)) = (0,1)$ holds. That is, either $\gamma^*(\beta^*(0,l)) = (1/2,1/2)$ or $\gamma^*(\beta^*(r,l)) = (1/2,1/2)$ holds. However, by Lemma 1, this is impossible, which is a contradiction.

Case (ii): Next, we suppose that $b = r$ and $p > 1/2$. Likewise, there are the three cases to be checked dependent on the voter's response to message $m = \beta^*(r,0)$. If either $\gamma^*(\beta^*(r,0)) = (1,0)$ or $(1/2,1/2)$, then we can derive contradictions by the same argument used in Case (i)-1 and (i)-3. Hence, it remains to check the scenario where $\gamma^*(\beta^*(r,0)) = (0,1)$. To support this equilibrium, $\gamma^*(m) = (0,1)$ should hold for any message $m \in M(r,0)$; otherwise, the outlet observing policy pair $z = (r, 0)$ deviates. Hence, it implies that $\gamma^*(\beta^*(z)) = (0, 1)$ should hold for any policy pair $z \in \bar{Z}_{12}$; otherwise, the outlet deviates to send a message containing $(r, 0)$. Now, candidate 2's winning probabilities from strategies $\alpha_2 = 0$ and r are $\sigma_2(0, 0) = 1 - p/2$ and $\sigma_2(0, r) = (1 + p)/2$, respectively. However, because $p > 1/2$, $\sigma_2(0, r) > \sigma_2(0, 0)$. That is, candidate 2 has an incentive to deviate, which is a contradiction.

(Sufficiency) **Case (i):** Suppose that $b \leq r/2$. Note that $Z_0(0, 0) = \{(0, 0)\}$, $Z_{11}(0, 0) = \{(0, l), (r, l)\}$, $Z_{22} = \{(r, 0)\}$ and $Z(0, 0) = Z_0(0, 0) \cup Z_{11}(0, 0) \cup Z_{22}(0, 0)$. We then show that there exists a $(0, 0)$ equilibrium supported by strategy β^* and off-the-equilibrium-path beliefs $\mathcal{P}^*$ given by (A.5), (A.6) and (A.7), respectively. First, it is obvious that γ^* is optimal given belief $\mathcal{P}^*$. It is worthwhile to remark that $\gamma^*(m) = (1/2, 1/2)$ holds for any message m such that $m \cap \bar{Z}_{12} \neq \emptyset$ and $m \cap \bar{Z}_{21} \neq \emptyset$. Second, we can show that the optimality of the outlet's strategy β^* by the similar argument used in the proof of Proposition 2.

Third, we show the optimality of the candidates' strategies α_1^* and α_2^* given the others' strategies. Note that candidate 1's winning probability from strategy $\alpha_1 = 0$ is $\sigma_1(0, 0) = 1 - p/2$. It is then sufficient to show that $\sigma_1(\alpha_1', 0) \leq 1 - p/2$ for any strategy $\alpha_1' \in \Delta(X)^*$. Candidate 1's winning probabilities given policies $x_2 = 0$ and l are $\mu_1(x_1, 0) \leq 1/2$ and $\mu_1(x_1, l) \leq 1$ for any $x_1 \in X$ because possible policy pairs lie in regions $Z_0 \cup Z_{22} \cup Z_{21}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_1' \in \Delta(X)^*$:

$$\sigma_1(\alpha_1', 0) \leq \frac{1}{2}p + (1 - p) = 1 - \frac{1}{2}p. \tag{A.8}$$

That is, candidate 1 has no incentive to deviate. Likewise, note that candidate 2's winning probability from strategy $\alpha_2 = 0$ is $\sigma_2(0, 0) = 1 - p/2$. It is then sufficient to show that $\sigma_2(0, \alpha_2') \leq 1 - p/2$ for any strategy $\alpha_2' \in \Delta(X)^*$. Candidate 2's winning probabilities given policies $x_1 = 0$ and r are $\mu_2(0, x_2) \leq 1/2$ and $\mu_2(r, x_2) \leq 1$ because possible policy pairs lie in regions $Z_0 \cup Z_{11} \cup Z_{12}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_2' \in \Delta(X)^*$:

$$\sigma_2(0, \alpha_2') \leq \frac{1}{2}p + (1 - p) = 1 - \frac{1}{2}p. \tag{A.9}$$

That is, candidate 2 has no incentive to deviate. Finally, because $\bar{Z}_{12}(0, 0) = \bar{Z}_{21}(0, 0) = \emptyset$, only the fully disclosure messages are used on the equilibrium path. That is, $\mathcal{P}^*$ is consistent with Bayes' rule. Thus, it is a PBE.

<u>**Case (ii):**</u> Suppose that $b = r$ and $p \leq 1/2$. We show that the following is a PBE: $\alpha_1^* = \alpha_2^* = 0$;

$$\beta^*(z) = \begin{cases} z & \text{if } z \in Z_0 \cup Z_{11} \cup Z_{22} \cup \{(r,0)\}, \\ \{z,(r,0)\} & \text{if } z \in \bar{Z}_{12}, \\ \{z,z'\} & \text{if } z \in \bar{Z}_{21} \backslash \{(r,0)\} \text{ where } z = (x_1,x_2) \text{ and } z' = (x_2,x_1) \in \bar{Z}_{12}; \end{cases} \tag{A.10}$$

for off-the-equilibrium-path message $m' \equiv \{z, z'\}$ with $z = (x_1, x_2) \in \bar{Z}_{21}$ and $z' = (x_2, x_1) \in \bar{Z}_{12}$:

$$\mathcal{P}^*(\tilde{z}|m') = \begin{cases} \frac{1}{2} & \text{if } \tilde{z} = z \text{ or } z', \\ 0 & \text{otherwise;} \end{cases} \tag{A.11}$$

for other off-the-equilibrium-path messages:

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} \{(r,0)\} & \text{if } m \in M(r,0), \\ m \cap \bar{Z}_{21} & \text{if } m \notin M(r,0), m \neq m', \text{ and } m \cap \bar{Z}_{21} \neq \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.12}$$

Note that $\gamma^*(m) = (0,1)$ for any message $m \in M(r,0)$, and $\gamma^*(m') = (1/2,1/2)$.

First, we show that the outlet never deviates from strategy β^*. It is obvious that the outlet observing a policy pair in region $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{12}$ has no incentive to deviate because following β^* induces her more preferred policy for certain. For the outlet observing a policy pair $z \in \bar{Z}_{21} \backslash \{(r,0)\}$, any available message cannot induce her preferred policy with probability more than 1/2. It is obvious that the outlet observing policy pair $z = (r,0)$ has no incentive to deviate because any message $m \in M(r,0)$ induces $\gamma^*(m) = (0,1)$. Thus, the outlet has no incentive to deviate from β^*.

Second, we show that candidate 1 never deviates from strategy α_1^*. Note that candidate 1's winning probability from strategy $\alpha_1 = 0$ is $\sigma_1(0,0) = 1 - p/2$. It is then sufficient to show that $\sigma_1(\alpha_1', 0) \leq 1 - p/2$ for any strategy $\alpha_1' \in \Delta(X)^*$. Candidate 1's winning probabilities given policies $x_2 = 0$ and l are $\mu_1(x_1, 0) \leq 1/2$ and $\mu_1(x_1, l) \leq 1$ for any $x_1 \in X$ because policy pair lie in regions $Z_0 \cup Z_{22} \cup Z_{21}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_1' \in \Delta(X)^*$:

$$\sigma_1(\alpha_1', 0) \leq \frac{1}{2}p + (1-p) = 1 - \frac{1}{2}p. \tag{A.13}$$

That is, candidate 1 has no incentive to deviate.

Third, we show that candidate 2 never deviates from strategy α_2^*. Note that candidate 2's winning probability from strategy $\alpha_2 = 0$ is $\sigma_2(0,0) = 1 - p/2$. For any policy $x_2 \in X$, possible policy pairs lie in regions $Z_0 \cup Z_{11} \cup Z_{12}$ if $x_1 = 0$, and regions $Z_0 \cup Z_{11} \cup \bar{Z}_{21}$ if $x_1 = r$. Hence, candidate 2's winning probabilities are determined as follows:

$$\mu_2(0,x_2)=\begin{cases}0 & \text{if } x_2\in[x^-,0)\cup[2b,x^+],\\ \frac{1}{2} & \text{if } x_2=0,\\ 1 & \text{if } x_2\in(0,2b);\end{cases} \tag{A.14}$$

$$\mu_2(r,x_2)=\begin{cases}0 & \text{if } x_2\in[x^-,-r)\cup(r,x^+],\\ \frac{1}{2} & \text{if } x_2\in[-r,r]\backslash\{0\},\\ 1 & \text{if } x_2=0.\end{cases} \tag{A.15}$$

That is, the maximum winning probability is either $1-p/2$ induced by strategy $\alpha_2=0$ or $(1+p)/2$ induced by strategy $\alpha_2\in(0,r]$. Because $p\leq 1/2$, candidate 2 has no incentive to deviate from α_2^*. Finally, it is obvious that belief $\mathcal{P}^*$ is consistent with Bayes' rule. Thus, it is a PBE. ■

A.4 Proof of Proposition 3

A.4.1 Preliminaries

First, define the following notation. Let $U_i:\Delta(X)^{*2}\times\bar{\Gamma}\to\mathbb{R}$ be opportunistic-type candidate i's expected utility when the induced outcome in the news-reporting stage is $\bar{\gamma}_c$, which is defined by:

$$U_i(\alpha_i,\alpha_j,\bar{\gamma}_c)\equiv\sum_{x_i\in S(\alpha_i)}\left(\sum_{x_j\in X}\Pr(y_i|\bar{\gamma}_c(x_i,x_j))\Pr(x_j|\alpha_j)\right)\Pr(x_i|\alpha_i,\theta_i=O). \tag{A.16}$$

Define $Z_{00}\equiv\{z\in Z|v(z,y_1)=v(z,y_2) \text{ and } w(z,y_1)=w(z,y_2)\}$.

A.4.2 Proof of Proposition 3

(i) Suppose, in contrast, that there exists USE $(\alpha_1^*,\alpha_2^*,\beta^*,\gamma^*;\mathcal{P}^*)$ such that the induced outcome of the news-reporting stage is $\bar{\gamma}_1$ when $b<r/2$. Hence:

$$U_1(\alpha_1^*,\alpha_2^*,\bar{\gamma}_1)=1-\frac{1}{2}\sum_{z\in Z_{00}}\Pr(z|\alpha_1^*,\alpha_2^*,\theta_1=O), \tag{A.17}$$

$$U_2(\alpha_1^*,\alpha_2^*,\bar{\gamma}_1)=\frac{1}{2}\sum_{z\in Z_{00}}\Pr(z|\alpha_1^*,\alpha_2^*,\theta_2=O)+(1-p). \tag{A.18}$$

To hold this equilibrium, $\sum_{z\in Z_{00}}\Pr(z|\alpha_1^*,\alpha_2^*,\theta_1=O)=0$ should hold; otherwise, candidate 1 deviates to strategy α_1' such that $S(\alpha_1')\cap S(\alpha_2^*)=\emptyset$. That is, $\sum_{z\in Z_{00}}\Pr(z|\alpha_1^*,\alpha_2^*,\theta_1=O,\theta_2=O)=0$. Furthermore, because $b\leq r/2$, $\sum_{z\in Z_{00}}\Pr(z|\alpha_1^*,\alpha_2^*,\theta_1=I,\theta_2=O)=0$. Thus, $U_2(\alpha_1^*,\alpha_2^*,\bar{\gamma}_1)=$

$1-p$. However, if candidate 2 deviates to strategy $\alpha_2 = \alpha_1^*$, then $U_2(\alpha_1^*, \alpha_1^*, \bar{\gamma}_1) > U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1)$ holds, which is a contradiction. Likewise, we can derive a contradiction when the induced outcome of the news-reporting stage is $\bar{\gamma}_0$. In other words, there exists no front-runner in this scenario.

(ii) Suppose, in contrast, that there exists UCE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ where the induced outcome of the news-reporting stage is $\bar{\gamma}_0$ when $b > r/2$.

Case (i): Suppose that $r/2 < b < |l|$. For any $\alpha_1, \alpha_2 \in \Delta([0,b])^*$:

$$U_1(\alpha_1, \alpha_2^*, \bar{\gamma}_0) = \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1, \alpha_2^*, \theta_1 = O) + (1-p). \tag{A.19}$$

$$U_2(\alpha_1^*, \alpha_2, \bar{\gamma}_0) = 1 - \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2, \theta_2 = O). \tag{A.20}$$

Because α_1^* is an equilibrium strategy, $U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \geq U_1(\alpha_1, \alpha_2^*, \bar{\gamma}_0)$ holds for any $\alpha_1 \in \Delta([0,b])^*$. That is, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) > 0$; otherwise, candidate 1 has an incentive to deviate to $\alpha_1 = \alpha_2^*$. Thus, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*) > 0$ holds. Note that:

$$\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*) = p \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) + (1-p) \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = I). \tag{A.21}$$

Because $\alpha_1^* \in \Delta([0,b])^*$, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) > 0$ should hold. In other words, either $S(\alpha_1^*) \cap S(\alpha_2^*) \neq \emptyset$ or $r \in S(\alpha_2^*)$ holds. Now, suppose that candidate 2 deviates to strategy α_2' such that $(S(\alpha_1^*) \cup \{r\}) \cap S(\alpha_2') = \emptyset$.[32] By Construction, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2', \theta_2 = O) = 0$. Hence, $U_2(\alpha_1^*, \alpha_2', \bar{\gamma}_0) = 1 > U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0)$. This means that candidate 2 has an incentive to deviate, which is a contradiction.

Case (ii): Suppose that $b \geq |l|$. For any $\alpha_2 \in \Delta([0,b])^*$, $U_2(\alpha_1^*, \alpha_2, \bar{\gamma}_0)$ is given by (A.20). Because $U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) \geq U_2(\alpha_1^*, \alpha_2, \bar{\gamma}_0)$ must hold for any $\alpha_2 \in \Delta([0,b])^*$, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) = 0$ should hold; otherwise, candidate 2 deviates to strategy α_2' such that $(S(\alpha_1^*) \cup \{r\}) \cap S(\alpha_2') = \emptyset$. Hence, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) = 0$ holds. That is:

$$\begin{aligned} U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) &= \frac{p}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) + \frac{1-p}{2} \alpha_1^*(|l|) + (1-p) \sum_{x_1 \in [0,|l|)} \alpha_1^*(x_1) \\ &= \frac{1-p}{2} \alpha_1^*(|l|) + (1-p) \sum_{x_1 \in [0,|l|)} \alpha_1(x_1)^*. \end{aligned} \tag{A.22}$$

Because $U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) \geq U_1(\alpha_1, \alpha_2^*, \bar{\gamma}_0)$ holds for any $\alpha_1 \in \Delta([0,b])^*$, $S(\alpha_1^*) \subseteq [0, |l|)$ must hold. In

[32] Because we restrict our attention to the scenario where any mixed strategies have finite supports, such α_2' always exists.

other words, $U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0) = 1-p$. Now, suppose, in contrast, that there exists $x_2 \in S(\alpha_2^*) \cap [0, |l|)$. Because $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) = 0$, $x_2 \notin S(\alpha_1^*)$. If candidate 1 deviates to strategy $\alpha_1 = x_2$ in this scenario, then his expected utility after this deviation is $U_1(x_2, \alpha_2^*, \bar{\gamma}) = p\alpha_2^*(x_2)/2 + 1 - p > 1 - p = U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_0)$, which is a contradiction. Thus, we can say that $S(\alpha_2^*) \subseteq [|l|, b]$ must hold. However, given such α_1^* and α_2^*, $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$. Hence, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1, 0)$ is the voter's best response supported by the consistent belief. That is, the induced outcome in the news-reporting stage should be $\bar{\gamma}_1$, which is a contradiction. Therefore, if there exists a USE with the front-runner, then the front-runner should be candidate 1. ■

A.5 Proof of Theorem 2

A.5.1 Key lemmas

Lemma 2 *Suppose that $r/2 < b < r$. Then, there exists no USE such that $S(\alpha_2^*) \cap [0, -r + 2b) \neq \emptyset$.*

Proof. Suppose, in contrast, that there exists USE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $S(\alpha_2^*) \cap [0, -r+2b) \neq \emptyset$ when $2/r < b < r$. By Proposition 3-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$.

Case (i): Suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_1$. Hence:

$$U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O). \tag{A.23}$$

Now, we show that $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) > 0$. Suppose, in contrast, that $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) = 0$. Hence, by the hypothesis and $(x_1, x_2) \notin Z_{22}$ for any $x_i \in S(\alpha_i^*)$:

$$\begin{aligned} U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) &= \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) + \sum_{z \in Z_{22}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_2 = O) \\ &= (1-p) \left(\frac{1}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = I, \theta_2 = O) + \sum_{z \in Z_{22}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = I, \theta_2 = O) \right) \\ &= (1-p) \sum_{x_2 \in [-r+2b, b]} \alpha_2^*(x_2). \end{aligned} \tag{A.24}$$

Because $U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \geq U_2(\alpha_1^*, \alpha_2, \bar{\gamma}_1)$ for any $\alpha_2 \in \Delta([0, b])^*$, $S(\alpha_2^*) \subseteq [-r + 2b, b]$ should hold, which is a contradiction to $S(\alpha_2^*) \cap [0, -r + 2b) \neq \emptyset$. Thus, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) > 0$ holds. That is, $U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_c) < 1$. However, if candidate 1 deviates to strategy α_1' such that $S(\alpha_1') \cap S(\alpha_2^*) = \emptyset$, then $U_1(\alpha_1', \alpha_2^*, \bar{\gamma}_1) = 1$, which is a contradiction.

Case (ii): Suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. Hence:

$$U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = \frac{1}{2}\left(p + (1-p)\sum_{x_2\in[0,-r+2b)} \alpha_2^*(x_2)\right) + (1-p)\sum_{x_2\in[-r+2b,b]} \alpha_2^*(x_2). \tag{A.25}$$

By the hypothesis, $\sum_{x_2\in[0,-r+2b)} \alpha_2^*(x_2) > 0$ holds. Now, if candidate 2 deviates to strategy $\alpha_2 = -r + 2b$, then $U_2(\alpha_1^*, -r+2b, \bar{\gamma}_{\frac{1}{2}}) = p/2 + (1-p) > U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{\frac{1}{2}})$. Therefore, candidate 2 has an incentive to deviate, which is a contradiction. ■

Lemma 3 *Suppose that $b \geq |l|$. If there exists a USE where $S(\alpha_1^*) \cap [|l|, b] \neq \emptyset$, then the induced outcome in the news-reporting stage is $\bar{\gamma}_1$.*

Proof. Suppose, in contrast, that there exists a USE in which $S(\alpha_1^*) \cap [|l|, b] \neq \emptyset$ and the induced outcome of the news-reporting stage is not $\bar{\gamma}_1$. By Proposition 3-(ii), the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. Hence:

$$U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = \frac{1}{2}p + (1-p)\left(\sum_{x_1\in[0,|l|)} \alpha_1^*(x_1)x_1 + \frac{1}{2}\sum_{x_1\in[|l|,b]} \alpha_1^*(x_1)x_1\right). \tag{A.26}$$

Because $S(\alpha_1^*) \cap [|l|, b] \neq \emptyset$, $\sum_{x_1\in[|l|,b]} \alpha_1^*(x_1)x_1 > 0$ holds. However, if candidate 1 deviates to strategy α_1' defined by:

$$\alpha_1'(x_1) \equiv \begin{cases} \alpha_1^*(0) + \sum_{x_1\in[|l|,b]} \alpha_1^*(x_1) & \text{if } x_1 = 0, \\ \alpha_1^*(x_1) & \text{if } x_1 \in (0, |l|), \\ 0 & \text{otherwise.} \end{cases} \tag{A.27}$$

Then, $U_1(\alpha_1', \alpha_2, \bar{\gamma}_{1/2}) = 1 - p/2 > U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2})$. Thus, candidate 1 has an incentive to deviate, which is a contradiction. ■

A.5.2 Proof of Theorem 2

Case (i): Suppose that $0 < b \leq r/2$. Note that if exactly one of the candidates is the ideological type, then the winner is the opportunistic-type candidate. Hence, it is sufficient to determine the conditional equilibrium outcome given that both candidates are opportunistic type. By Proposition 3-(i), the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, then it should be that $\alpha_1^* = \alpha_2^* = x \in [0, b]$. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then $\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 = \sum_{x_2\in[0,b]} \alpha_2^*(x_2)x_2$ should hold because of the consistency of belief $\mathcal{P}^*$. That is, the conditional equilibrium outcome given that the both candidates are the opportunistic type is given by $\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1$. Therefore:

$$
\begin{aligned}
D(e) &= p^2 \sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 + p(1-p)\sum_{x_2\in[0,b]} \alpha_2^*(x_2)x_2 + p(1-p)\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 \\
&= p\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 + p(1-p)\sum_{x_2\in[0,b]} \alpha_2^*(x_2)x_2. \qquad \text{(A.28)}
\end{aligned}
$$

Case (ii): Suppose that $r/2 < b \leq r$. By Proposition 3-(ii) and Lemma 2, if exactly one of the candidates is the ideological type, then the winner is the opportunistic-type candidate. Hence, similar to Case (i), it is sufficient to determine the conditional equilibrium outcome given that both candidates are the opportunistic type. If the induced outcome of the news-reporting stage is $\bar{\gamma}_1$, then it is obviously given by $\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1$. If the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$, then we can show that the conditional equilibrium outcome is also given by $\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1$ by the similar argument used in Case (i). Therefore:

$$
\begin{aligned}
D(e) &= p^2 \sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 + p(1-p)\sum_{x_2\in[-r+2b,b]} \alpha_2^*(x_2)x_2 + p(1-p)\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 \\
&= p\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 + p(1-p)\sum_{x_2\in[-r+2b,b]} \alpha_2^*(x_2)x_2. \qquad \text{(A.29)}
\end{aligned}
$$

Case (iii): Suppose that $r < b < |l|$. In this scenario, policy pair could be in the disagreement regions $Z_{12} \cup Z_{21}$ even though candidate 1 is the ideological type. By Proposition 3-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$. If the induced outcome of the news-reporting stage is $\bar{\gamma}_1$, then candidate 1 wins for certain unless policy pairs lie in region Z_{00}. Hence:

$$
\begin{aligned}
D(e) &= p^2 \sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 + p(1-p)r + p(1-p)\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 \\
&= p\sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 + p(1-p)r. \qquad \text{(A.30)}
\end{aligned}
$$

Next, suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, then $\alpha_1^* = \alpha_2^* = r$ should hold. If $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \cup \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$, then the consistent belief implies that $p\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+(1-p)r=\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2$. Therefore:

$$\begin{aligned}D(e)&=p^2\left(\frac{1}{2}\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+\frac{1}{2}\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2\right)+p(1-p)\left(\frac{1}{2}r+\frac{1}{2}\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2\right)\\&\qquad+p(1-p)\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1\\&=\frac{p}{2}\left((2-p)\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+(1-p)r+\sum_{x_2\in[0,b]}\alpha_2^*(x_2)x_2\right)\\&=p\sum_{x_1\in[0,b]}\alpha_1^*(x_1)x_1+p(1-p)r.\end{aligned}\tag{A.31}$$

Case (iv): Suppose that $b\geq|l|$. By Proposition 3-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$. Furthermore, by Lemma 3, candidate 1 wins for certain when $\theta_1=O$ and $\theta_2=I$. Thus, the representation of $D(e)$ is identical to that in Case (iii). ■

A.6 Proof of Corollary 1

A.6.1 Key Lemmas: construction of USEs

Lemma 4 *Suppose that* $0<b\leq r/2$. *Then, for any* $\alpha\in\Delta([0,b])$, *there exists a UCE such that* $\alpha_1^*=\alpha_2^*=\alpha$.

Proof. Fix $\alpha\in\Delta([0,b])^*$, arbitrarily. We show that the following is a USE: $\alpha_1^*=\alpha_2^*$, and the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.5), (A.6) and (A.7), respectively. It is obvious that $\beta^*\in B$ and $\gamma^*\in\Gamma$. If α is a degenerate distribution, then $\bar{Z}_{12}(\alpha_1^*,\alpha_2^*)\cup\bar{Z}_{21}(\alpha_1^*,\alpha_2^*)=\emptyset$. Hence, we can show the optimality of β^* and γ^* by the similar argument used in the proof of Proposition 2-(i). If α is a nondegenerate distribution, then $Z_{12}(\alpha_1^*,\alpha_2^*)\neq\emptyset$ and $Z_{21}(\alpha_1^*,\alpha_2^*)\neq\emptyset$. Hence, the consistent belief implies that $\gamma^*(\bar{Z}_{12}\cup\bar{Z}_{21})=(1/2,1/2)$ because of the symmetry between strategies α_1^*, and α_2^*. In this scenario, it is also obvious that β^* is the outlet's best response because the outlet who observes policy pairs in the disagreement regions cannot induce his preferred outcome with probability more than 1/2. Thus, we can say that, in both scenarios, $\gamma^*(\beta^*(z))=(1/2,1/2)$ holds for any $z\in[0,b]^2$. Given β^* and γ^*, any pure strategy $\alpha_i\in[0,b]$ is indifferent for each candidate, which means that strategy $\alpha_i=\alpha$ is a best response to the others' strategies. Therefore, it is a USE. ■

Lemma 5 *Suppose that* $r/2 < b \leq r$.

(i) For any $\alpha \in \Delta([-r+2b, b])^*$, *there exists a USE where* $\alpha_1^* = \alpha_2^* = \alpha$.

(ii) If $b \neq r$, *then there exists a USE such that (1)* $S(\alpha_1^*) = \{0, \varepsilon_1, \cdots, \varepsilon_{N-1}\}$ *with* $\alpha_1(x_1) = 1/N$ *for any* $x_1 \in S(\alpha_1^*)$ *and* $\varepsilon_i \in [0, -r+2b)$ *for any* i; *and (2)* $\alpha_2^* \in \Delta([-r+2b, b])^*$, *where* N *satisfies* $2(N-1)/(1+2(N-1)) < p \leq 2N/(1+2N)$.

(iii) If $b = r$, *then there exists a USE where* $\alpha_1^* = 0$ *and* $\alpha_2^* = r(1-p)$.

Proof. (i) Fix $\alpha \in \Delta([-r+2b, b])^*$, arbitrarily. We show that the following is a USE: $\alpha_1^* = \alpha_2^* = \alpha$, and the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.5), (A.6) and (A.7), respectively. It is obvious that $\beta^* \in B$ and $\gamma^* \in \Gamma$, and then it is sufficient to show that it is a PBE. Because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, we can show the optimality of β^* and γ^* by the similar argument used in the proof of Proposition 2-(i). It remains to show the optimality of α_i^*. Note that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$. For candidate 1, $\sigma_1(\alpha, \alpha) = 1 - p/2$. Note that $\mu_1(x_1, x_2) = 1/2$ for any $x_1 \in [0, b]$ and $x_2 \in [-r+2b, b]$ because possible policy pairs lie in region $Z_0 \cup Z_{12} \cup Z_{21}$. Likewise, $\mu_1(x_1, l) = 1$ for any $x_1 \in [0, b]$ because $(x_1, l) \in Z_{11}$. Thus, for any $\alpha_1 \in \Delta([0, b])^*$, $\sigma_1(\alpha_1, \alpha) = 1 - p/2$. Therefore, candidate 1 has no incentive to deviate. For candidate 2, $\sigma_2(\alpha, \alpha) = 1 - p/2$. Note that $\mu_2(x_1, x_2) = 1/2$ for any $x_1 \in [-r+2b, b]$ and $x_2 \in [0, b]$ because possible policy pairs lie in region $Z_0 \cup Z_{12} \cup Z_{21}$. Furthermore, $\mu_2(r, x_2) \leq 1$ for any $x_2 \in [0, b]$ because possible policy pairs lie in region $Z_{21} \cup Z_{22}$. Thus, for any $\alpha_2 \in \Delta([0, b])^*$, $\sigma_2(\alpha, \alpha_2^*) \leq 1 - p/2$. That is, candidate 2 also never deviates. Therefore, it is a USE.

(ii) We show that the following is a USE: α_1^* and α_2^* satisfy condition (1) and (2); the outlet's strategy β^* is given by:

$$\beta^* = \begin{cases} \bar{Z}_{12} \cup \bar{Z}_{21} & \text{if } z \in Z_{12} \cup \bar{Z}_{21}, \\ z & \text{otherwise;} \end{cases} \tag{A.32}$$

the voter's belief for off-the equilibrium-path messages is given by:

$$S(\mathcal{P}^*(\cdot|m)) \subseteq \begin{cases} m \cap Z_{12} & \text{if } m \cap Z_{12} \neq \emptyset, \\ m \cap Z_{02} & \text{if } m \cap Z_{12} = \emptyset \text{ and } m \cap Z_{02} \neq \emptyset, \\ m & \text{otherwise.} \end{cases} \tag{A.33}$$

It is obvious that $\beta^* \in B$ and $\gamma^* \in \Gamma$. Note that because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$, the voter's consistent belief induces that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1, 0)$, and then the induced outcome in the news-reporting stage is $\bar{\gamma}_1$. Hence, it is straightforward that β^* is the outlet's best response.

It remains to show the optimality of α_i^*. Because $S(\alpha_2^*) \subseteq [-r+2b, b]$, $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) = 0$. Thus, $U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1$; that is, candidate 1 has no incentive to deviate from α_1^*. Because $b \neq r$ and $S(\alpha_2^*) \subseteq [-r+2b, b]$, $U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1-p$. Suppose that candidate 2 deviates to α_2' such that $S(\alpha_1^*) \cap S(\alpha_2') \neq \emptyset$. Hence:

$$U_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1) = \frac{1}{2} \sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2'', \theta_2 = O) + (1-p)(1-q), \tag{A.34}$$

where $q \equiv \sum_{x_2\in[0,-r+2b)} \alpha_2'(x_2)$. Now, consider the following strategy α_2'' given by:

$$\alpha_2''(x_2) \equiv \begin{cases} q & \text{if } x_2 = 0, \\ 0 & \text{if } x_2 \in (0, -r+2b), \\ \alpha_2'(x_2) & \text{otherwise.} \end{cases} \tag{A.35}$$

Then:

$$U_2(\alpha_1^*, \alpha_2'', \bar{\gamma}_1) = \frac{pq}{2N} + (1-p)(1-q). \tag{A.36}$$

By construction of α_1^* and α_2'', $U_2(\alpha_1^*, \alpha_2'', \bar{\gamma}_1) \geq U_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1)$ should hold. Furthermore, note that:

$$\begin{aligned} U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) - U_2(\alpha_1^*, \alpha_2'', \bar{\gamma}_1) &= (1-p) - \frac{pq}{2N} - (1-p)(1-q) \\ &= q\left(1 - \frac{2N+1}{2N}p\right) \geq 0. \end{aligned} \tag{A.37}$$

That is, $U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) \geq U_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1)$ holds, which means that candidate 2 has no incentive to deviate to such strategy α_2'. If candidate 2 deviate to α_2' such that $S(\alpha_2') \subseteq [-r+2b, b]$ and $\alpha_2' \neq \alpha_2^*$, then $U_2(\alpha_1^*, \alpha_2', \bar{\gamma}_1) = 1-p$. As a result, candidate 2 has no incentive to deviate from α_2^*. Therefore, it is a USE.

(iii) We show that the following is a USE: $\alpha_1^* = 0$, $\alpha_2^* = r(1-p)$, and the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.5), (A.6) and (A.7), respectively. It is obvious that $\beta^* \in B$ and $\gamma^* \in \Gamma$. Because $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \{(0, r(1-p))\}$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \{(r, r(1-p))\}$, the voter's consistent belief implies that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$, and then the induced outcome in the news reporting stage is $\bar{\gamma}_{1/2}$. Given γ^*, it is straightforward that β^* is the outlet's best response. Furthermore, because $\bar{\gamma}_{1/2}(z) = (1/2, 1/2)$ for any $z \in [0, r]^2$, any strategy $\alpha_i \in \Delta([0, r])^*$ is indifferent for each candidate; that is, each candidate has no incentive to deviate. Thus, it is a USE. ■

Lemma 6 *Suppose that $r < b < |l|$. Then, there exist USEs where (i) $\alpha_1^* = 0$ and $\alpha_2^* = (1-p)r$, and (ii) $\alpha_1^* = b$ and $\alpha_2^* = pb + (1-p)r$.*

Proof. We show by constructing equilibria where the outlet's strategy β^* and the voter's belief $\mathcal{P}^*$ for off-the-equilibrium-path messages are given by (A.5), (A.6) and (A.7), respectively, and the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. By the same argument used in the proof of Proposition 2-(i), the optimality of β^* and γ^* except for message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ can be shown. Hence, it remains to show the optimality of α_i^* and $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21})$.

(i) Notice that $\bar{\gamma}_{1/2}(z) = (1/2, 1/2)$ holds for any $z \in [0,b]^2$. Hence, for any $\alpha_1 \in \Delta([0,b])^*$, $U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = U_1(\alpha_1, \alpha_2^*, \bar{\gamma}_{1/2}) = 1 - p/2$. That is, candidate 1 has no incentive to deviate. Likewise, because $b > r$, $U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) = U_2(\alpha_1^*, \alpha_2, \bar{\gamma}_{1/2}) = 1/2$ for any $\alpha_2 \in \Delta([0,b])^*$. Hence, candidate 2 also has no incentive to deviate. Note that $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \{(0, (1-p)r)\}$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \{(r, (1-p)r)\}$, so message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is used on the equilibrium path. Hence, $\mathcal{P}^*(0, (1-p)r | \bar{Z}_{12} \cup \bar{Z}_{21}) = p$ and $\mathcal{P}^*(r, (1-p)r | \bar{Z}_{12} \cup \bar{Z}_{21}) = 1 - p$. Given $\mathcal{P}^*(\cdot | \bar{Z}_{12} \cup \bar{Z}_{21})$, actions $y = y_1$ and y_2 are indifferent for the voter. Thus, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21})$ is optimal. Therefore, it is a USE.

(ii) By the same argument used in the proof of (i), we can show that each candidate has no incentive to deviate. Note that $\bar{Z}_{12}(\alpha_1^*, \alpha_2^*) = \{(r, pb + (1-p)r)\}$ and $\bar{Z}_{21}(\alpha_1^*, \alpha_2^*) = \{(b, pb + (1-p)r)\}$, so message $m = \bar{Z}_{12} \cup \bar{Z}_{21}$ is used on the equilibrium path. Hence, $\mathcal{P}^*(r, pb + (1-p)r | \bar{Z}_{12} \cup \bar{Z}_{21}) = 1 - p$ and $\mathcal{P}^*(b, pb + (1-p)r | \bar{Z}_{12} \cup \bar{Z}_{21}) = p$. Given $\mathcal{P}^*(\cdot | \bar{Z}_{12} \cup \bar{Z}_{21})$, actions $y = y_1$ and y_2 are indifferent for the voter. Thus, $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21})$ is optimal. Therefore, it is a USE. ■

Lemma 7 *Suppose that $b \geq |l|$. Then, there exist USEs where (i) $\alpha_1^* = 0$ and $\alpha_2^* = (1-p)r$, and (ii) $\alpha_1^* = x_1$ and $\alpha_2^* = px_1 + (1-p)r$ for any $x_1 \in [0, |l|)$.*

Proof. We can show this lemma by the similar argument used in the proof of Lemma 6.[33] ■

Lemma 8 *Suppose that $b > r$. If there exists a USE where the induced outcome of the news-reporting stage is $\bar{\gamma}_1$, then $\alpha_2^* = r$ holds.*

Proof. Because the induced outcome of the news-reporting stage is $\bar{\gamma}_1$:

$$U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1 - \frac{1}{2} \sum_{z \in Z_{00}} \Pr(z | \alpha_1^*, \alpha_2^*, \theta_1 = O). \tag{A.38}$$

[33] The detail is available from the author upon the request.

Hence, to be an equilibrium, $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) = 0$ should hold; otherwise, candidate 1 has an incentive to deviate to strategy α_1' such that $S(\alpha_1') \cap S(\alpha_2^*) = \emptyset$. That is, $\sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) = 0$ holds. Because $b > r$:

$$\begin{aligned} U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) &= \frac{1}{2}\left(p \sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O, \theta_2 = O) + (1-p) \sum_{z\in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = I, \theta_2 = O)\right) \\ &= \frac{1}{2}(1-p)\alpha_2^*(r). \end{aligned} \tag{A.39}$$

Because $U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_{1/2}) \geq U_2(\alpha_1^*, \alpha_2, \bar{\gamma}_{1/2})$ holds for any $\alpha_2 \in \Delta(X)^*$, $\alpha_2^*(r) = 1$ should hold. ■

A.6.2 Proof of Corollary 1

Case (i): Suppose that $0 < b \leq r/2$. By Theorem 2, the degree of distortion $D(e)$ is bounded as follows: $0 \leq D(e) \leq p(2-p)b$. By Lemma 4, there exist $(0,0)$ and (b,b) equilibria, which are denoted by e^- and e^+, respectively. By algebra, $D(e^-) = 0$ and $D(e^+) = p(2-p)b$.

Case (ii): Suppose that $r/2 < b \leq r$. By Theorem 2, the degree of distortion $D(e)$ is bounded as follows: $p(1-p)(-r+2b) \leq D(e) \leq p(2-p)b$. By Lemma 5-(i), there exists (b,b) equilibrium, which is denoted by e^+. It is obvious by algebra that $D(e^+) = p(2-p)b$. Now, suppose that $b \neq r$, and let e^- be the equilibrium characterized in Lemma 5-(ii). If $p \leq 2/3$, then e^- is equivalent to $(0, -r+2b)$ equilibrium. That is, $D(e^-) = p(1-p)(-r+2b)$. If $p > 2/3$, then the degree of distortion of equilibrium e^- is:

$$D(e^-) = p(1-p)(-r+2b) + p\sum_{k=1}^{N-1} \frac{\varepsilon_k}{N}. \tag{A.40}$$

Notice that each ε_k can be arbitrarily small. Hence, $D(e^-) \to p(1-p)(-r+2b)$ as $\alpha_1^* \to 0$. Thus, we can say that $\inf D(e) = p(1-p)(-r+2b)$. Next, suppose that $b = r$, and let e^- be the equilibrium characterized in Lemma 5-(iii). It is obvious that $D(e^-) = p(1-p)r$.

Case (iii): Suppose that $r < b < |l|$. By Theorem 2, the degree of distortion is bounded as follows: $p(1-p)r \leq D(e) \leq pb + p(1-p)r$. By Lemma 6, there exist $(0, (1-p)r)$ and $(b, pb+(1-p)r)$ equilibria, which are denoted by e^- and e^+, respectively. It is obvious that $D(e^-) = p(1-p)r$ and $D(e^+) = pb + (1-p)r$.

Case (iv): Suppose that $b \geq |l|$. By Theorem 2, the degree of distortion $D(e)$ is given by $D(e) = \sum_{x_1\in[0,b]} \alpha_1^*(x_1)x_1 + p(1-p)r$. By Lemma 7-(i), there exists $(0, (1-p)r)$ equilibrium, which is denoted by e^-. Hence, it is obvious that $D(e^-) = p(1-p)r \leq D(e)$ for any USE e. Now, we consider the upper bound. By Lemma 7-(ii), there exists $(x_1, px_1 + (1-p)r)$ equilibrium for any $x_1 \in [0, |l|)$, which is represented by e^+. By algebra, $D(e^+) = px_1 + p(1-p)r$, and then $\lim_{x_1 \to |l|} D(e^+) = p|l| + p(1-p)r$. It is sufficient to show that $D(e) \leq p|l| + p(1-p)r$ for any USE $e = (\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$. There are the following two cases depending on α_1^*.

Case (iv)-1: Suppose that $S(\alpha_1^*) \subseteq [0, |l|)$. Note that:

$$p|l| + p(1-p)r - D(e) = p\left(|l| - \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1)\right) > 0, \tag{A.41}$$

where the last inequality comes from $S(\alpha_1^*) \subseteq [0, |l|)$.

Case (iv)-2: Suppose that $S(\alpha_1^*) \cap [|l|, b] \neq \emptyset$. By Lemmas 3 and 8, we can restrict our attention to the scenario where the induced outcome of the news-reporting stage is $\bar{\gamma}_1$ and $\alpha_2^* = r$ without loss of generality. To support $\bar{\gamma}_1$ in equilibrium, the following condition should hold:

$$\begin{aligned}
&p^2 \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) + p(1-p)r + p(1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^* \\
&\qquad < p \sum_{x_2 \in [0,b]} x_2 \alpha_2^*(x_2) + p(1-p)|l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1). \\
\iff & p \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) + (1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^* < pr + (1-p)|l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1).
\end{aligned} \tag{A.42}$$

Now, we show that $\sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) \leq |l|$. Suppose, in contrast, that $\sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) > |l|$. Then:

$$\begin{aligned}
p|l| + (1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^*(x_1) &< p \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) + (1-p) \sum_{x_1 \in [|l|,b]} x_1 \alpha_1^*(x_1) \\
&< pr + (1-p)|l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1),
\end{aligned} \tag{A.43}$$

where the last inequality comes from (A.42). Because $\sum_{x_1 \in [|l|,b]} x_1 \alpha_1^*(x_1) \geq |l| \sum_{x_1 \in [|l|,b]} \alpha_1^*(x_1)$, $|l| < r$ should hold for satisfying (A.43), which is a contradiction. Thus, $\sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1) \leq |l|$ holds. Hence:

$$p|l| + p(1-p)r - D(e) = p\left(|l| - \sum_{x_1 \in [0,b]} x_1 \alpha_1^*(x_1)\right) \geq 0. \tag{A.44}$$

Therefore, $\sup D(e) = p|l| + p(1-p)r$ holds. ■

A.7 Proof of Theorem 3

A.7.1 Key lemma

Lemma 9 *Suppose that $r/2 < b \leq r$. If the induced outcome of the news-reporting stage is $\bar{\gamma}_1$, then $S(\alpha_1^*) \subseteq [0, -r+2b)$.*

Proof. Suppose, in contrast, that there exists a USE where the induced outcome of the news-reporting stage is $\bar{\gamma}_1$ and there exists $x_1' \in S(\alpha_1^*) \cap [-r+2b, b]$. To hold this equilibrium, $U_1(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1$ should hold. That is, $\sum_{z \in Z_{00}} \Pr(z|\alpha_1^*, \alpha_2^*, \theta_1 = O) = 0$, and then $x_1' \notin S(\alpha_2^*)$. Hence, by Lemma 2, $U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1) = 1-p$ holds. Now, if candidate 2 deviates to strategy $\alpha_2 = x_1'$, then $U_2(\alpha_1^*, x_1', \bar{\gamma}_1) = p\alpha_1^*(x_1')/2 + 1 - p > U_2(\alpha_1^*, \alpha_2^*, \bar{\gamma}_1)$, which is a contradiction. ■

A.7.2 Proof of Theorem 3

(i) Suppose, without loss of generality, that $Z_{12}(\alpha_1^*, \alpha_2^*) \neq \emptyset$ and $Z_{21}(\alpha_1^*, \alpha_2^*) \neq \emptyset$.

Case (i)-1: Suppose that $0 < b \leq r$. Because $c = 1/2$, $\sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 = \sum_{x_2 \in [0,b]} \alpha_2^*(x_2)x_2$ holds, or still:

$$\sum_{(x_1,x_2) \in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2 - x_1) = \sum_{(x_1,x_2) \in Z_{21}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_1 - x_2). \tag{A.45}$$

Therefore:

$$\begin{aligned} d(e) &= \frac{p^2}{2}\left(\sum_{(x_1,x_2) \in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2 - x_1) + \sum_{(x_1,x_2) \in Z_{21}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_1 - x_2)\right) \\ &= p^2 \sum_{(x_1,x_2) \in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2 - x_1). \end{aligned} \tag{A.46}$$

Case (i)-2: Suppose that $b > r$. In this scenario, $c = 1/2$ implies that $p^2 \sum_{x_1 \in [0,b]} \alpha_1^*(x_1)x_1 + p(1-p)r = p\sum_{x_2 \in [0,b]} \alpha_2^*(x_2)x_2$, which is equivalent to:

$$\begin{aligned} &p^2 \sum_{(x_1,x_2) \in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2 - x_1) + p(1-p) \sum_{x_2 \in (r,b]} \alpha_2^*(x_2)(x_2 - r) \\ &= p^2 \sum_{(x_1,x_2) \in Z_{21}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_1 - x_2) + p(1-p) \sum_{x_2 \in [0,r)} \alpha_2^*(x_2)(r - x_2). \end{aligned} \tag{A.47}$$

Thus:

$$\begin{aligned} d(e) &= \frac{p^2}{2}\left(\sum_{(x_1,x_2)\in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2-x_1) + \sum_{(x_1,x_2)\in Z_{21}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_1-x_2)\right) \\ &\quad + \frac{p(1-p)}{2}\left(\sum_{x_2\in(r,b]} \alpha_2^*(x_2)(x_2-r) + \sum_{x_2\in[0,r)} \alpha_2^*(x_2)(r-x_2)\right) \\ &= p^2 \sum_{(x_1,x_2)\in Z_{12}} \alpha_1^*(x_1)\alpha_2^*(x_2)(x_2-x_1) + p(1-p)\sum_{x_2\in(r,b]} \alpha_2^*(x_2)(x_2-r). \end{aligned} \tag{A.48}$$

(ii) By Proposition 3-(i), there exists no USE with $\bar{\gamma}_1$ when $0 < b \leq r/2$. Hence, we can focus on the scenario where $b > r/2$.

Case (ii)-1: Suppose that $r/2 < b \leq r$. By Lemmas 2 and 9, $Z_{21}(\alpha_1^*, \alpha_2^*) = \emptyset$. Hence, $d(e) = 0$.

Case (ii)-2: Suppose that $r < b < |l|$. By Lemma 8, $\alpha_2^* = r$. Hence:

$$d(e) = \sum_{(x_1,x_2)\in Z_{21}} (x_1 - x_2)\Pr(x_1, x_2|\alpha_1^*, \alpha_2^*) = p^2 \sum_{x_1\in(r,b]} \alpha_1^*(x_1)(x_1 - r). \tag{A.49}$$

Case (ii)-3: Suppose that $b \geq |l|$. By the similar argument used in Case (ii)-2:

$$\begin{aligned} d(e) &= \sum_{(x_1,x_2)\in Z_{21}} (x_1 - x_2)\Pr(x_1, x_2|\alpha_1^*, \alpha_2^*) \\ &= p^2 \sum_{x_1\in(r,b]} \alpha_1^*(x_1)(x_1 - r) + p(1-p)\sum_{x_1\in(|l|,b]} \alpha_1^*(x_1)(x_1 - |l|). \blacksquare \end{aligned} \tag{A.50}$$

A.8 Proof of Corollary 2

A.8.1 Key lemma

Lemma 10 $\min d(e) = 0$ *for any* b.

Proof. We show that there exists USE e such that $d(e) = 0$ by construction. First, suppose that $0 < b \leq r/2$. By Theorem 1, there exists a $(0, 0)$ equilibrium, which is denoted by e'. Hence, it is obvious that $d(e') = 0$. Next, suppose that $r/2 < b \leq r$. By Lemma 5-(i), there exists USE e'' such that $\alpha_1^* = \alpha_2^* \in [-r + 2b, b]$. By Theorem 3, $d(e'') = 0$. Finally, suppose that $b > r$. We can show that there exists (r, r) equilibrium denoted by e''' by the similar argument used in the proof of Lemma 4. By Theorem 3, $d(e''') = 0$. $\blacksquare$

A.8.2 Proof of Corollary 2

(i) Suppose, in contrast, that there exists USE e^- such that for any $\varepsilon > 0$, $D(e^-) - \inf D(e) \leq \varepsilon$. By Lemma 10, $d(e^-) = 0$. First, we suppose that $r/2 < b \leq r$. By Theorem 3 and Lemma 2, e^- should be a USE where $\alpha_1^* = \alpha_2^* \in [-r + 2b, b]$. However, by Theorem 2 and Corollary 1:

$$D(e^-) = p(2-p)(-r+2b) > p(1-p)(-r+2b) = \inf D(e), \tag{A.51}$$

which is a contradiction. Second, we suppose that $b > r$. By Theorem 3, e^- should be a USE with $\alpha_1^* = \alpha_2^* = r$. Hence, by the similar argument used in the above scenario, we can derive a contradiction.

(ii) By Corollary 1, the supremum of $D(e)$ is attained by (b, b) equilibrium. It is straightforward that the direct distortion of (b, b) equilibrium is 0. ■

Appendix B: Supplementary Materials

B.1 Proof of Proposition 1

(i) (Existence) Because there is no media manipulation, $\beta^*(z) = z$ for any $z \in Z$. Hence, it is sufficient to show that the following is a PBE: $\alpha_1^* = \alpha_2^* = 0$ and $\gamma^*(z) = y^v(z)$ because there is no media manipulation. It is straightforward that γ^* is undominated and optimal for the voter. Given γ^* and α_2^*, strategy α_1^* is optimal for candidate 1; that is, the winning probability from α_1^* is $1/2$, but his winning probabilities from other strategies are less than $1/2$. The same argument holds for candidate 2. Thus, it is a PBE.

(Uniqueness) It is obvious that for any policy pair z, the voter's unique undominated strategy is given by $\gamma^*(z) = y^v(z)$. Then, suppose, in contrast, that there exists an equilibrium where either $\alpha_1^* \neq 0$ or $\alpha_2^* \neq 0$ holds. Without loss of generality, assume that $\alpha_1^* \neq 0$. However, candidate 1 can strictly improve his winning probability by proposing policy 0 for certain whatever candidate 2's strategy is, which is a contradiction. Therefore, the $(0, 0)$ equilibrium is the unique one.

(ii) It is obvious from Table 1. ■

B.2 Proof of Proposition 2-(ii)

Suppose, in contrast, that there exists PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ such that $\gamma^*(\beta^*(z)) = y^v(z)$ for any $z \in Z$. Fix $z \in Z_{12}$ and $z' \in Z_{21}$, arbitrarily. Hence, $\gamma^*(\beta^*(z)) = (1, 0)$ and $\gamma^*(\beta^*(z')) = (0, 1)$. To be incentive compatible, the following conditions must hold: (1) $\gamma^*(m) = (0, 1)$ for any message $m \in M(z)$, and (2) $\gamma^*(m') = (1, 0)$ for any message $m' \in M(z')$. However, there is no incentive compatible reaction to message $m = \{z, z'\} \in M(z) \cap M(z')$, which is a contradiction. ■

B.3 Proof of Claim 1

We show that the following is a PBE. For any $q \in (0, p)$:

$$\alpha_1^*(x_1) = \begin{cases} q/p & \text{if } x_1 = 0, \\ 1 - q/p & \text{if } x_1 = r, \\ 0 & \text{otherwise;} \end{cases}$$
$$\alpha_2^*(x_2) = \begin{cases} q & \text{if } x_2 = 0, \\ 1 - q & \text{if } x_2 = r, \\ 0 & \text{otherwise;} \end{cases} \tag{B.1}$$
$$\mathcal{P}^*(\tilde{z}|m) = \begin{cases} 1 & \text{if } m = z \text{ and } \tilde{z} = z, \\ \frac{1}{2} & \text{if } m = \bar{Z}_{12} \cup \bar{Z}_{21} \text{ and } \tilde{z} = (0, r) \text{ or } (r, 0), \\ 0 & \text{otherwise;} \end{cases}$$

The outlet's strategy β^* and the voter's belief for off-the-equilibrium-path messages are given by (A.5), (A.6) and (A.7), respectively. The optimality of strategies β^* and γ^* can be shown by the same argument used in the proof of Proposition 2. Hence, it remains to show the optimality of strategies α_1^* and α_2^*, and the consistency of belief $\mathcal{P}^*$. It is worthwhile to remark that $\gamma^*(\bar{Z}_{12} \cup \bar{Z}_{21}) = (1/2, 1/2)$.

First, consider candidate 1's behavior. Candidate 1's winning probabilities from strategies $\alpha_1 = 0$ and r are $\sigma_1(0, \alpha_2^*) = \sigma_1(r, \alpha_2^*) = 1 - p/2$, respectively. It is then sufficient to show that $\sigma_1(\alpha_1', \alpha_2^*) \leq 1 - p/2$ for any strategy $\alpha_1' \in \Delta(X)^*$. Candidate 1's winning probabilities given policies $x_2 = 0$, r and l are $\mu_1(x_1, 0) \leq 1/2$, $\mu_1(x_1, r) \leq 1/2$ and $\mu_1(x_1, l) \leq 1$ for any $x_1 \in X$ because possible policy pairs lie in regions $Z_0 \cup Z_{22} \cup Z_{21}$, $Z_0 \cup Z_{22} \cup \bar{Z}_{12} \cup Z_{21}$ and $Z_0 \cup Z_{11} \cup Z_{22} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_1' \in \Delta(X)^*$:

$$\sigma_1(\alpha_1', \alpha_2^*) \leq \frac{1}{2}pq + \frac{1}{2}p(1-q) + (1-p) = 1 - \frac{1}{2}p. \tag{B.2}$$

That is, α_1^* is optimal for candidate 1.

Next, consider candidate 2's behavior. Candidate 2's winning probabilities from strategies $\alpha_2 = 0$ and r are $\sigma_2(\alpha_1^*, 0) = \sigma_2(\alpha_1^*, r) = 1/2$, respectively. It is then sufficient to show that $\sigma_2(\alpha_1^*, \alpha_2') \leq 1/2$ for any strategy $\alpha_2' \in \Delta(X)^*$. Candidate 2's winning probabilities given policies $x_1 = 0$ and r are $\mu_2(0, x_2) \leq 1/2$ and $\mu_2(r, x_2) \leq 1/2$ for any $x_2 \in X$ because possible policy pairs lie in regions $Z_0 \cup Z_{11} \cup Z_{12}$ and $Z_0 \cup Z_{11} \cup Z_{12} \cup \bar{Z}_{21}$, respectively. Hence, for any $\alpha_2' \in \Delta(X)^*$:

$$\sigma_2(\alpha_1^*, \alpha_2') \leq \frac{1}{2}q + \frac{1}{2}(1-q) = \frac{1}{2}. \tag{B.3}$$

That is, α_2^* is optimal for candidate 2. Finally, it is straightforward that belief $\mathcal{P}^*$ is consistent with Bayes' rule. Thus, it is a PBE. ■

B.4 Justification for the USE

B.4.1 Certifiable dominance

In this subsection, we formally define the notion of certifiable dominance, and provide some related results. The following analysis is based on Miura (2014b). Let $B^0 \equiv \{\beta \in \Delta(M)^Z | S(\beta(z)) \subseteq M(z) \text{ for any } z \in Z\}$. Define:

$$W(z, \gamma(m)) \equiv \sum_{y \in Y} w(z, y) \Pr(y|\gamma(m)), \tag{B.4}$$

$$\Omega(\alpha_1, \alpha_2, \beta, \gamma) \equiv \sum_{z \in Z(\alpha_1, \alpha_2)} \left(\sum_{m \in M} W(z, \gamma(m)) \Pr(m|\beta(z)) \right) \Pr(z|\alpha_1, \alpha_2). \tag{B.5}$$

Definition 3 ***Certifiable dominance (Miura, 2014b)***
The outlet's strategy $\beta \in B^0$ is certifiably undominated if it is (weakly) undominated in $\Delta(X)^{*2} \times B^0 \times \Gamma$.

The certifiable dominance is a modified version of the weak dominance that is consistent with the assumption that the outlet's private information is fully certifiable. That is, because Γ is the strategy space that is consistent with this assumption as mentioned in the body of the paper, the voter's strategy space should be restricted to Γ when we apply the dominance criterion. First, we can say that the outlet who observes policy pairs in the agreement regions never send "careless" messages that could induce her unfavorite outcome with positive probability.

Lemma 11 *If the outlet's strategy β is certifiably undominated, then $S(\beta(z)) \subseteq 2^{Z_{11} \cup Z_{12}}$ (resp. $2^{Z_{22} \cup Z_{21}}$) holds for any $z \in Z_{11}$ (resp. Z_{22}).*

Proof. Suppose, in contrast, that strategy β is certifiably undominated, but there exists $z' \in Z_{11}$ such that $S(\beta(z')) \not\subseteq 2^{Z_{11} \cup Z_{12}}$. Without loss of generality, we assume that $\beta(z') = \{z', \tilde{z}\}$ such that

$\tilde{z} \in Z \backslash (Z_{11} \cup Z_{12})$. Now, we consider the following strategy β^* defined by:

$$\beta^*(z) = \begin{cases} z' & \text{if } z = z', \\ \beta(z) & \text{otherwise.} \end{cases} \tag{B.6}$$

Note that if $\gamma \in \Gamma$, then $S(\gamma(\{z', \tilde{z}\})) \subseteq \{y_1, y_2\}$. Fix α_1, α_2 and $\gamma \in \Gamma$, arbitrarily. Then:

$$\Omega(\alpha_1, \alpha_2, \beta^*, \gamma) - \Omega(\alpha_1, \alpha_2, \beta, \gamma) = \left\{ w(z', y^v(z')) - \sum_{m \in M} W(z', \gamma(m)) \Pr(m|\beta(z)) \right\} \Pr(z'|\alpha_1, \alpha_2). \tag{B.7}$$

If either $\Pr(z'|\alpha_1, \alpha_2) = 0$ or $\Pr(z'|\alpha_1, \alpha_2) \neq 0$ with $S(\gamma(\{z', \tilde{z}\})) = \{y_1\}$ holds, then (B.7) is 0. Then, we suppose that $\Pr(z'|\alpha_1, \alpha_2) \neq 0$, and either $S(\gamma(\{z', \tilde{z}\})) = \{y_2\}$ or $\{y_1, y_2\}$. Because $z' \in Z_{11}$, $w(z', y^v(z')) - \sum_{m \in M} W(z', \gamma(m)) \Pr(m|\beta(z')) > 0$ holds. Hence, (B.7) is positive. That is, strategy β is certifiably dominated by strategy β^*, which is a contradiction. Thus, such a $z' \in Z_{11}$ never exists. Likewise, we can show that $S(\beta(z)) \subseteq 2^{Z_{22} \cup Z_{21}}$ holds for any $z \in Z_{22}$. ■

Now, we impose the following additional requirement upon off-the-equilibrium-path beliefs.

Requirement 3 *If $\beta^{-1}(m) \neq \emptyset$, then $S(\mathcal{P}(\cdot|m)) \subseteq \beta^{-1}(m)$ holds.*

This requirement means that if there exist policy pairs in which message m is sent under strategy β, then the posterior belief after observing message m must be a probability distribution over the set of such policy pairs even though that message is sent off the equilibrium path. Intuitively, we require that the voter's belief is consistent with Bayes' rule as much as possible. Notice that the consistency is automatically satisfied if message m is sent on the equilibrium path, i.e., for message m such that $m \in S(\beta^*(z))$ for some policy pair $z \in Z(\alpha_1^*, \alpha_2^*)$. However, it might not hold for message m' sent by the outlet who observes policy pairs off the equilibrium path, i.e., for message m' such that $m \in S(\beta^*(z))$ for some policy pair $z \notin Z(\alpha_1^*, \alpha_2^*)$. This requirement extends that consistency up to the later scenario. We call a PBE satisfying Requirement 3 *strong perfect Bayesian equilibrium* (hereafter, SPBE), which is defined as follows.

Definition 4 ***SPBE***
A PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is an SPBE if $\mathcal{P}^*$ satisfies Requirement 3.[34]

[34]The SPBE is associated with the PBE defined in Fudenberg and Tirole (1991) and Gibbons (1992). Because the candidates' actions are unobservable to the voter and there is no prior distribution over policy pairs, the posterior belief after observing message m' mentioned above is not uniquely pinned down.

Once we focus on SPBEs where the outlet's strategy is certifiably undominated, the equilibrium outcomes over the disagreement regions must be constant as shown in the following proposition. It is worthwhile emphasizing that those equilibrium outcomes can be replicated by PBEs where the outlet's strategy is simple. Hence, without loss of generality in the above sense, we can restrict our attention to PBEs in which the outlet's strategy is simple.

Proposition 4 *If $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is an SPBE where β^* is certifiably undominated, then $\gamma^*(\beta^*(z)) = \gamma^*(\beta^*(z'))$ holds for any $z, z' \in Z_{12} \cup Z_{21}$.*

Proof. Suppose, in contrast, that there exists SPBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ where there exist $z, z' \in Z_{12} \cup Z_{21}$ such that $\gamma^*(\beta^*(z)) \neq \gamma^*(\beta^*(z'))$. Let $\beta^*(z) \equiv m$ and $\beta^*(z') \equiv m'$.[35]

<u>**Case (i):**</u> Suppose that $z, z' \in Z_{12}$ or Z_{21}. Without loss of generality, assume that $z, z' \in Z_{12}$.

Case (i)-1: Suppose that $\gamma^*(\beta^*(z)) = (1, 0)$. To hold this equilibrium, $\gamma^*(m) = (1, 0)$ should hold for any $m \in M(z)$. Hence, $\gamma^*(\beta^*(z'')) = (1, 0)$ must hold for any $z'' \in Z_{21}$. Without loss of generality, assume that $\gamma^*(\beta^*(z')) = (0, 1)$. Hence, by Requirement 3, $\beta^{-1}(m') \cap (Z_{22} \cup Z_{21}) \neq \emptyset$ holds. Because of $\beta^{-1}(m') \cap Z_{22} = \emptyset$ by Lemma 11, $\beta^{-1}(m') \cap Z_{21} \neq \emptyset$ must hold. However, because $\gamma^*(\beta^*(z'')) = (1, 0)$ holds for any $z'' \in Z_{21}$, $\beta^{-1}(m') \cap Z_{21} = \emptyset$, which is a contradiction.

Case (i)-2: Suppose that $\gamma^*(\beta^*(z)) = (0, 1)$. By Requirement 3 and Lemma 11, $\beta^{-1}(m) \cap Z_{21} \neq \emptyset$ must hold. That is, there exists $\hat{z} \in Z_{21}$ such that $\beta^*(\hat{z}) = m$, which induces that $\gamma^*(\beta^*(\hat{z})) = (0, 1)$. To hold this equilibrium, $\gamma^*(\hat{m}) = (0, 1)$ should hold for any $\hat{m} \in M(\hat{z})$. However, because $\gamma^*(\beta^*(z')) \neq (0, 1)$, the outlet observing policy pair z' has an incentive to send a message including policy pair $\hat{z}$, which is contradiction.

Case (i)-3: Suppose that $\gamma^*(\beta^*(z)) = (1/2, 1/2)$. We can derive a contradiction by the similar argument used in Cases (i)-1 and (i)-2.

<u>**Case (ii):**</u> Suppose that $z \in Z_{12}$ and $z' \in Z_{21}$. By Proposition 2-(ii), either $\gamma^*(\beta^*(z)) \neq (1, 0)$ or $\gamma^*(\beta^*(z')) \neq (0, 1)$ holds. Without loss of generality, assume that $\gamma^*(\beta^*(z)) = (0, 1)$.[36] By the similar argument used in Case (i)-2, there exists $\hat{z} \in Z_{21}$ such that $\gamma^*(\hat{m}) = (0, 1)$ holds for any $\hat{m} \in M(\hat{z})$. Suppose that $\gamma^*(\beta^*(z')) = (1, 0)$. By Requirement 3 and Lemma 11, $\beta^{-1}(m') \cap Z_{12} \neq \emptyset$ holds. That is, there exists $\tilde{z} \in Z_{12}$ such that $\beta^*(\tilde{z}) = m'$, which induces that $\gamma^*(\beta^*(\tilde{z})) = (1, 0)$. However, the outlet observing policy pair $\hat{z}$ has an incentive to deviate to a message containing policy pair $\hat{z}$, which is a contradiction. Likewise, we can derive a contradiction in the scenario where $\gamma^*(\beta^*(z')) = (1/2, 1/2)$. ■

[35] Without loss of generality, we can restrict our attention to the scenario where the outlet adopts a pure strategy.
[36] We can derive contradictions in other cases by the similar argument used here.

B.4.2 The candidates' undominated strategies

In this subsection, we characterize the set of undominated strategies of the candidates under the restriction mentioned in the body of the paper. We say that strategy $\alpha_i \in \Delta(X)^*$ is *weakly dominated with respect to* $\bar{\Gamma}$ by strategy $\alpha_i' \in \Delta(X)^*$ if $U_i(\alpha_i', \alpha_j, \bar{\gamma}_c) \geq U_i(\alpha_i, \alpha_j, \bar{\gamma}_c)$ holds for any $\alpha_j \in \Delta(X)^*$ and $\bar{\gamma}_c \in \bar{\Gamma}$ with strict inequality for some α_j' and $\bar{\gamma}_c'$.[37] We say that strategy $\alpha_i \in \Delta(X)^*$ is *undominated with respect to* $\bar{\Gamma}$ if it is not weakly dominated by other strategies. Let A_i be the set of candidate i's undominated strategies with respect to $\bar{\Gamma}$, and $A \equiv A_1 \times A_2$. The set of undominated strategies A_i can be characterized as follows. Notice that this characterization is irrelevant to the magnitude of the preference bias.

Proposition 5 $A_i = \Delta([0, b])^*$ *holds for any* $i \in \{1, 2\}$.

Proof. ($A_i \subseteq \Delta([0, b])^*$) Suppose, in contrast, that there exists $\alpha_i \in A_i$ such that $\alpha_i \notin \Delta([0, b])^*$. That is, there exists policy $x_i' \in S(\alpha_i)$ such that $x_i' \in [x^-, 0) \cup (b, x^+]$. Without loss of generality, we assume that $i = 1$.

Case (i): Suppose that $x_1' \in [x^-, 0)$. Without loss of generality, we assume that $|x_1'| < 2b$. Consider the following strategy $\hat{\alpha}_1$ defined by:

$$\hat{\alpha}_1(x_1) \equiv \begin{cases} 0 & \text{if } x_1 = x_1', \\ \alpha_1(0) + \alpha_1(x_1') & \text{if } x_1 = 0, \\ \alpha_1(x_1) & \text{otherwise.} \end{cases} \tag{B.8}$$

It is sufficient to show that $\mu_1(0, x_2) \geq \mu_1(x_1', x_2)$ for any $x_2 \in X$ and any $\bar{\gamma}_c \in \bar{\Gamma}$ with strictly inequality for some x_2' and $\bar{\gamma}_c'$.

(1) If $x_2 \in [2b, x^+]$, then $(0, x_2) \in Z_0 \cup Z_{11}$. Hence, $\mu_1(0, x_2) = 1 \geq \mu_1(x_1', x_2)$.

(2) If $x_2 \in (-x_1', 2b)$, then $(0, x_2), (x_1', x_2) \in Z_{12}$. Hence, $\mu_1(0, x_2) = \mu_1(x_1', x_2)$.

(3) If $x_2 = -x_1'$, then $(0, x_2) \in Z_{12}$ and $(x_1', x_2) \in Z_{02}$. Note that if $c \leq 1/2$, then $\bar{\gamma}_c(x_1', x_2) = (c, 1 - c)$. Hence, $\mu_1(0, x_2) = \mu_1(x_1', x_2) = c$. If $c > 1/2$, then $\bar{\gamma}_c(x_1', x_2) = (1/2, 1/2)$. Hence, $\mu_1(0, x_2) = c > \mu_1(x_1', x_2) = 1/2$.

(4) If $x_2 \in [0, -x_1')$, then $(x_1', x_2) \in Z_{22}$. Hence, $\mu_1(0, x_2) \geq \mu_1(x_1', x_2) = 0$.

(5) If $x_2 \in (x_1', 0)$, then $(0, x_2) \in Z_{11}$ and $(x_1', x_2) \in Z_{22}$. Hence, $\mu(0, x_2) > \mu_1(x_1', x_2)$.

[37] Definition of $\bar{\gamma}_c$, $\bar{\Gamma}$ and U_i is in Appendix A.

(6) If $x_2 \in [x^-, x_1']$, then $(0, x_2) \in Z_{11}$. Hence, $\mu_1(0, x_2) \geq \mu_1(x_1', x_2)$.

That is, strategy α_1 is weakly dominated with respect to $\bar{\Gamma}$ by strategy $\hat{\alpha}_1$, which is a contradiction.

Case (ii): Suppose that $x_1' \in (b, x^+]$. Without loss of generality, we assume that $|x_1'| < 3b$. Similar to Case 1, we consider the following strategy $\hat{\alpha}_1$ defined by:

$$\hat{\alpha}_1(x_1) \equiv \begin{cases} 0 & \text{if } x_1 = x_1', \\ \alpha_1(b) + \alpha_1(x_1') & \text{if } x_1 = b, \\ \alpha_1(x_1) & \text{otherwise.} \end{cases} \tag{B.9}$$

It is sufficient to show that $\mu_1(b, x_2) \geq \mu_1(x_1', x_2)$ for any $x_2 \in X$ and any $\bar{\gamma}_c$ with strict inequality for some $x_2' \in X$ and $\bar{\gamma}_c'$.

(1) If $x_2 \in (x_1', x^+]$, then $(b, x_2) \in Z_{11}$. Hence, $\mu_1(b, x_2) = 1 \geq \mu_1(x_1', x_2)$.

(2) If $x_2 = x_1'$, then $(b, x_2) \in Z_{11}$ and $(x_1', x_2) \in Z_0$. Note that $\gamma^*(\beta^*(x_1', x_2)) = (1/2, 1/2)$. Hence, $\mu_1(b, x_2) = 1 > \mu_1(x_1', x_2) = 1/2$.

(3) If $x_2 \in [-x_1' + 2b, x_1')$, then $(x_1', x_2) \in Z_{22}$. Hence, $\mu_1(b, x_2) \geq \mu_1(x_1', x_2) = 0$.

(4) If $x_2 \in (-b, -x_1' + 2b)$, then $(b, x_2), (x_1', x_2) \in Z_{21}$. Hence, $\mu_1(b, x_2) = \mu_1(x_1', x_2)$.

(5) If $x_2 = -b$, then $(b, x_2) \in Z_{01}$ and $(x_1', x_2) \in Z_{21}$. Note that if $c \geq 1/2$, then $\bar{\gamma}_c(b, x_2) = (c, 1-c)$. Hence, $\mu_1(b, x_2) = \mu_1(x_1', x_2) = c$. If $c < 1/2$, then $\bar{\gamma}_c(b, x_2) = (1/2, 1/2)$. Hence, $\mu_1(b, x_2) = 1/2 > \mu_1(x_1', x_2) = c$.

(6) If $x_2 \in [x^-, -b)$, then $(b, x_2) \in Z_{11}$. Hence, $\mu_1(b, x_2) = 1 \geq \mu_1(x_1', x_2)$.

Thus, strategy $\hat{\alpha}_1$ weakly dominates strategy α_1, which is a contradiction.

($\Delta([0, b])^* \subseteq A_i$) **Case (i):** Suppose that $0 < b \leq r/2$. Without loss of generality, assume that $i = 1$, and fix $\alpha_1 \in \Delta([0, b])^*$ and $\alpha_1' \in \Delta(X)^*$ with $\alpha_1 \neq \alpha_1'$, arbitrarily. Note that $(x_1, l) \in Z_{11}$ for any $x_1 \in [0, b]$. That is, as long as we focus on the induced outcome of the news-reporting stage $\bar{\gamma}_c \in \bar{\Gamma}$, the opportunistic-type candidate 1 wins for certain if candidate 2 is the ideological type. Hence, it is sufficient to compare the winning probabilities when candidate 2 is also the opportunistic-type. Because $\alpha_1 \neq \alpha_1'$, there exists $x_1' \in S(\alpha_1)$ such that $\alpha_1(x_1') \neq \alpha_1'(x_1')$. If $\alpha_1(x_1') > \alpha_1'(x_1')$, then take $\alpha_2 = x_1'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. By construction, $\mu_1(x_1, x_1') = 1/2$ if $x_1 = x_1'$ and 0 otherwise. Hence:

$$U_1(\alpha_1, \alpha_2, \bar{\gamma}_0) - U_1(\alpha_1', \alpha_2, \bar{\gamma}_0) = \frac{1}{2} p(\alpha_1(x_1') - \alpha_1'(x_1')) > 0. \tag{B.10}$$

If $\alpha_1(x_1') < \alpha_1'(x_1')$, then take $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. By construction, $\mu_1(x_1, x_1') = 1/2$ if $x_1 = x_1'$ and 1 otherwise. Hence:

$$U_1(\alpha_1, \alpha_2, \bar{\gamma}_1) - U_1(\alpha_1', \alpha_2, \bar{\gamma}_1) = \left(1 - \frac{1}{2}p\alpha_1(x_1')\right) - \left(1 - \frac{1}{2}p\alpha_1'(x_1')\right)$$
$$= \frac{1}{2}p(\alpha_1'(x_1') - \alpha_1(x_1')) > 0. \tag{B.11}$$

Thus, because α_1' is arbitrary, strategy α_1 is undominated with respect to $\bar{\Gamma}$; that is, $\alpha_1 \in A_1$ holds. Because α_1 is arbitrary, $\Delta([0,b])^* \subseteq A_1$ holds.

Case (ii): Suppose that $r/2 < b \le r$. By the similar argument used in Case (i), we can show that $\Delta([0,b])^* \subseteq A_1$. Next, we suppose that $i = 2$. Fix $\alpha_2 \in \Delta([0,b])^*$ and $\alpha_2' \in \Delta(X)^*$ with $\alpha_2 \neq \alpha_2'$, arbitrarily. Define $E(\alpha_2, \alpha_2') \equiv \{x_2 \in S(\alpha_2) | \alpha_2(x_2) \neq \alpha_2'(x_2)\}$.

Case (ii)-1: Suppose that there exists $x_2' \in E(\alpha_2, \alpha_2') \cap [-r+2b, b]$ such that $\alpha_2(x_2') < \alpha_2'(x_2')$. Fix $\alpha_1 = x_2'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. In this scenario, $(r, x_2) \in Z_{22} \cup Z_0 \cup Z_{21}$ for any $x_2 \in [0,b]$. Hence, $\bar{\gamma}_0(r, x_2) = (0,1)$ for any $x_2 \in [0,b]$; that is, candidate 2 wins for certain when candidate 1 is the ideological type. It is then sufficient to compare the winning probabilities when candidate 1 is opportunistic type. By construction:

$$U_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - U_2(\alpha_1, \alpha_2', \bar{\gamma}_0) = \left(1 - \frac{1}{2}p\alpha_2(x_2')\right) - \left(1 - \frac{1}{2}p\alpha_2'(x_2')\right)$$
$$= \frac{1}{2}p(\alpha_2'(x_2') - \alpha_2(x_2')) > 0. \tag{B.12}$$

Case (ii)-2: Suppose that $E(\alpha_2, \alpha_2') \cap [-r+2b, b] \neq \emptyset$ and $\alpha_2(x_2) > \alpha_2'(x_2)$ holds for any $x_2 \in E(\alpha_2, \alpha_2') \cap [-r+2b, b]$. In this scenario, there exists $x_2' \in E(\alpha_2, \alpha_2') \cap [0, -r+2b)$ such that $\alpha_2(x_2') < \alpha_2'(x_2')$; otherwise, either $\sum_{x_2 \in X} \alpha_2(x_2) > 1$ or $\sum_{x_2 \in X} \alpha_2'(x_2) < 1$ holds. Fix $\alpha_1 = x_2'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Note that $\bar{\gamma}_0(r, x_2) = (0,1)$ for any $x_2 \in [0,b]$. Hence, by the same argument used in Case (ii)-1, we can show that $U_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - U_2(\alpha_1, \alpha_2', \bar{\gamma}_0) > 0$.

Case (ii)-3: Suppose that $E(\alpha_2, \alpha_2') \cap [-r+2b, b] = \emptyset$. In this scenario, there exists $x_2' \in E(\alpha_2, \alpha_2') \cap [0, -r+2b)$. Furthermore, $\alpha(x_2) = \alpha_2'(x_2)$ for any $x_2 \in [-r+2b, b]$. That is, candidate 2's winning probabilities from strategies α_2 and α_2' when candidate 1 is the ideological type are equivalent. It is then sufficient to compare the winning probabilities when candidate 1 is the opportunistic type. By the similar argument used in Case (i), we can show that there exists $\alpha_1 \in \Delta(X)^*$ and $\bar{\gamma}_c \in \bar{\Gamma}$ such that $U_2(\alpha_1, \alpha_2, \bar{\gamma}_c) > U_2(\alpha_1, \alpha_2', \bar{\gamma}_c)$. Because α_2' is arbitrarily, strategy α_2 is undominated with respect to $\bar{\Gamma}$; that is, $\alpha_2 \in A_2$. Because α_2 is arbitrarily,

$\Delta([0,b])^* \subseteq A_2$.

<u>**Case (iii):**</u> Suppose that $r < b < |l|$. By the similar argument used in Case (i), we can show that $\Delta([0,b])^* \subseteq A_1$. Next, we suppose that $i = 2$. Fix $\alpha_2 \in \Delta([0,b])^*$ and $\alpha_2' \in \Delta(X)^*$ with $\alpha_2 \neq \alpha_2'$, arbitrarily. Note that as long as $x_1 \in [0,b]$, any possible policy pair lies in regions $Z_0 \cup Z_{12} \cup Z_{21}$.

Case (iii)-1: Suppose that $\alpha_2(r) \neq \alpha_2'(r)$. If $\alpha_2(r) < \alpha_2'(r)$, then fix $\alpha_1 = r$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Hence:

$$\begin{aligned} U_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - U_2(\alpha_1, \alpha_2', \bar{\gamma}_0) &= \left(1 - \frac{1}{2}\alpha_2(r)\right) - \left(1 - \frac{1}{2}\alpha_2'(r)\right) \\ &= \frac{1}{2}(\alpha_2'(r) - \alpha_2(r)) > 0. \end{aligned} \tag{B.13}$$

If $\alpha_2(r) > \alpha_2'(r)$, then fix $\alpha_1 = r$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence:

$$U_2(\alpha_1, \alpha_2, \bar{\gamma}_1) - U_2(\alpha_1, \alpha_2', \bar{\gamma}_1) = \frac{1}{2}(\alpha_2(r) - \alpha_2'(r)) > 0. \tag{B.14}$$

Case (iii)-2: Suppose that $\alpha_2(r) = \alpha_2'(r)$. Because $\alpha_2 \neq \alpha_2'$, there exists $x_2' \in S(\alpha_2)$ such that $\alpha_2(x_2') \neq \alpha_2'(x_2')$ and $x_2' \neq r$. If $\alpha_2(x_2') < \alpha_2'(x_2')$, then fix $\alpha_1 = x_2'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Hence:

$$\begin{aligned} U_2(\alpha_1, \alpha_2, \bar{\gamma}_0) - U_2(\alpha_1, \alpha_2', \bar{\gamma}_0) &= \left(1 - \frac{1}{2}(p\alpha_2(x_2') + (1-p)\alpha_2(r))\right) - \left(1 - \frac{1}{2}(p\alpha_2'(x_2') + (1-p)\alpha_2'(r))\right) \\ &= \frac{1}{2}p(\alpha_2'(x_2') - \alpha_2(x_2')) > 0. \end{aligned} \tag{B.15}$$

If $\alpha_2(x_2') > \alpha_2'(x_2')$, then fix $\alpha_1 = x_2'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence:

$$\begin{aligned} U_2(\alpha_1, \alpha_2, \bar{\gamma}_1) - U_2(\alpha_1, \alpha_2', \bar{\gamma}_1) &= \frac{1}{2}\left(p\alpha_2(x_2') + (1-p)\alpha_2(r)\right) - \frac{1}{2}(p\alpha_2'(x_2') + (1-p)\alpha_2'(r)) \\ &= \frac{1}{2}p(\alpha_2(x_2') - \alpha_2'(x_2')) > 0. \end{aligned} \tag{B.16}$$

Because α_2' is arbitrary, strategy α_2 is undominated with respect to $\bar{\Gamma}$; that is $\alpha_2 \in A_2$. Because α_2 is arbitrary, $\Delta([0,b])^* \subseteq A_2$.

<u>**Case (iv):**</u> Suppose that $b \geq |l|$. First, we can show that $\Delta([0,b])^* \subseteq A_2$ by the similar argument used in Case (iii). Next, we suppose that $i = 1$. Fix $\alpha_1 \in \Delta([0,b])^*$ and $\alpha_1' \in \Delta(X)^*$ with $\alpha_1 \neq \alpha_1'$, arbitrarily. Define $E(\alpha_1, \alpha_1') \equiv \{x_1 \in S(\alpha_1) | \alpha_1(x_1) \neq \alpha_1'(x_1)\}$.

Case (iv)-1: Suppose that there exists $x_1' \in E(\alpha_1, \alpha_1') \cap [0, |l|)$ such that $\alpha_1(x_1') < \alpha_1'(x_1')$. Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Note that because $(x_1, l) \in Z_{11} \cup \bar{Z}_{21}$ for any $x_1 \in [0,b]$, $\bar{\gamma}_1(x_1, l) = (1, 0)$

holds for any $x_1 \in [0,b]$; that is, candidate 1 wins for certain when candidate 2 is the ideological type. It is then sufficient to compare the winning probabilities when candidate 2 is the opportunistic type. Hence:

$$\begin{aligned} U_1(\alpha_1,\alpha_2,\bar{\gamma}_1) - U_1(\alpha_1',\alpha_2,\bar{\gamma}_1) &= \left(1-\frac{1}{2}p\alpha_1(x_1')\right) - \left(1-\frac{1}{2}p\alpha_1'(x_1')\right) \\ &= \frac{1}{2}p(\alpha_1'(x_1') - \alpha_1(x_1')) > 0. \end{aligned} \tag{B.17}$$

Case (iv)-2: Suppose that $E(\alpha_1,\alpha_1') \cap [0,|l|) \neq \emptyset$ and $\alpha_1(x_1) > \alpha_1'(x_1)$ holds for any $x_1 \in E(\alpha_1,\alpha_1') \cap [0,|l|)$. In this scenario, there exists $x_1' \in E(\alpha_1,\alpha_1') \cap [|l|,b]$ such that $\alpha_1(x_1') < \alpha_1'(x_1')$; otherwise, either $\sum_{x_1\in X}\alpha_1(x_1) > 1$ or $\sum_{x_1\in X}\alpha_1'(x_1) < 1$ holds. Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence, $\bar{\gamma}_1(x_1,l) = (1,0)$ holds for any $x_1 \in [0,b]$. That is, we can show that $U_1(\alpha_1,\alpha_2,\bar{\gamma}_1) - U_1(\alpha_1',\alpha_2,\bar{\gamma}_1) > 0$ by the same argument used in Case (iv)-1.

Case (iv)-3: Suppose that $E(\alpha_1,\alpha_1') \cap [0,|l|) = \emptyset$. In this scenario, there exists $x_1' \in E(\alpha_1,\alpha_1') \cap [|l|,b]$. Note that because $E(\alpha_1,\alpha_1') \cap [0,|l|) = \emptyset$, $\alpha_1(x_1) = \alpha_1'(x_1)$ holds for any $x_1 \in [0,|l|)$. Then, $\sum_{x_1\in[0,|l|)}\alpha_1(x_1) = \sum_{x_1\in[0,|l|)}\alpha_1'(x_1)$. There are the following three cases to be checked. First, suppose that $\alpha_1(x_1') < \alpha_1'(x_1')$. Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Hence, $\bar{\gamma}_1(x_1,l) = (1,0)$ holds for any $x_1 \in [0,b]$. That is, we can show that $U_1(\alpha_1,\alpha_2,\bar{\gamma}_1) - U_1(\alpha_1',\alpha_2,\bar{\gamma}_1) > 0$ by the same argument used in Case (iv)-1. Second, suppose that $\alpha_1(x_1') > \alpha_1'(x_1')$ and $\alpha_1(|l|) < \alpha_1(|l|)$. Fix $\alpha_2 = |l|$ and $\bar{\gamma}_1 \in \bar{\Gamma}$. Note that $\bar{\gamma}_1(x_1,l) = (1,0)$ for any $x_1 \in [0,b]$. Hence, we can show that $U_1(\alpha_1,\alpha_2,\bar{\gamma}_1) - U_1(\alpha_1',\alpha_2,\bar{\gamma}_1) > 0$ by the similar argument used in Case (iv)-1. Finally, suppose that $\alpha_1(x_1') > \alpha_1'(x_1')$ and $\alpha_1(|l|) \geq \alpha_1'(|l|)$. Fix $\alpha_2 = x_1'$ and $\bar{\gamma}_0 \in \bar{\Gamma}$. Note that $\bar{\gamma}_0(|l|,l) = (1/2,1/2)$, and $\bar{\gamma}_0(x_1,l) = (0,1)$ for any $x_1 \in (|l|,b]$. Hence:

$$\begin{aligned} U_1(\alpha_1,\alpha_2,\bar{\gamma}_0) - U_1(\alpha_1',\alpha_2,\bar{\gamma}_0) = &\left(\frac{1}{2}p\alpha_1(x_1') + \frac{1}{2}(1-p)\alpha_1(|l|) + (1-p)\sum_{x_1\in[0,|l|)}\alpha_1(x_1)\right) \\ &- \left(\frac{1}{2}p\alpha_1'(x_1') + \frac{1}{2}(1-p)\alpha_1'(|l|) + (1-p)\sum_{x_1\in[0,|l|)}\alpha_1'(x_1)\right) \\ = &\frac{1}{2}p(\alpha_1(x_1') - \alpha_1'(x_1')) + \frac{1}{2}(1-p)(\alpha_1(|l|) - \alpha_1'(|l|)) > 0. \end{aligned} \tag{B.18}$$

Because α_1' is arbitrarily, strategy α_1 is undominated with respect to $\bar{\Gamma}$; that is, $\alpha_1 \in A_1$. Because α_1 is arbitrary, $\Delta([0,b])^* \subseteq A_1$. ■

B.5 Robustness

In this section, we discuss the robustness of the results by relaxing the assumption of (i) single media outlet, (ii) asymmetry between the candidates, (iii) the tie-breaking rules, and (iv) the nonstrategicness of the ideological type. We demonstrate that the $(0, 0)$ equilibrium is still fragile under relaxing assumptions (i), (iii) and (iv), which suggests that we can obtain the qualitatively same results in these scenarios. That is, assumptions (i), (iii) and (iv) are not essential to the results. On the other hand, assumption (ii) is crucial to the results in the sense that symmetry between the candidates makes the $(0, 0)$ equilibrium more persistent. Hereafter, we use USE as a solution concept, so the message space is modified: $M(z) = \{z, \phi\}$ for any $z \in Z$ where $m = \phi$ means suppression of the information.

B.5.1 Multiple media outlets

The assumption of single media outlet is hard to justify in the real world, but it seems crucial to the results of the baseline model. Imagine a scenario where there exist multiple media outlets whose preferences are opposing biased. Because the outlets have opposing-biased preferences, then the voter certainly learns the truth by observing both messages. In other words, if one outlet has an incentive to suppress information, then the other outlet definitely has an incentive to disclose it. Because the voter learns the true information, equilibrium outcomes are never distorted.[38] Thus, because ideologically different media outlets coexist in mature democracies, electoral outcomes seems to be little biased in practice. However, this conjecture is based on an implicit assumption that both media outlets are sufficiently influential. In other words, the mechanism of mutual checking does not work well if the influence of one outlet dominates that of the other as demonstrated in the following example.[39]

Example 1. There exist outlets L and R whose preference bias is b_L and b_R with $b_L < 0 < b_R$ and $|b_L| > |l|/2$ and $b_R > r/2$, respectively. Each outlet correctly observes policy pair z, and then simultaneously sends messages m_L and m_R. However, the voter might not recognize the messages. Let s_j be the voter's observation from outlet j. We assume that the voter observes $s_R = m_R$ for certain, but he observes $s_L = m_L$ and ϕ with probabilities $2/5$ and $3/5$, respectively. That is, outlet R is more influential than outlet L in the sense that messages from outlet R is more likely to be recognized by the voter than those from outlet L. We assume that the voter cannot

[38] This phenomenon is well-known in the literature of persuasion games. See, for example, Milgrom and Roberts (1986) and Lipman and Seppi (1995).

[39] See the companion paper Miura (2013) for the detailed analysis.

distinguish whether message $m_j = \phi$ is sent or message $m_j = z$ does not reach after observing $s_j = \phi$. Furthermore, we assume that $p = 9/10$. The remaining setup is identical to that of the baseline model.

Claim 2 *There does not exist* $(0, 0)$ *equilibrium in Example 1.*

Proof. Suppose, in contrast, that there exists a $(0, 0)$ equilibrium. Notice that if $s \equiv (s_L, s_R) \neq (\phi, \phi)$, then the voter observes the truth. Hence, the voter's posterior after observing $s = (\phi, \phi)$ is $\mathcal{P}^*(z|\phi, \phi) = 1$ if $z = (r, 0)$ and 0 otherwise. Hence, the voter's best response to observation $s = (\phi, \phi)$ is choosing candidate 2 for certain. As a result, candidate 2's equilibrium winning probability is 11/20. However, if candidate 2 deviates to $\alpha_2 = r$, then his winning probability is 59/100. That is, candidate 2 has an incentive to deviate from $\alpha_2^* = 0$, which is a contradiction. ■

As demonstrated, coexistence of ideologically different outlets could not prevent distortion. When one outlet is sufficiently more influential than the other, the direct and the indirect distortion appear because of the same reason in the baseline scenario. Hence, the single-outlet model can be interpreted as a reduced form of a multiple-outlet model whose influence is unbalanced as discussed in Section 2.2.7. That is, the single outlet in the baseline model is a representative outlet in a country where the aggregate media coverage is not neutral, which is frequently observed even in democratic counties.

B.5.2 Asymmetry between the candidates

We have assumed that the candidates are asymmetric in the sense that the preferred policies of the ideological-type candidates differ. In order to consider the importance of the asymmetry, we first consider the model where the candidates are completely symmetric in the following sense; for $i \in \{1, 2\}$, if candidate i is the ideological type, then he proposes $x_i = r$ for certain. Except for this modification, the setup is identical to that in the baseline model. The result is as follows.

Proposition 6 *Consider the manipulated news model with symmetric candidates. Then, there exists a* $(0, 0)$ *equilibrium if and only if* $b \notin (r/2, r)$.

Proof. (Necessity) Suppose, in contrast, that there exists a $(0, 0)$ equilibrium when $r/2 \leq b < r$. Because we focus on USE and the candidates are symmetric, the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. That is, candidate 1's winning probability from $\alpha_1^* = 0$ is 1/2. However, if candidate 1 deviates to strategy $\alpha_1 = b$, then his winning probability is $1 - p/2$. Because $p < 1$, candidate 1 has an incentive to deviate, which is a contradiction.

(Sufficiency) Suppose that $b \notin (r/2, r)$. By the same argument used in the proof of Theorem 1, we can show that there exists a $(0,0)$ equilibrium supported by strategy β^* and off-the-equilibrium-path beliefs $\mathcal{P}^*$ given by (A.5), (A.6) and (A.7), respectively. ■

In contrast with the baseline model, a $(0,0)$ equilibrium exists except for $b \in (r/2, r)$.[40] The persistence of the policy convergence result is a consequence of symmetric candidates. Because the candidates are symmetric, the front-runner never exists; that is, the voter is indifferent between the candidates under message $m = \phi$. Hence, the candidates do not have a sufficient indirect-appealing incentive that is the main force breaking down the $(0,0)$ equilibrium. In other words, the asymmetry between the candidates reinforces the indirect-appealing incentives. Therefore, we can conclude that the asymmetric setup is essential for the results.

While asymmetry between the candidates is essential, we emphasize that this requirement is mild in the sense that excluding the symmetric case is sufficient to derive the baseline results. In the baseline model, we have assumed the following two kinds of asymmetry between the candidates. The first is *asymmetry in distance* in the sense that the voter has a strict preference for the policy pair given by the ideological-type candidates. The second is *asymmetry in direction* in the sense that one candidate prefers a positive policy, but the other prefers a negative policy when they are of the ideological type. We hereafter show that it is unnecessary to distinguish the difference; that is, either one of these asymmetries is sufficient to generate strong indirect-appealing incentives.

First, we consider the asymmetry in distance using the following one-sided setup. We assume that if candidate 2 is the ideological type, then he always proposes policy $x_2 = r'$ with $r' > r$. The remaining setup is identical to the baseline model. Similar to the baseline model, the $(0,0)$ equilibrium is still fragile.

Proposition 7 *Consider the manipulated news model with the candidates being only asymmetric in distance. Then, there exists a* $(0,0)$ *equilibrium if and only if* $b \leq r/2$.

Proof. (Necessity) Suppose, in contrast, that there exists a $(0,0)$ equilibrium when $b > r/2$. First, suppose that $r/2 < b \leq r$. In this scenario, candidate 2 is the front-runner, and then he has an incentive to deviate to $\alpha_2 = b$, which is a contradiction. Next, suppose that $b > r$. In this scenario, the induced outcome of the news-reporting stage is either $\bar{\gamma}_0$ or $\bar{\gamma}_1$. However, in each case, the

[40] Notice that there exist multiple equilibria as in the baseline model because the benefit from the direct appealing is discounted.

front-runner has an incentive to deviate $\alpha_i = \varepsilon > 0$ where ε is sufficiently small.

(Sufficiency) The construction used in the proof of Theorem 1 is still valid when $b \leq r/2$. ■

It is worth mentioning the difference in applicability between the baseline and one-sided setups. The baseline model is reasonable to describe a situation where the candidates and the outlet have an ideological conflict. Conversely, the one-sided setup is more appropriate to represent the situation where they have quantitative conflicts. For instance, they agree about a tax increase, but disagree about the level of the tax increase. The manipulated news model can apply to both scenarios and predicts the same distortion mechanism.

Next, we discuss the asymmetry in direction by considering the following setup. We assume that candidate 2 of the ideological type always proposes policy $x_2 = -r$. Except for this modification, the setup is identical to that in the baseline model. Note that the candidates are only asymmetric in direction. Again, the $(0, 0)$ equilibrium is fragile in this symmetric two-sided setup when the preference bias is not small.

Proposition 8 *Consider the manipulated news model with the candidates being only asymmetric in direction. Then, there exists a* $(0, 0)$ *equilibrium if and only if* $b \leq r/2$.

Proof We can show the statement by the similar argument used in the proof of Proposition 7. ■

In summary, difference between the policies proposed by the ideological candidates 1 and 2 are necessary for inducing the fragility of the $(0, 0)$ equilibrium. However, as long as the candidates are asymmetric in the above sense, the $(0, 0)$ equilibrium becomes fragile. In other words, we do not require a particular structure of asymmetry, and then the $(0, 0)$ equilibrium is generically fragile. Therefore, we can conclude that the assumption of asymmetric candidates is a mild requirement.

B.5.3 Tie-breaking rules

The tie-breaking rules specified in Requirement 2 seem crucial to the results. While the tie-breaking rule for the voter is well accepted in the literature, for the outlet it would seem much more controversial. We have assumed that the outlet discloses the information whenever the proposed policies are convergent, but there is no strong justification for this behavior. However, if the outlet suppresses the information, even when the proposed policies are convergent, then the serious multiplicity of equilibria occurs such that any strategy $\alpha_i \in \Delta([0, b])^*$ can be an equilibrium strategy.

Although this multiplicity is serious, most of the equilibria are not robust with respect to a small perturbation in the outlet's behavior. Instead of assuming full disclosure, we thus assume that the outlet discloses the information about convergent policies with probability $\varepsilon \in (0, 1]$. That is, the outlet that observes the convergent policy pairs randomizes disclosure and suppression.[41] For easy reference, we call this tie-breaking rule the *ε-randomization rule*, and the original the *disclosure rule*. We can show that even if the probability of disclosure ε is sufficiently small, then the set of equilibrium policy pairs under the ε-randomization rule is equivalent to that under the disclosure rule. Therefore, we can justify focusing on the equilibria satisfying the disclosure rule from the viewpoint of robustness.

Let us introduce additional notation, and then modify USE as follows. With abuse of notation, let $\beta(z) = (t, 1-t)$ represent that the outlet who observes policy pair z sends messages $m = z$ and ϕ with probabilities t and $1-t$, respectively. We say that the outlet's strategy β satisfies the ε-randomization rule if $\beta(z) = (\varepsilon, 1-\varepsilon)$ for any $z \in Z_{00}$. We say that the outlet's strategy β^ε is *ε-simple* if $\beta^\varepsilon(z) = (1, 0)$ for any $z \in Z_{11} \cup Z_{22} \cup Z_0 \backslash Z_{00}$, $(\varepsilon, 1-\varepsilon)$ for any $z \in Z_{00}$, and $(0, 1)$ for any $z \in Z_{12} \cup Z_{21}$. Let B^ε be the set of the ε-simple strategy of the outlet. We say that PBE $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ is an *ε-USE* if (i) $\gamma^* \in \Gamma$; (ii) $\beta^* \in B^\varepsilon$; and (iii) $(\alpha_1^*, \alpha_2^*) \in \Delta([0, b])^{*2}$. Notice that the disclosure rule is 1-randomization rule, and then USE is equivalent to 1-USE. Let $EP^\varepsilon \equiv \{(\alpha_1^*, \alpha_2^*) \in \Delta([0, b])^{*2} \mid (\alpha_1^*, \alpha_2^*)$ can be supported in an ε-USE$\}$ be the set of equilibrium strategies of the opportunistic-type candidates under the ε-randomization rule.

Proposition 9 *Consider the manipulated news model. Then, $EP^\varepsilon = EP^1$ for any $\varepsilon \in (0, 1]$.*

Proof. Without loss of generality, assume that $r < b < |l|$, and fix $\varepsilon \in (0, 1)$, arbitrarily.[42] First, we show that $EP^1 \subseteq EP^\varepsilon$. Take $(\alpha_1, \alpha_2) \in EP^1$, arbitrarily. That is, there exists β^1, γ^1, and $\mathcal{P}^1$ such that $(\alpha_1, \alpha_2, \beta^1, \gamma^1, \mathcal{P}^1)$ is a USE. By Proposition 3-(ii), the induced outcome of the news-reporting stage is either $\bar{\gamma}_1$ or $\bar{\gamma}_{1/2}$.

Case (i): Suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_1$. Now, we show that given α_1, α_2 and β^ε, γ^1 is the voter's best response. Let $\mathcal{P}^\varepsilon$ be the voter's consistent posterior given α_1, α_2 and β^ε. If $m = z$, then it is obvious that $\gamma^1(z) = y^v(z)$ is optimal. Because $\gamma^1(\phi) =$

[41] Because the result of the election is indifferent for the outlet when the proposed policy is convergent, such randomization can be supported as one of the best responses for the outlet.

[42] By the similar argument used in this case, we can show that this statement holds in other cases. The detail is available from the author upon the request.

(1, 0):

$$\begin{aligned}
&\sum_{z\in Z}|x_1|\mathcal{P}^1(z|\phi) < \sum_{z\in Z}|x_2|\mathcal{P}^1(z|\phi).\\
&\iff \sum_{z\in Z_{12}\cup Z_{21}}|x_1|\Pr(z|\alpha_1,\alpha_2) < \sum_{z\in Z_{12}\cup Z_{21}}|x_2|\Pr(z|\alpha_1,\alpha_2).\\
&\iff \sum_{z\in Z_{12}\cup Z_{21}}|x_1|\Pr(z|\alpha_1,\alpha_2) + (1-\varepsilon)\sum_{z\in Z_{00}}|x_1|\Pr(z|\alpha_1,\alpha_2) \qquad \text{(B.19)}\\
&\qquad\qquad < \sum_{z\in Z_{12}\cup Z_{21}}|x_2|\Pr(z|\alpha_1,\alpha_2) + (1-\varepsilon)\sum_{z\in Z_{00}}|x_2|\Pr(z|\alpha_1,\alpha_2).\\
&\iff \sum_{z\in Z}|x_1|\mathcal{P}^\varepsilon(z|\phi) < \sum_{z\in Z}|x_2|\mathcal{P}^\varepsilon(z|\phi).
\end{aligned}$$

Therefore, we can say that γ^1 is the voter's best response under the ε-randomization rule.

Next, we show that given $\alpha_j, \beta^\varepsilon$ and γ^1, α_i is the best response of candidate i. Notice that the candidates' winning probability from strategies α_1 and α_2 in the disclosure rule are:

$$\begin{aligned}
U_1(\alpha_1,\alpha_2,\bar{\gamma}_1) &= 1 - \frac{1}{2}\sum_{z\in Z_{00}}\Pr(z|\alpha_1,\alpha_2,\theta_1=O)\\
&= \sum_{z\in[0,b]^2\setminus Z_{00}}\Pr(z|\alpha_1,\alpha_2,\theta_1=O) + \frac{1}{2}\left(1 - \sum_{z\in[0,b]^2\setminus Z_{00}}\Pr(z|\alpha_1,\alpha_2,\theta_1=O)\right).
\end{aligned} \tag{B.20}$$

$$U_2(\alpha_1,\alpha_2,\bar{\gamma}_1) = \frac{1}{2}\sum_{z\in Z_{00}}\Pr(z|\alpha_1,\alpha_2,\theta_2=O). \tag{B.21}$$

Because $(\alpha_,\alpha_2) \in EP^1$, the following conditions should hold:

$$\sum_{z\in[0,b]^2\setminus Z_{00}}\Pr(z|\alpha_1,\alpha_2,\theta_1=O) \geq \sum_{z\in[0,b]^2\setminus Z_{00}}\Pr(z|\alpha_1',\alpha_2,\theta_1=O) \text{ for any } \alpha_1' \in \Delta([0,b])^*. \tag{B.22}$$

$$\sum_{z\in Z_{00}}\Pr(z|\alpha_1,\alpha_2,\theta_2=O) \geq \sum_{z\in Z_{00}}\Pr(z|\alpha_1,\alpha_2',\theta_2=O) \text{ for any } \alpha_2' \in \Delta([0,b])^*. \tag{B.23}$$

Likewise, the candidates' winning probabilities under the ε-randomization rule are as follows:

$$U_1(\alpha_1, \alpha_2, \bar{\gamma}_1^{\varepsilon}) = \sum_{z \in [0,b]^2 \setminus Z_{00}} \Pr(z|\alpha_1, \alpha_2, \theta_1 = O) + \left(1 - \frac{\varepsilon}{2}\right)\left(1 - \sum_{z \in [0,b]^2 \setminus Z_{00}} \Pr(z|\alpha_1, \alpha_2, \theta_1 = O)\right), \tag{B.24}$$

$$U_2(\alpha_1, \alpha_2, \bar{\gamma}_1^{\varepsilon}) = \frac{\varepsilon}{2} \sum_{z \in Z_{00}} \Pr(z|\alpha_1, \alpha_2, \theta_2 = O), \tag{B.25}$$

where $\bar{\gamma}_1^{\varepsilon}$ is the induced outcome of the news-reporting strategy under the ε-randomization rule. By (B.22) and $(B.23)$, we can say that α_i is a best response to $\alpha_j, \beta^{\varepsilon}$ and γ^1. Therefore, $(\alpha_1, \alpha_2, \beta^{\varepsilon}, \gamma^1; \mathcal{P}^{\varepsilon})$ is an ε-USE; that is, $(\alpha_1, \alpha_2) \in EP^{\varepsilon}$.

Case (ii) Suppose that the induced outcome of the news-reporting stage is $\bar{\gamma}_{1/2}$. By the similar argument used in Case (i), we can show that γ^1 is the voter's best response given α_1, α_2 and β^{ε}. Hence, the candidates' winning probabilities under the ε-randomization rule are $U_1(\alpha_1, \alpha_2, \bar{\gamma}_{1/2}^{\varepsilon}) = 1 - p/2$ and $U_2(\alpha_1, \alpha_2, \bar{\gamma}_{1/2}^{\varepsilon}) = 1/2$. That is, each candidate has no incentive to deviate from strategy α_i. Hence, $(\alpha_1, \alpha_2, \beta^{\varepsilon}, \gamma^1; \mathcal{P}^{\varepsilon})$ is an ε-USE, which means that $(\alpha_1, \alpha_2) \in EP^{\varepsilon}$. Because (α_1, α_2) is arbitrary, we can conclude that $EP^1 \subseteq EP^{\varepsilon}$. By the similar argument, we can also show that $EP^{\varepsilon} \subseteq EP^1$.[43] Therefore, $EP^{\varepsilon} = EP^1$ holds for any $\varepsilon \in (0, 1]$.

B.5.4 Fully strategic candidates

One of the most important factors in deriving the distortion mechanism is the voter's uncertainty about how the candidates behave. In other words, the nonstrategicness of the ideological type is irrelevant to the results. In the baseline model, we have assumed that there are two types of candidates, one of which is nonstrategic. In this subsection, we instead assume that the candidates are fully rational and office-motivated, but the candidates face uncertainty about the voter's preference. We then show the fragility of policy convergence in this new setup; that is, nonstrategicness of the ideological candidates is not essential.

The baseline model is modified as follows. Let $d \in \{l, 0, r\}$ be the voter's ideal policy. We assume that ideal policy d is private information of the voter and the outlet; that is, the candidates do not know it. However, we do not assume common prior on d. Instead, we represent the players' beliefs by a type space of Harsanyi (1967-68) denoted by $\mathcal{T} \equiv (T_1, T_2, T_o, T_v; \lambda_1, \lambda_2, \lambda_o, \lambda_v)$. For any $i \in \{1, 2, o, v\}$, player i's type t_i is an element of measurable set T_i, and player i's belief is represented by a measurable function $\lambda_i : T_i \to \Delta(T_{-i})$.

[43] This statement does not hold if $\varepsilon = 0$. Furthermore, we can show that candidate 2 cannot be the front-runner even under the ε-randomization rule by the similar argument used in the proof of Proposition 3.

We assume that the Harsanyi's type space $\mathcal{T}$ satisfies the following properties. For $i \in \{o, v\}$, let $d_i(t_i) \in \{l, 0, r\}$ be the ideal policy of the voter when player i is type t_i, and let $T_i^j \equiv \{t_i \in T_i | d_i(t_i) = j\}$ be the set of type t_i where the voter's ideal policy is $j \in \{l, 0, r\}$. That is, $T_i = T_i^l \cup T_i^0 \cup T_i^l$ holds. Because both the voter and the outlet know the voter's true ideal policy for certain, the following condition should be satisfied.

Assumption 1 *For any $(t_o, t_v) \in T_o \times T_v$, $d_o(t_o) = d_v(t_v)$ holds.*

We then assume that the candidates' beliefs over ideal policy d satisfy the following properties.

Assumption 2

(i) For any $t_1 \in T_1$, either exactly one of the following condition holds: (1) $\int_{t_{-1}:t_v \in T_v^0} \lambda_1(t_{-1}|t_1)dt_{-1} = 1$ or (2) $\int_{t_{-1}:t_v \in T_v^r} \lambda_1(t_{-1}|t_1)dt_{-1} = 1$.

(ii) For any $t_2 \in T_2$, either exactly one of the following condition holds: (1) $\int_{t_{-2}:t_v \in T_v^0} \lambda_2(t_{-2}|t_2)dt_{-2} = 1$ or (2) $\int_{t_{-2}:t_v \in T_v^l} \lambda_2(t_{-2}|t_2)dt_{-2} = 1$.

In other words, we assume that candidate 1 (resp. 2) believes either (i) $d = 0$ for certain, or (ii) $d = r$ (resp. l) for certain. We refer to the former as *moderate type* (M), and the latter as *extreme type* (E). For candidate $i \in \{1, 2\}$, let $f_i(t_i) \in \{M, E\}$ represent that candidate with type t_i is whether the moderate or the extremist. For any $i \in \{1, 2\}$ and $j \in \{M, E\}$, define $T_i^j \equiv \{t_i^j \in T_i | f_i(t_i) = j\}$, and then $T_i = T_i^M \cup T_i^E$ holds. Finally we assume the following properties.

Assumption 3

(i) For any $t_v \in T_v$ and $i \in \{1, 2\}$, $0 < \int_{t_{-v}:t_i \in T_i^M} \lambda_v(t_{-v}|t_v)dt_{-v} < 1$.

(ii) There exists $t_2 \in T_2^M$ such that $\int_{t_{-2}:t_1 \in T_1^M} \lambda_2(t_{-2}|t_2)dt_{-2} > 0$.

The first condition means that any type of the voter cannot pin down the type of the candidates. This condition is associated with the voter's uncertainty to the candidates' types in the baseline model. The second condition excludes the scenario where any type of candidate 2 certainly believes that candidate 1 is the extremist.

The candidates' preference is still given by (1). The voter's von Neumann-Morgenstern utility function $u : Z \times Y \times T_v \to \mathbb{R}$ is defined as follows:

$$v(z, y, t_v) \equiv \begin{cases} -|x_1 - d_v(t_v)| & \text{if } y = y_1, \\ -|x_2 - d_v(t_v)| & \text{if } y = y_2. \end{cases} \tag{B.26}$$

The timing of the game is identical to that of the baseline model except that there is no nature's move. Then, the players' strategies and beliefs are defines as follows. Let $\alpha_i : T_i \to \Delta(Z)^*$ be candidate i's strategy, $\beta : T_o \times Z \to M$ be the outlet's strategy, and $\gamma : T_v \times M \to \Delta(Y)$ be the voter's strategy.[44] Let $\mathcal{P} : T_v \times M \to \Delta(Z)$ be the voter's posterior belief over the policy pair space Z. Because of this modification, we adopt PBE with restrictions that (i) the voter's strategy is in set Γ, and (ii) the outlet's strategy is simple as a solution concept. Except for these modifications, this new setup is identical to that of the baseline model.

As a benchmark, we briefly discuss the situation where there is no media manipulation. Because the voter correctly observes policy pair z, the candidates directly appeal to the voter. That is, there exists a PBE where the candidates' strategies are the following:[45]

$$\alpha_1^*(t_1) = \begin{cases} 0 & \text{if } t_1 \in T_1^M, \\ r & \text{if } t_1 \in T_1^E; \end{cases} \tag{B.27}$$

$$\alpha_2^*(t_2) = \begin{cases} 0 & \text{if } t_2 \in T_2^M, \\ l & \text{if } t_2 \in T_2^E. \end{cases} \tag{B.28}$$

In this equilibrium, each candidate directly appeals to the voter upon his belief. Thus, the situation where the voter's ideal policy is 0 is associated with the baseline model; that is, from the perspective of the voter with type $t_v \in T_v^0$, candidate 1 (resp. 2) proposes policy $x_1 = 0$ (resp. $x_2 = 0$) with probability $p_1(t_v)$ (resp. $p_2(t_v)$) and policy $x_1 = r$ (resp. $x_2 = l$) with probability $1 - p_1(t_v)$ (resp. $1 - p_2(t_v)$) where $p_i(t_v) \equiv \int_{t_{-v}:t_i \in T_i^M} \lambda_v(t_{-v}|t_v)dt_{-v}$ for $i \in \{1, 2\}$. Hence, the moderate and the extremist are associated with the opportunistic-type and the ideological-type candidates in the baseline model, respectively. We call this equilibrium the *direct-appealing equilibrium*, and adopt it as a reference point instead of the $(0, 0)$ equilibrium in the baseline model.

Now, we move back to the scenario where the outlet behaves strategically. Because of the indirect-appealing incentive of the candidates, the sincere equilibrium brakes down as the baseline model.

Proposition 10 *Consider the manipulated news model with fully rational candidates. Then, there exists the direct-appealing equilibrium if and only if $b \leq r/2$.*

Proof. (Necessity) Suppose, in contrast, that there exists the direct-appealing equilibrium $(\alpha_1^*, \alpha_2^*, \beta^*, \gamma^*; \mathcal{P}^*)$ when $b > r/2$. Notice that $Z(\alpha_1^*, \alpha_2^*) \equiv \{(\alpha_1^*(t_1), \alpha_2^*(t_2))|(t_1, t_2) \in T_1 \times T_2\} = \{(0, 0), (0, l), (r, 0), (r, l)\}$,

[44] The outlet is restricted to the simple strategies.
[45] The detail is available from the author upon the request.

and any type of the voter believes that it is the set of possible policy pairs on the equilibrium path by Assumption 3-(i). If $t_v \in T_v^0$, then only policy pair $z = (r, 0)$ exists in the disagreement region. That is, candidate 2 is the front-runner for any $t_v \in T_v$. By Assumptions 2 and 3-(ii), there exists type $t_2' \in T_2^M$ who believes that (i) the voter's ideal policy is 0 for certain, and (ii) candidate 1 is moderate with positive probability $q(t_2') \equiv \int_{t_{-2}:t_1 \in T_1^M} \lambda_2(t_{-2}|t_2')dt_{-2}$. Hence, the winning probability of type t_2' from $\alpha_2^*(t_2') = 0$ is $1 - q(t_2')/2$. However, if he proposes policy $x_2 = \varepsilon > 0$ where ε is small enough, then his winning probability is 1. That is, candidate 2 of type t_2' has an incentive to deviate, which is a contradiction.

(Sufficiency) Suppose that $b \leq r/2$, and we show that the following is a PBE: α_1^* and α_2^* are given by (B.27) and (B.28), respectively:

$$\beta^*(t_o, z) = \begin{cases} \phi & \text{if } [t_o \in T_o^0 \cup T_o^l \text{ and } z \in \bar{Z}_{12} \cup Z_{21}] \text{ or } [t_o \in T_o^r \text{ and } z \in Z_{12} \cup \bar{Z}_{21}], \\ z & \text{otherwise;} \end{cases}$$

$$\gamma^*(t_v, m) = \begin{cases} y^v(z) & \text{if } m = z, \\ (1, 0) & \text{if } m = \phi \text{ and } t_v \in T_v^r, \\ (1/2, 1/2) & \text{if } m = \phi \text{ and } t_v \in T_v^0, \\ (0, 1) & \text{if } m = \phi \text{ and } t_v \in T_v^l, \end{cases} \tag{B.29}$$

$$S(\mathcal{P}^*(\cdot|t_v, \phi)) \subseteq \begin{cases} \{(b, b)\} & \text{if } t_v \in T_v^0, \\ \{(r, 0)\} & \text{if } t_v \in T_v^r, \\ Z_{21} & \text{if } t_v \in T_v^l. \end{cases}$$

It is obvious that $\mathcal{P}^*$ is consistent with Bayes' rule given α_1^*, α_2^* and β^*, and γ^* is optimal to the voter given $\mathcal{P}^*$. Also, given γ^*, it is obvious that β^* is optimal to the outlet. Thus, it is sufficient to show that α_1^* and α_2^* are optimal for any type of the candidates.

First, consider candidate 1's behavior. For any type $t_1 \in T_1^M$, the winning probability from $\alpha_1^*(t_1) = 0$ is $1 - q_1(t_1)/2$ where $q(t_1) \equiv \int_{t_{-1}:t_2 \in T_2^M} \lambda_1(t_{-1}|t_1)dt_{-1}$. Notice that there is no front-runner in this scenario. Hence, for any $x_1 \in X$, $\mu_1(x_1, 0) \leq 1/2$ and $\mu_1(x_1, l) \leq 1$. That is, any type $t_1 \in T_1^M$ of candidate 1 has no incentive to deviate. For any $t_1 \in T_1^E$, the winning probability from $\alpha_1^*(t_1) = r$ is 1; that is, he has no incentive to deviate. Therefore, α_1^* is optimal to candidate 1. Likewise, we can show that α_2^* is optimal to candidate 2. ■

Appendix C: Figures

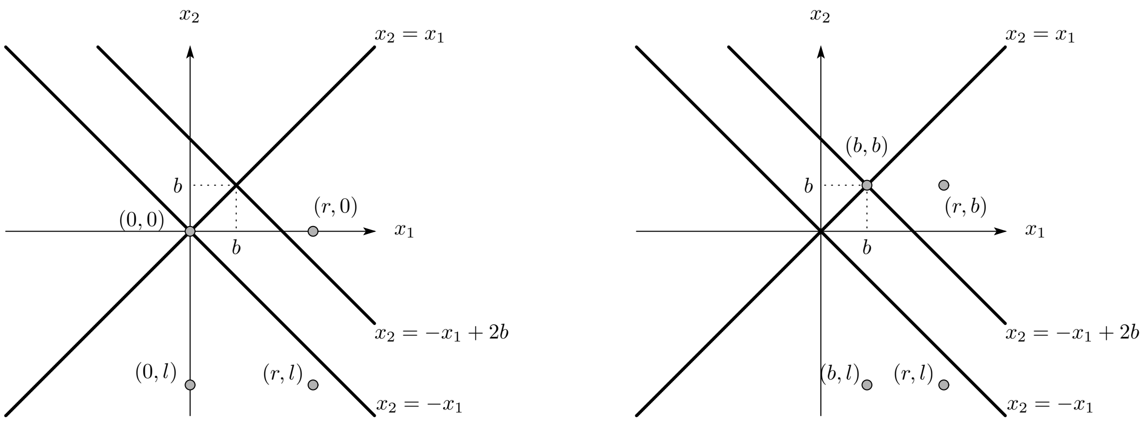

Figure 5: Case (i): lower bound (left) and upper bound (right)

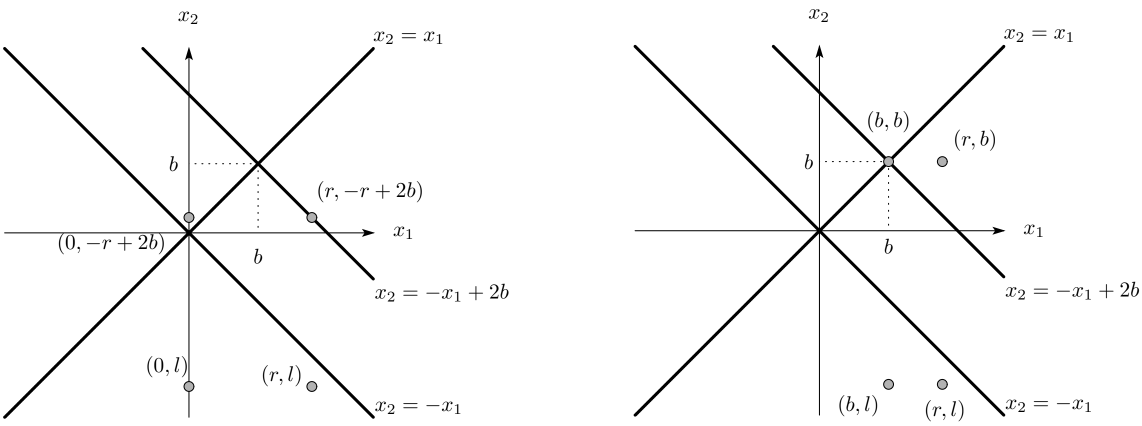

Figure 6: Case (ii): lower bound (left) and upper bound (right)

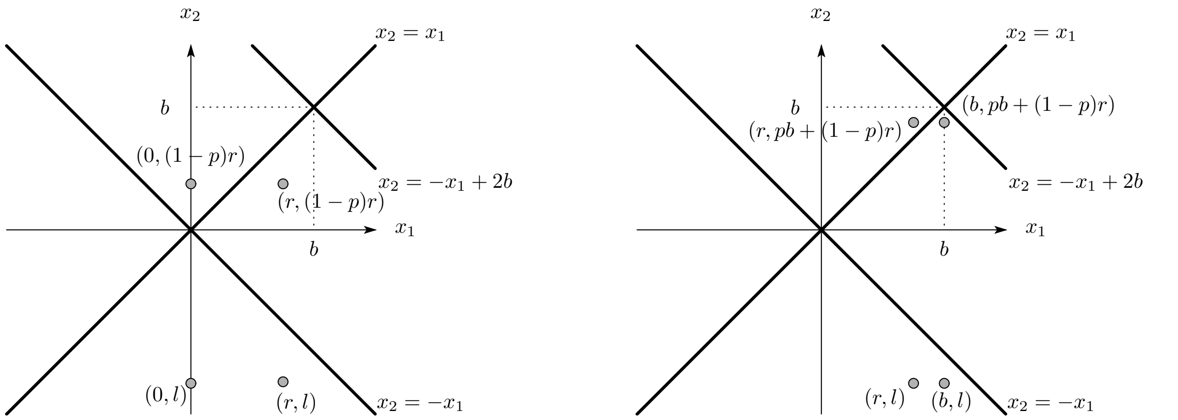

Figure 7: Case (iii): lower bound (left) and upper bound (right)

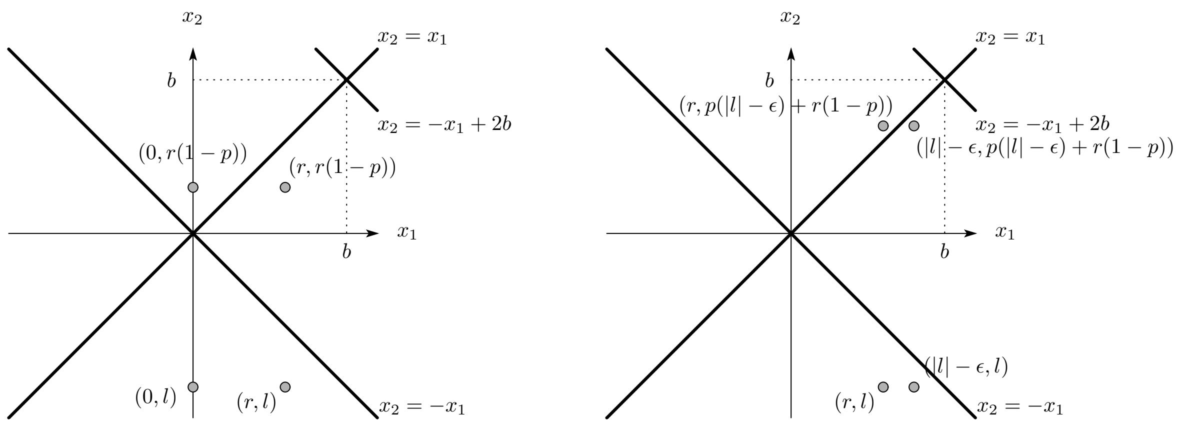

Figure 8: Case (iv): lower bound (left) and upper bound (right)