Non-cooperative equilibrium with multiple deviators

Dmitry Levando,

National Research University - Higher School of Economics, Moscow

CEPET Workshop 2016
University of Udine, Italy

24 June, 2016

The problem:

1. Nash equilibrium: every player considers oneself as the only deviator
   - 1.1 if the same player assumes existence of other deviators, what is an appropriate equilibrium concept then?
2. strong Nash equilibrium - equilibrium robust to a deviation of a coalition of any size
   - 2.1 $N = 3$, initial state - a grand coalition, $\{1, 2, 3\}$
   - 2.2 *all players deviate* in different groups: $\Big\{\{1\}, \{2\}, \{3\}\Big\}$, $\Big\{\{12\}, \{3\}\Big\}$, $\Big\{\{13\}, \{2\}\Big\}$, $\Big\{\{1\}, \{23\}\Big\}$
   - 2.3 Do these partitions (coalition structures) describe different deviating coalitions or not?
   - 2.4 Does strong Nash discriminate between these deviating coalition structures?
   - 2.5 What are the externalities between/within the players during the deviation, finally occurred in different groups of the same partition?
3. Nash bargaining solution: a central planner ("an umpire") implements pre-play agreements with axioms on payoffs.

Comment on the word "an umpire"

Nash (1951): *A less obvious type of application (of non-cooperative games) is to the study of coop-erative games. By a cooperative game we mean a situation involving a set of players, pure strategies, and payoffs as usual; but with the assumption that the players can and will collaborate as they do in the von Neumann and Morgenstern theory. This means the players may communicate and form coalitions which will be enforced by* **an umpire**. *It is unnecessarily restrictive, however, to assume any transferability or even comparability of the pay-offs [which should be in utility units] to different players."*

The suggested approach changes the role of the umpire or a central planner, more further.

Applied motivation: European currency system:

1. European Currency Union was designed as an equilibrium system. By 2016 there are 19 member countries, with a debt/GNP ratio membership criterium (simplified).
2. Nash approach: if there is only one deviator, then all other country-members are able to penalize a deviator.
3. By 2016 more than half from 19 members are deviators.
   - 3.1 Can deviators penalize themselves?
   - 3.2 Can non-deviating countries have enough power to penalize the deviators?
4. What is a maximum number of deviators, when a desirable European Currency Union equilibrium is still implementable?
5. Externalities between deviators and others are costly.
6. Shapley value approach is inappropriate: it does consider an opportunistic behaviour of member countries (besides other limitations). An alternative - further.
7. Similar problems exist in international emission agreements.

Applied motivation: herding behavior:

1. Herding is an individual action, may be correlated between agents
2. Herding is a deviation of one group or deviations of many different groups ?
3. How big should be a total herding set(s) to become contagious for the rest at the market?

Similar problems:

1. What is a coalition structure of workers in a union need to start a strike?
2. International refugee migration ...

Choice of a tool: non-cooperative games

Maskin (**2011**)[1]: "I believe that there are three (related) reasons for the historical shift from cooperative to noncooperative theory: (a) most cooperative theory ignores externalities, the possibility that a coalition can be affected by the actions of those not in the coalition; (b) it assumes that a Pareto efficient outcome will be reached; and (c) it supposes that the grand coalition (the coalition of all players) will form."
Further: "These features of cooperative theory are problematic because most applications of game theory to economics involve settings in which externalities are important, Pareto inefficiency arises, and the grand coalition does not form. "

Presenting paper uses and develops *only* non-cooperative approach

[1] Maskin, E., *Commentary: Nash equilibrium and mechanism design*, Games and Economic Behavior. 2011. v. 71. N. 1. pp. 9-11.

Completed research agenda:

1. Constructed a non-cooperative game with modified individual strategy sets and imbedded mechanism design features.
2. The game has intra/inter group externalities and individual payoffs.
3. **No** assumptions present: efficiency of a final equilibrium, existence of a pre-play communication, axioms for payoffs, super-additivity,....
4. Only individual (non-bounded) rationality (rationalisable bounded rationality is left for the future paper)
5. Constructed a family of equilibrium concepts for different number of deviators.
6. Suggested a way to measure a trust over a non-cooperative equilibrium.
7. Apply to " a stag or a hare" game.

Standard notation

1. N, a finite set of players, also a number of players, $2 \leq N < \infty$
2. $S_i \in R^1_+$, a compact for every $i \in N$, ex. $S_i = [0, 1]$
3. $U_i : S \equiv \times_{i \in N} S_i \mapsto R$, $U_i \leq M < \infty$, a set of payoffs, a bounded integrable functions for every $i \in N$,
4. $\Gamma = \langle N, (S_i, U_i)_{i \in N} \rangle$ a non-cooperative game
5. $\Delta(S_i)$ - an infinite-dimensional set of probability measures over S_i (set of mixed strategies), uniformly bounded, uniformly continuous set, all measures are regular and tight
6. $EU_i = \int_S U_i(s_i, s_{-i}) d\sigma_i d\sigma_{-i}$ expected utility (linear operator or a scalar product) for i

$\sigma^* = (\sigma_i^*)_{i \in N}$ - an equilibrium mixed strategy,

$$EU_i(\sigma_i^*, \sigma_{-i}^*) \geq EU_i(\sigma_i, \sigma_{-i}^*), \ \sigma_i \neq \sigma_i^*, \text{ for all } i \in N.$$

EU_i - uniformly bounded, compact, uniformly continuous set. Existence of σ^* follows from properties of compact continuous mappings of compact continuous sets.

Changes for a central planner in comparison to a cooperative game of Nash:

Nash coalition approach: a central planner enforces implementation of pre-play agreements.

Suggested approach: to include a mechanism design into a definition of a game with a specially designed strategy set:

1. to announce a fixed number of deviators, $K = 1, N$,
2. to announce $\mathcal{P}(K) = \{P^{\beta}(K) : \beta\}$, a list of coalition structures or eligible "partitions" (eligible partitions of players into groups) or jurisdictions in terms of shared equilibrium
3. to announce a family of rules $\mathcal{R}(K)$ to form partitions from $\mathcal{P}(K)$ and partition-specific strategy sets $\{S_{\beta}\}$ such that $\forall s \in S_{\beta} \mid s \in R^{N}$, $\forall \beta$, $\mathcal{R}(K)$ properties are not studied in this paper
4. to enforce the rules.

New question appears - social welfare maximization coalition rules.

A non-cooperative simultaneous game

1. K - number of deviators, $K = 1, \ldots, N$,
2. $\mathcal{P}(K) = \{P^{\beta}(K) \colon \beta, K = 1, N\}$ - a partition of N, when a maximum size of a group does not exceed K,
   $G(K) = \#\mathcal{P}(K)$ - size of $\mathcal{P}(K)$,
3. a strategy set of i for $\mathcal{P}(K)$ is

$$S_i(K) = \left\{ s_{\beta_i} \in S_{\beta_i} \colon \beta_i = 1, \ldots, G(K), S_{\beta_i} \equiv S_i \subset R \right\},$$

   a list of compacts $S_i(K) = \{\underbrace{[0,1]_{\beta_1=1}, \ldots, [0,1]_{\beta_1=G(K)}}_{G(K) \text{ time}}\}$ a set i's of strategies in all possible partitions
4. a player i chooses a partition β_i, and an action inside, s_{β_i}. A final partition is $P^{\beta} \in \mathcal{P}(K)$, β depends on profile $(s_{\beta_i})_{i \in N}$. A corresponding action s_{i,β_i} from the subset $S_{i,\beta_i} \in S_i(K)$. Final partition P^{β} may differ for i choosing β_i. Size of a chosen group can not exceed K. Only single group participation.
5. $s = (s_{\beta_i})_{i \in N}$ - a strategy profile of all players, $s \in S$,

Family of group formation rules

1. $\mathcal{R}(K)$ - a family of group formation rules over a set of all strategies of all players $S = \times_{i \in N} S_i(K)$, $K = 1, N$
2. $\mathcal{R}(K) : S \mapsto \Big\{ \{S_\beta\} : \beta = 1, G(K), \quad S = \cup_\beta S_\beta, \quad S_{\bar{\beta}} \cap S_{\hat{\beta}} = \{\emptyset\}, \Big\}$,
3. s_{β_i} - a strategy of i for a partition P^{β_i}, $s_{i,\beta}$ is a final strategy and a partition assigned for i, contingent on $(s_{\beta_j})_{j \neq i}$
4. $\boxed{\forall (s_{\beta_i})_{i \in N} \in (S_i(K))_{i \in N} \quad \exists! \beta \; s_\beta = (s_{i,\beta})_{i \in N} \in S_\beta}$
5. any $s_\beta = (s_{j,\beta})_{i \in N} \in S_\beta$ may have more than one pre-image $(s_i)_{i \in N} \in S$, where $s_i \equiv s_{\beta_j}$
6. every S_β - a bounded compact in R^N, (may be partially open)
7. every $s \in S$ has only one final partition P^β,
8. $S_{\bar{\beta}} \cap S_{\hat{\beta}} = \{\emptyset\}$, S_β may have a composed structure, ex. below
9. $S = \cup_\beta S_\beta$

$\mathcal{P}(K), \mathcal{R}(K)$ is a mechanism design to include into a non-cooperative game. An example further

Example: construction of a strategy set $S_i(2)$, $N = 3$:

- Players $= 1, 2, 3$; $N = 3$; $K = 2$, a family of partitions is $\mathcal{P}(K) = \{P^{\beta=1} = \{1, 2, 3\}, P^{\beta=2} = \{\{12\}, 3\}, P^{\beta=3} = \{\{13\}, 2\}, P^{\beta=4} = \{1, \{23\}\}\}$, $G(K) = 4$; $S_{\beta_i} = S_i \equiv [0, 1]$, as above
- $S_1(2) = \{s_1 : s_1 \in [0, 1]_{\beta_1=1} \sqcup [0, 1]_{\beta_1=2} \sqcup [0, 1]_{\beta_1=3} \sqcup [0, 1]_{\beta_1=4}\}$.
- $\sqcup$ - a disjoint union symbol
- Players individually choose groups and strategies without coordination, guided only by private pre-game beliefs.
- If $i = 1$ believes he deviates with 2, then $i = 1$ chooses partition $P^{\beta=2}$, but 2 has his own plans.
- If $i = 1$ believes he deviates with 3,then $i = 1$ chooses partition $P^{\beta=3}$, but 3 has his own plans.
- If $i = 1$ believes he deviates alone, then $i = 1$ chooses either partition $P^{\beta=1}$ or $P^{\beta=4}$.
- Conclusion: a choice of a partner is not enough unless a partition is specified.

Family of group formation rules and a game:

1. Example. no more than K players in a group, a group is formed from a unanimous choice of the group, otherwise a player obtains a singleton.
2. Central planner supports group formation rules $\mathcal{R}(K)$ given a strategy profile of all players.
3. Ex. Strategy sets $(S_{1,\{\{1,3\},\{2\}\}}, S_{2,\{\{1\},\{2,3\}\}}, S_{3,\{\{1\},\{2,3\}\}})$ lead to the same partition $\{\{1\},\{2,3\}\}$, with a partition-specific strategy profile $S_{\beta=\{\{1\},\{2,3\}\}} = \Big\{(s_i)_{i\in N} \in S_{1,\{\{1,3\},\{2\}\}} \times S_{2,\{\{1\},\{23\}\}} \times S_{3,\{\{1\},\{23\}\}} = [0,1]^3$
4. the same partition can be obtained from other choices of player $i = 1$!

Partition contingent payoffs, $\forall\beta$: $U_i^\beta(K)\Big(s_{\beta_1},\ldots,s_{\beta_N}\Big) : S_\beta \mapsto R$, $U_i^\beta(K), \forall i \in N$, are bounded, integrable(!) almost everywhere on S_β formed by strategy sets for every formed partition P^β.
$\Gamma(K) = \langle N, K, \mathcal{P}(K), \mathcal{R}(K), (S_i(K), U_i(K))_{i\in N}\rangle$ a non-cooperative game with a mechanism design, $\mathcal{P}(K), \mathcal{R}(K)$,

Mixed strategies in Γ:

1. $\Delta(S_i(K))$ - an infinite-dimensional probability space over $S_i(K)$, or a set of mixed strategies $\{\sigma_i(K)\}$ (probability measures) over $S_i(K)$. Example, $N = 3$, $K = 2$, for every i, $\forall\beta_i \in \{1,2,3,4\}$: $S_{\beta_i} = [0,1]$, four partitions to choose: $\Delta\Big(S_i(2)\Big) = \Delta\Big(\{[0,1]_{\beta_i=1}, [0,1]_{\beta_i=2}, [0,1]_{\beta_i=3}, [0,1]_{\beta_i=4}\}\Big)$,

$$\int_{S_{\beta_i=1}} d\sigma_i(s_i) + \int_{S_{\beta_i=2}} d\sigma_i(s_i) + \int_{S_{\beta_i=3}} d\sigma_i(s_i) + \int_{S_{\beta_i=4}} d\sigma_i(s_i) = 1$$

2. $\Delta\Big(S_i(K)\Big)$ is a bounded, compact, complete and uniformly convex set (Lux, 2002), with a general element $\sigma_i(K)$
3. $EU_i^{\mathcal{R}(K)}\Big(\sigma_i(K), \sigma_{-i}(K)\Big) : \Delta\Big(S_i(K)\Big) \times \Big(\times_{j\neq i}\, \Delta S_j(K)\Big) \mapsto R$ - expected utility of i, bounded, complete and uniform continuous linear mapping (a linear operator) over $\Delta(S_i(K))$, given $\sigma_{-i}(K) \in \Big(\times_{j\neq i}\, \Delta S_j(K)\Big)$
4. also next slide

Expected utility in details

$EU_i^{\mathcal{R}(K)}(\sigma_i(K), \sigma_{-i}(K))$, in terms of choosable partitions, $\beta_1, \ldots, \beta_N$:

$$\sum_{j=1,N;\beta_j=1,\ldots,G_j(K)} \int_{S_{\beta_1} \times \ldots S_{\beta_j} \ldots \times S_{\beta_N}} U_i(s_i, s_{-i}) d\sigma_i(K) d\sigma_{-i}(K).$$

Mixed strategies can be defined in terms of initial strategy set (above) or in terms of partition specific strategy sets $(S_\beta)_{\beta=1,G(K)}$.
$\mathcal{R}(K)$ - a family of group formation rules, $\mathcal{R}(K) \colon (S_i(K))_{i \in N} \mapsto \left\{ S_\beta \colon \beta = 1, G(K), \ \cup_\beta S_\beta = S, S_{\bar{\beta}} \cap S_{\hat{\beta}} = \{\emptyset\} \right\}$,
$\mathcal{R}(K)$ transports a measure of choice strategies $(\sigma_i)_{i \in N}$ from $\times_{i \in N} S_i(K)$ to measures of final partition-specific strategies $\cup_\beta S_\beta$
See Billingsley (1965, 1999), Dudley (2005), Schilling (2002) on measure transportation

Unanimous rule formation

If all players from group g^{β_0} choose the partition P^{β_0}, then formation still depends on partitions chosen by all other players. But any final partition $P^{\bar{\beta}}$ will contain the unanimously chosen group $g^{\beta_0} \in P^{\bar{\beta}}$.

If there is a mismatch in choices of players i_1, i_2, who choose different partitions $P^{\bar{\beta}}$ and $P^{\bar{\bar{\beta}}_2}$, where $i_1 \in g^{\bar{\beta}}$, $i_2 \in g^{\bar{\bar{\beta}}}$, $i_1, i_2 \in g^{\bar{\bar{\beta}}}$ but $i_1 \notin g^{\bar{\bar{\beta}}}$. Then partition $P^{\bar{\beta}}$ can not be formed from a unanimous rule, and in any final partition a player i_2 obtains a singleton.

Example: transformation of $S(K)$ into partition specific domains, $N = 3$, for a unanimous group formation rule.

K	$i = 1$	$i = 2$	$i = 3$	
1, 2, 3	$[0,1]_{\beta_1=\{\{1\},\{2\},\{3\}\}}$	$[0,1]_{\beta_2=\{\{1\},\{2\},\{3\}\}}$	$[0,1]_{\beta_3=\{\{1\},\{2\},\{3\}\}}$	$[0,1$
2, 3	$[0,1]_{\beta_1=any}$	$[0,1]_{\beta_2=\{\{1\},\{2\},\{3\}\}}$	$[0,1]_{\beta_3=\{\{1\},\{2\},\{3\}\}}$	$[0,1$
2, 3	$[0,1]_{\beta_1=\{\{1\},\{2\},\{3\}\}}$	$[0,1]_{\beta_2=any}$	$[0,1]_{\beta_3=\{\{1\},\{2\},\{3\}\}}$	$[0,1$
2, 3	$[0,1]_{\beta_1=\{\{1\},\{2\},\{3\}\}}$	$[0,1]_{\beta_2=\{\{1\},\{2\},\{3\}\}}$	$[0,1]_{\beta_3=any}$	$[0,1$
2, 3	$[0,1]_{\beta_1=\{\{1,2\},\{3\}\}}$	$[0,1]_{\beta_2=\{\{1,2\},\{3\}\}}$	$[0,1]_{\beta_3=any}$	$[0,1$
2, 3	$[0,1]_{\beta_1=\{\{1,3\},\{2\}\}}$	$[0,1]_{\beta_2=any}$	$[0,1]_{\beta_3=\{\{1,3\},\{2\}\}}$	$[0,1$
2, 3	$[0,1]_{\beta_1=any}$	$[0,1]_{\beta_2=\{\{1\},\{2,3\}\}}$	$[0,1]_{\beta_3=\{\{1\},\{2,3\}\}}$	$[0,1$
3	$[0,1]_{\beta_1=\{1,2,3\}}$	$[0,1]_{\beta_2=\{1,2,3\}}$	$[0,1]_{\beta_3=\{1,2,3\}}$	$[0$

$$S_{\beta=\{\{1\},\{2\},\{3\}\}} = \left\{ \{[0,1]^3\}_r \mid r \text{ such that at least two players choose} \left\{ \{1\}, \{2\}, \{3\} \right\} \right\}$$

$$S_{\beta=\{\{1,2\},\{3\}\}} = \left\{ \{[0,1]^3\} \mid \text{player 3 chooses any } \beta_3 \right\}$$

Comments for the previous slide

1. for $K = 2$ a partition of singletons $\{\{1\}, \{3\}, \{3\}\}$ appears from few different partition choices of players
2. $S_{\{\{1\},\{3\},\{3\}\}}$ has a compound structure and includes all these cases
3. the same for some other final partitions
4. group formation rules anticipates all cases of partition choices

Payoffs in a unanimous group formation game

Sizes of partitions depend on K

Table : Individual partition-contingent payoff depends only on i's strategy, however a realization of a partition depends on a whole strategy profile s.

K	partition	U_1^β	U_2^β	U_3^β
$1,2,3$	$\{\{1\},\{2\},\{3\}\}$	$s_{1,\beta=1}+2$	$s_{2,\beta=1}+2$	$s_{3,\beta=1}+2$
$2,3$	$\{\{1,2\},\{3\}\}$	$s_{1,\beta=2}+5$	$s_{2,\beta=2}+7$	$s_{3,\beta=2}+6$
$2,3$	$\{\{1,3\},\{2\}\}$	$s_{1,\beta=3}+6$	$s_{2,\beta=3}+5$	$s_{3,\beta=3}+7$
$2,3$	$\{\{1\},\{2,3\}\}$	$s_{1,\beta=4}+7$	$s_{2,\beta=4}+6$	$s_{3,\beta=4}+5$
3	$\{1,2,3\}$	$s_{1,\beta=5}-1$	$s_{2,\beta=5}-1$	$s_{3,\beta=5}-1$

An increase in K adds new subsets due to nested property of strategy sets, $S_i(K=1) \subset S_i(K=2) \subset S_i(K=3)$, thus after an increase in K we need to define payoffs only for the newly added areas. Payoffs in the game are constructed in such a way that if $K=3$ neither player is not interested to choose a grand coalition $\{1,2,3\}$. An increase in number of deviators from $K=2$ to $K=3$ does not result in better off for the players.

Equilibrium

Definition (equilibrium with no more than K deviators)

For the game $\Gamma(K)$ a K-Nash equilibrium is a mixed strategy profile $\sigma^*(K) = (\sigma_i^*(K))_{i \in N}$ such that for every $i \in N$ and every $\sigma_i(K) \neq \sigma_i^*(K)$ there is

$$EU_i^{\mathcal{R}(K)}\Big(\sigma_i^*(K), \sigma_{-i}^*(K)\Big) \geq EU_i^{\mathcal{R}(K)}\Big(\sigma_i(K), \sigma_{-i}^*(K)\Big).$$

Theorem

K-Nash equilibrium exists.

Existence: from compactness, completeness and uniform continuity of a domain and a range of the mapping $(\Psi_i)_{i \in N}$ such that $\forall i \in N$

$$\Psi_i = \arg\max_{\sigma_i(K) \in \Delta(S_i(K))} EU_i^{\mathcal{R}(K)}\Big(\sigma_i(K), \sigma_{-i}(K)\Big) \mapsto \Delta\Big(S_i(K)\Big),$$

$\sigma_{-i}(K) \in \Delta\Big(S_j(K)\Big)_{j \neq i}$, with a final application of the Shauder fixed point theorem.

Ex. Equilibrium *pure* strategies

Table : Payoff maximizing strategies of players for a unanimous group formation rule with different number of deviators K

players	$s_i^*(K=1)$	$s_i^*(K=2)$	$s_i^*(K=3)$
$i=1$	$1_{\beta_1=1}$	$1_{\beta_1=2}$ $1_{\beta_1=3}$	the same
$i=2$	$1_{\beta_2=1}$	$1_{\beta_2=2}$ $1_{\beta_2=4}$	the same
$i=3$	$1_{\beta_3=1}$	$1_{\beta_3=1}$ $1_{\beta_3=4}$	the same

Not unique payoff maximizing strategy profiles for $K > 1$.

Ex. Unanimous group formation: mixed strategies

Table : Equilibrium mixed strategies for players in the unanimous group formation game

partition	$i = 1$	$i = 2$	$i = 3$
$\{\{12\},\{3\}\}$	$\frac{7}{5+6+7}$	$\frac{5}{5+6+7}$	$\frac{6}{5+6+7}$
$\{\{13\},\{2\}\}$	$\frac{6}{5+6+7}$	$\frac{7}{5+6+7}$	$\frac{7}{5+6+7}$
$\{\{1\},\{23\}\}$	$\frac{5}{5+6+7}$	$\frac{6}{5+6+7}$	$\frac{5}{5+6+7}$

General case - application of Fourier analysis for several disjoint domains is not a trivial task

Comparison with strong Nash equilibrium and other non-cooperative equilibrium concepts:

K-Nash equilibrium can be defined for every group formation set of rule $\mathcal{R}(K)$ and for every K
It differs from strong Nash equilibrium, coalition-proof and k-equilibrium:

1. there is no restriction that all deviators move as one group,
2. presence of externalities for all players, independently of a coalition structure of any partition (also the difference with coalition-proof and k-equilibrium concepts). This is a response to Maskin (2011) research program.

For $K = N$ and any well-defined group formation rule $\mathcal{R}(N)$ the introduced equilibrium concept seams reacher than a strong Nash equilibrium

Reconsideting the equilibrium: 1

1. In a simultaneous game need to anticipate opportunism or formation of blocking coalitions (*sic!* plural)
2. Opportunism means - a desire of a set of players from N to deviate *after* the game is played - robustness of an equilibrium to a change in K
3. Need to compare expected payoff from two games (an example of unanimous group formation rule above)
4. $\mathcal{R}(K)$, $\mathcal{P}(K)$ are defined for $K = 1, N$, but every game is constructed for a fixed K
5. Below - construction of an equilibrium criterium for compatible group formation rules
6. Clear definition of group partitions and formation rules a separate research program for mechanism design in social choice theory.

Reconsidering the equilibrium: 2

$$EU_i^{\mathcal{R}(\mathcal{K}_{\mathcal{IN}})}\left(\sigma_i^*(K_{IN}), \sigma_{-i}^*(K_{IN})\right) \geq EU_i^{\mathcal{R}(\mathcal{K}_{\mathcal{OUT}})}\left(\sigma_i(K_{OUT}), \sigma_{-i}^*(K_{OUT})\right).$$

- Equilibrium is an incentive compatibility constraint for every player: no gain from a deviation for a given K
- Fix group formation rules $\mathcal{R}(K)$ in the left-side, but vary K, $\mathcal{R}(K)$ in right-side
- Left-side - expected payoff from entering into an equilibrium within a set of no more than (K_{IN}) players
- Right-side - expected payoff from quitting an equilibrium within a set of no more that K_{OUT} players, for $\mathcal{R}(K_{OUT})$
- For the given equilibrium strategy profile $\sigma^*(K_{IN})$, what is a maximum K_{OUT} when $\sigma^*(K_{IN})$ is still an equilibrium strategy profile?

Non-cooperative stability criterium of an equilibrium to an increase in number of deviators

Forming equilibrium has two sides - to join and to prevent opportunistic behavior later

$$K^* = \max_{\substack{K_{IN}<K \\ i\in N}} \left\{ EU_i^{\mathcal{R}(K_{IN})}\left(\sigma_i^*(K_{IN}), \sigma_{-i}^*(K_{IN})\right) \geq EU_i^{\mathcal{R}(K)}\left(\sigma_i^*(K), \sigma_{-i}^*(K)\right) \right\}$$

K^* is a maximum number of deviators, when $\sigma^*(K_{IN})$ is still an equilibrium

1. If $K > K^*$ then some deviating players which may have a profitable deviation:
   $EU_i^{\mathcal{R}(K)}\left(\sigma_i^*(K), \sigma_{-i}^*(K)\right) > EU_i^{\mathcal{R}(K_{IN})}\left(\sigma_i^*(K_{IN}), \sigma_{-i}^*(K_{IN})\right)$
2. Unanimous group formation example above - the same principle for $\mathcal{R}(K)$, $K = 1, 2, 3$
3. K^* - a suggested measure of trust (or of commitment) of players to the equilibrium strategy profile $\sigma^*(K_{IN})$, based on comparison of equilibrium payoffs from games $\Gamma(K_{IN})$ and $\Gamma(K)$, $K > K_{IN}$
4. Can be applied to compare different families of group formation

For discussion: selection between payoff ordered equilibria

Table : A variant of "a stag or a hare" hunting game. A strategy - a target and a group size,

	hare, alone	stag, alone	hare, together	stag, together
hare, alone	(8;8)	(8;0)	(8;8)	(8;0)
stag, alone	(0;8)	(0;0)	(0;8)	(0;0)
hare, together	(8;8)	(8;0)	(4;4)	(8;0)
stag, together	(0;8)	(0;0)	(0;8)	(100;100)

1. $\Gamma(K = 1)$ (alone strategies) one equilibrium in weakly-dominating strategies)
2. $\Gamma(K = 2)$ (all strategies), three equilibria in weakly and one in strictly dominating strategies, where both players are unwilling to deviate paralelly
3. But, $\Gamma(K = 1) \subset \Gamma(K = 2)$, which game is played by each player, an uncertainty about K ... ?

Conclusion:

1. Number of potential deviators is included into the structure of a non-cooperative game along with a mechanism design.
2. Constructed game is parametrized by a number of deviators
3. Proved existence of an equilibrium concept for this game
4. The equilibrium concepts differs from existing by more subtle structure of deviations
5. Constructed a stability criterium in terms of a number of potential deviators
6. "Self-enforcing" feature of a non-cooperative equilibrium depends on K^*, as well as on induced partitions $\mathcal{P}(K)$ and allowed family of group formation rules $\mathcal{R}(K)$

New questions appear

Next

Immediate

1. Application to networks games
2. Application to GE
3. Application to repeated games
4. Application dynamical GE

Further

1. Strategic market games with group formation as a set of eligible markets
2. Structure of group formation rules and evolution of group formation rules
3. Construction endogenous common prior beliefs and endogenous bounded rationality
4. Non-cooperative social welfare mechanism design and social choice theory

Thank you