Some Problems of Controllability of Distributed Systems Governed by Integrodifferential Equations

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We consider one - dimensional problem of boundary controllability:

$$\theta_t(t,x) + \int_0^t K(t-s)\theta_{xxxx}(s,x)ds = 0, \tag{1}$$

$$t > 0, \quad x \in (0,\pi).$$

$$\theta|_{t=0} = \xi(x), \tag{2}$$

$$\theta(t,0) = \theta(t,\pi) = 0, \tag{3}$$

$$\theta_{xx}|_{x=0} = v(t), \quad \theta_{xx}|_{x=\pi} = 0. \tag{4}$$

$$K(t) = \sum_{j=1}^{N} c_j e^{-\gamma_j t},\ c_j, \gamma_j > 0.$$

$$\theta(t,x) = \sum_{n=1}^{+\infty} \theta_n(t)\sqrt{\frac{2}{\pi}}\sin(nx)$$

$$\theta_{tt}(t,x) + K(0)\theta_{xxxx}(s,x) + \int_0^t K'(t-s)\theta_{xxxx}(s,x)ds = 0, \qquad (5)$$

$$t > 0, \quad x \in (0,\pi).$$

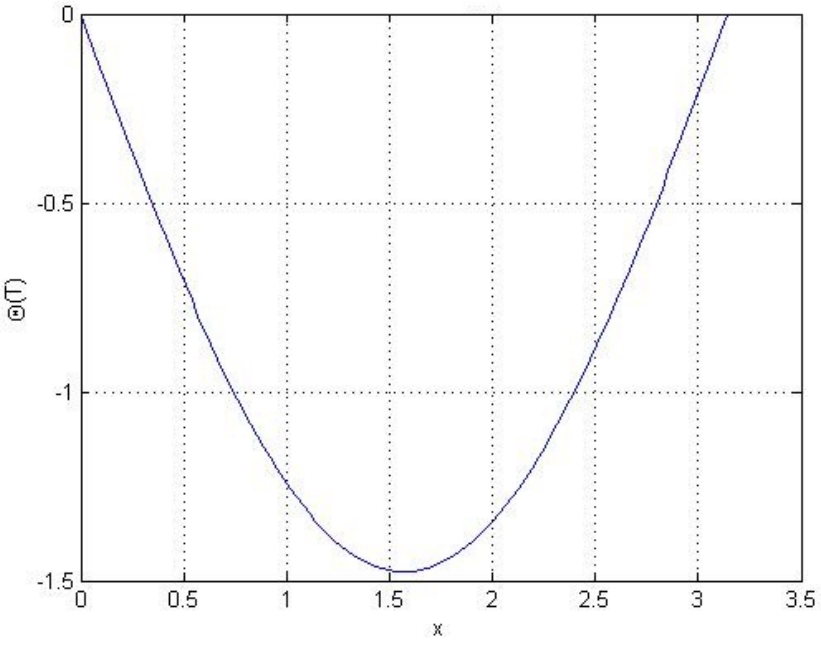
$$\theta|_{t=0} = \xi(x), \quad \theta_t|_{t=0} = 0, \qquad (6)$$

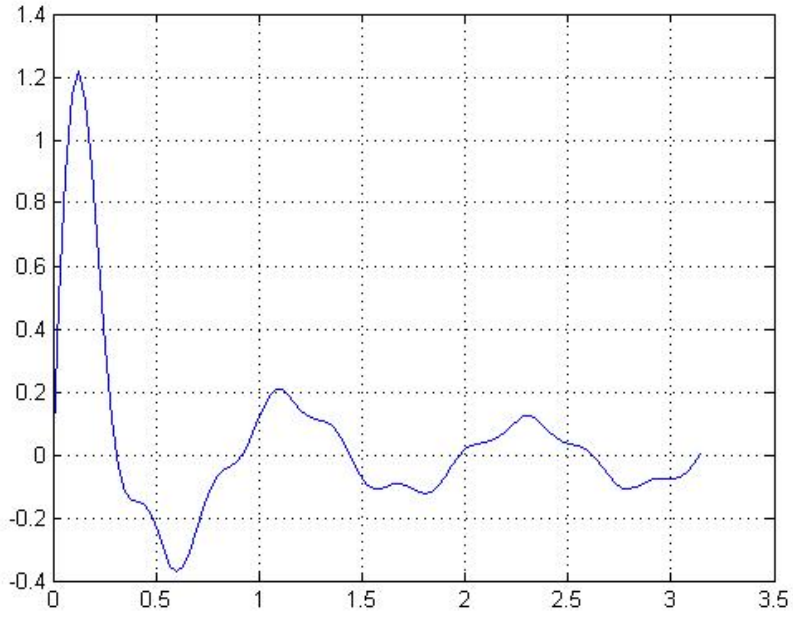
$$\theta(t,0) = \theta(t,\pi) = 0, \qquad (7)$$

$$\theta_{xx}|_{x=0} = v(t), \quad \theta_{xx}|_{x=\pi} = 0. \qquad (8)$$

$$K(t) = \sum_{j=1}^{N} c_j e^{-\gamma_j t}, \; c_j, \gamma_j > 0.$$

$$\theta(t,x) = \sum_{n=1}^{+\infty} \theta_n(t)\sqrt{\frac{2}{\pi}}\sin(nx)$$


0
-0.5
-1
-1.5
Θ(T)
0
0.5
1
1.5
2
2.5
3
3.5
x


1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
0
0.5
1
1.5
2
2.5
3
3.5

Let see one - dimensional problem of boundary controllability:

$$\theta_t(t,x) - \int_0^t K(t-s)\theta_{xx}(s,x)ds = 0, \tag{9}$$

$$t > 0, \quad x \in (0,\pi).$$

$$\theta|_{t=0} = \xi(x), \tag{10}$$

$$\theta|_{x=0} = v(t), \quad \theta|_{x=\pi} = 0. \tag{11}$$

$$K(t) = \sum_{j=1}^{N} c_j e^{-\gamma_j t},\ c_j, \gamma_j > 0.$$

$$\theta(t,x) = \sum_{n=1}^{+\infty} \theta_n(t)\sqrt{\frac{2}{\pi}}\sin(nx)$$

There is a following problem of distributed controllability:

$$\theta_t(t,x) - \int_0^t K(t-s)\theta_{xx}(s,x)ds = u(t,x), \tag{12}$$

$$t > 0, \quad x \in (0,\pi).$$

$$\theta|_{t=0} = \xi(x), \tag{13}$$

$$\theta(t,0) = \theta(t,\pi) = 0. \tag{14}$$

$$u(t,x) = \sum_{n=1}^{+\infty} u_n(t)\sqrt{\frac{2}{\pi}}\sin(nx),$$

$\mu_{1,n}$, $\mu_{2,n}$, ... $\mu_{N+1,n}$ — roots of an equation $\lambda + n^2\hat{K}(\lambda) = 0$.

$$\int_0^T u_n(s)e^{-\mu_{i,n}s}ds = -\xi_n,\ i = 1,2,...,N+1.$$

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Thank you for attention!