Need to Know?

On Information Systems in Firms

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Abstract

This paper theoretically explores the role of executives in acquiring and sharing information with insiders and outsiders. We view our analysis as a first step towards a theory of a firm's information system, which, as described by Cyert and March (1963), stipulates how a firm generates and condenses information, and which information is distributed internally and externally under partial conflict of interest. We develop a cheap-talk model with information acquisition and multiple audiences, in the context of a firm that has an investment project. We show that information acquisition and communication interact. The executive's incentive to overstate firm value distorts communication to a limited extent. Instead, it reduces information acquisition. Furthermore, we find that for firms, transparency is a necessary evil. Transparency allows for influential communication to outsiders, but constrains internal communication. Theoretically, we contribute by showing that the forward induction refinement excludes babbling as an equilibrium outcome if non-babbling equilibria exist.

Keywords: Information system, Information acquisition, Cheap-talk, Transparency, Firm behavior

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1 Introduction

Recently, Bandiera et al. (2017) observed that CEOs spend on average 70% of their time in meetings, and serve as a linking pin between insiders and outsiders of their firms. Almost 50 years ago, Mintzberg (1971) analyzed daily activities of top managers and reached a similar conclusion: top managers are pre-dominantly involved in collecting and sharing information. Hence his characterization of top managers as the 'nerve centre' of their organisations.

In the context of firm decision-making, several motives for acquiring and sharing information exist. First, information serves to make better decisions. Influential management scholar Peter Drucker stressed the importance of obtaining proper information for executives, as well as of providing it: "Effective executives ... share their plans with and ask for comments from all their colleagues—superiors, subordinates, and peers. At the same time, they let each person know what information they'll need to get the job done." (Drucker 2004). Second, communication serves to impress. Information can be utilized to improve outsiders' perceptions of the firm. Third, collecting and sharing information serve to persuade. In the words of Cyert and March (1963, p. 79): "Where different parts of the organization have responsibility for different pieces of information relevant to a decision, we would expect some bias in information transmitted...as a device for manipulating the decision."[2]

This paper theoretically explores the role of executives in acquiring and sharing information with insiders and outsiders.[3] We view our analysis as a first step towards a theory of, what Cyert and March refer to as a firm's information system. An information system stipulates how a firm generates and condenses information, and how and what information is distributed internally and externally under partial

[2]Besides the three motives discussed here, monitoring and evaluation are prominent motives for acquiring and communicating information. We abstract from these. They have been analyzed extensively in the literature on incentive pay following Holmström (1979).

[3]Our paper mostly considers acquisition and communication of strategic, forward-looking information. Measurement and reporting of firms' past performance is heavily regulated. Still, the accounting literature documents substantial earnings management in reporting, often linked to managerial incentives (Watts and Zimmermann 1986, Habib and Hansen 2008). Information gathered and reported for decision-making is far less subject to regulation. For instance, in the US, the Private Securities Litigation Reform Act of 1995 shelters firms from possible litigation if forecasts turn out to be ill predictions ex post.

Theories on disclosure and reporting typically assume that firms or managers possess, rather than acquire, private information, see e.g. Diamond (1985), Dye (1985), Stocken and Verrecchia (2004), Goldman and Slezak (2006), Crocker and Slemrod (2007), and Hermalin and Weisbach (2012). Notable exceptions are Pae (1999), Hughes and Pae (2004), Einhorn and Ziv (2007).

conflict of interest (Cyert and March, 1963, p.80 and p.127).

We develop a cheap-talk model à la Crawford and Sobel (1982) with information acquisition and multiple audiences, in the context of a firm that has an investment project. The value of the project depends on two random variables. One variable relates to the value of the firm's ongoing activities, the other is project-specific. The executive of the firm determines how accurately she learns about the value of ongoing activities. Next, she sends a public cheap-talk message to three receivers: an insider, an outsider, and the public. Further exploration of the project requires approval from the outsider. Investment plans often need support of, or authorization from external stakeholders, like banks and other providers of debt, external board members, or regulators.[4] The outsider only approves if expected project value is sufficiently high. This gives the executive an incentive to overstate firm value after learning that firm value is low. Through this channel, we capture the persuasion motive. If the outsider approves, the final implementation decision is made by the insider, who possesses information about the realization of the project-specific variable. The preferences of the insider and the executive are well aligned. In this way, we capture the informational motive of information acquisition and communication. Lastly, the executive cares about the firm's stock price, which depends on the beliefs of the public about firm value. This impression motive gives another incentive to overstate firm value, which arises for any realization of firm value.[5]

The main theme of this paper is that information acquisition and communication interact. One key result is that the executive's incentive to overstate firm value to raise the firm's stock price leads to limited distortions in communication, if any. Instead, it reduces information acquisition, even if information acquisition is costless. In our model, the executive is tempted to moderately exaggerate firm value. This would impress the public and might persuade the outsider, while the insider´s deci-

[4]As illustrated by the quote from Cyert and March (1963), the persuasion motive can also arise within the firm's boundaries. For instance, it is possible to interpret the outsider as an internal auditor, legal unit, or ethical board. Crucially, the outsider can deny approval, but cannot directly impose restrictions on the firm's information acquisition, communication, or decision-making procedures.

[5]The persuasion and impression motive both originate from the relation between the executive's payoff and beliefs held by others. We use persuasion to refer to situations where the executive's payoff depends on actions taken by people with non-aligned preferences in the decision-making process. The impression motive captures settings where the executive directly benefits from more favorable beliefs held by people not involved in the decision-making process.

sion would be distorted to a limited extent. Obtaining accurate information enables the executive to exaggerate moderately. However, she anticipates that in equilibrium the public and the outsider cannot be fooled, implying that accurate information acquisition yields distorted communication and, in turn, a less-informed decision by the insider. Coarse information allows for vast overstatements only, which yield big distortions in the insider's decision. Hence, by acquiring less information, the executive (fully or partially) abstains from exaggerating, which makes communication more informative. Of course, less accurate information also has a cost. It means inferior decision making.[6]

Another important result of this paper is that the persuasion motive and impression motive have a different effect on communication and information collection. We show that they are imperfect substitutes in hindering communication and reducing information acquisition. The impression motive gives the executive an incentive to overstate firm value independent of actual firm value. Acquiring less information is an effective response, as it reduces the incentive to overstate for all realizations of firm value. Instead, the binary nature of the approval decision implies that the persuasion motive gives an incentive to overstate firm value only if actual firm value is low. If firm value is high, the executive prefers to send an accurate report. We show that in equilibrium at most one report can induce the outsider to reject exploration

[6] The Enron scandal illustrates the impression motive at work at an extreme. In the years leading up to their demise in 2001, Enron's executives obsessively focused on raising the stock price. They convinced analysts and investors that Enron's prospects were glorious; Fortune named Enron the most innovative company for six consecutive years up to 2000. Internally, they demanded ever-higher revenues, which led to a series of bad investment decisions. Despite such setbacks, Enron's executives kept expressing confidence in the firm's value and prospects to the outside world. Besides stock-based incentives, observers attribute the executives' behavior largely to their desire to impress others (McLean and Elkind 2013; Eichenwald 2005). Employees who criticized projects were removed from these projects, and internal warnings on malpractice were ignored (Behr and Witt 2002a, 2002b; Free and Macintosh 2008). Recalling how CEO Kenneth Lay handled internal warnings, a former CFO of one of Enron's units noted "[he] has always been hands off even in his best days. ... My surmise is he didn't want to be informed. His attitude was, 'I don't want to know' " (Behr and Witt 2002b). In Enron's case, reputational concerns led to decision makers being poorly informed.

Our model also speaks to less extreme situations. For instance, an increasing number of firms provide forward-looking statements, such as management earnings forecasts, often based on non-verifiable information (Bozanic et al. 2017). Despite the widespread concern that such statements can deliberately be misleading, investors and analysts do respond to this information (Patell 1976, Penman 1980, Waymire 1986, and Jennings 1987). Investors primarily respond to credible information (Bamber and Cheon 1998, Hutton et al. 2003, Dzieliński et al. 2017). In line with our results, Graham et al. (2005) present survey evidence suggesting that managers are willing to make decisions that reduce project quality if this prevents a negative financial report.

of the project. The project's option value gives the executive a strong incentive not to send this report.[7] Generally, the executive's optimal response is to reduce information acquisition, and to distort communication if firm value is low but less so, if at all, if firm value is high.

Our paper also highlights the importance of transparency for the working of information systems. Initially, we assume that messages are public, meaning that insiders and outsiders receive the same message. The implication of this assumption is that the executive faces a trade-off. Overstating firm value may raise the firm's stock price or may persuade an outsider to approve with the project on the one hand, but it distorts internal decision making on the other. When the executive could send private messages to each receiver, the executive shares her information fully with insiders. This serves the informational motive. Because there are no costs of sending distorted messages to outsiders, outsiders do not receive informative messages.[8] For a firm transparency is a necessary evil. Without it, outsiders cannot be persuaded. In the absence of the persuasion motive, however, the possibility of privately informing insiders typically leads to proper decision making based on information that is optimal from the firm's point of view.

Even though we regard our paper as applied theory, we also make a theoretical contribution. We show that using forward induction as an equilibrium refinement excludes babbling as an equilibrium outcome if non-babbling equilibria exist. Loosely speaking, forward induction imposes that previous actions are rational. In our model this implies that if babbling is an equilibrium outcome, the executive

[7] The persuasion motive played an important role in the Volkswagen scandal. Increased emission standards forced Volkswagen to improve its diesel engines. Experts were doubtful about the possibility to meet the standards, but Volkswagen kept exploring and ultimately claimed to have found a solution. The new engines received regulatory approval, and Volkswagen's executives expressed their confidence in the new technology (Volkswagen Group 2012). In 2015, fraudulent software was exposed. The software made engines appear cleaner during regulatory tests than during regular driving.

[8] If the executive could make the implementation decision herself rather than the insider, communication to the outsider and the public is also non-informative. Hence, as anticipated by Cyert and March (1963), the effects of an information system depend on decision-making processes. Aghion and Tirole (1997) model the interaction between the decision-making structure and information acquisition in firms, and Dessein (2002) models the interaction between decision-making and communication. More recently, several papers study (de-)centralization and communication in situations where local units possess private information and potential benefits of coordination and adaptation exist (see, for example, Alonso et al., 2008, Rantakari, 2008, and Swank and Visser, 2015). A key difference between these studies and ours is that they take the distribution of information as given, whereas in our model part of the information has to be acquired.

has not acquired any information. It would be a pure waste. Yet, it also implies that by acquiring some information, the executive can avoid a babbling equilibrium. This result is interesting in itself, in the sense that virtually all papers that use a cheap-talk model a la Crawford and Sobel (1982) acknowledge that one equilibrium of their models is the babbling one. Argenziano et al. (2016) even use the babbling equilibrium as an off-equilibrium-path punishment by the receiver, which induces the sender to overinvest in information collection. If forward induction is imposed, babbling is no longer a credible threat. We derive conditions under which forward induction selects a unique equilibrium.

Our results are derived from a cheap-talk model with information acquisition and multiple receivers. As one of our objectives is to better understand executives' incentives to manipulate information - one of the observations by Cyert and March - our choice for a cheap-talk model seems natural. A Bayesian Persuasion model à la Gentzkow and Kamenica (2011 and 2014) is more suitable for studying settings where firms are legally obliged to reveal all information gathered. One way of looking at our results is that in a cheap-talk setting the need to persuade or the desire to impress is costly for firms. It leads to internal decisions based on too little information. As a result, firms may look for other ways of making messages to outsiders credible, for example, by hiring auditors. To examine those settings, the model of Dewatripont and Tirole (1999) seems appropriate. Following Milgrom and Roberts (1986), they assume verifiable information that can be concealed, but not manipulated. Then, the need to persuade strengthens incentives to gather information.

Farrell and Gibbons (1989) were the first to analyze a cheap-talk model with multiple receivers. Comparing public and private communication, they showed that a public message can be more informative than separate, private messages if preferences are sufficiently mis-aligned. This carries over to our model, where informative communication to the public and the outsider requires an informational motive to lead to informative communication. Goltsman and Pavlov (2011) generalize Farrell and Gibbons (1989) by allowing for a more general distribution of sender types. By adding an information acquisition stage, the number of sender types is endogenous in our model. Taking information acquisition into account, we show that public communication is more informative than private communication if the sender needs to persuade, but may lead to less information acquisition if the sender only wants to impress.[9]

Di Pei (2015) and Argenziano et al. (2016) consider information acquisition in a cheap-talk model with one receiver. In Di Pei (2015), the sender can first segment the state space in any arbitrary way, and subsequently learns in which segment the true state lies. Finer segmentation is more costly. The main result is that the sender never collects more precise information than she can communicate in equilibrium. In our model, the sender also segments the state space, but it is assumed that all segments are equally large. In Argenziano et al. (2016), the sender chooses the accuracy of information by deciding how many Bernoulli trials to conduct. Their way of modeling information acquisition can be regarded as a micro foundation of the technology we assume.[10]

We are aware that our model does not capture all aspects of an information system distinguished by Cyert and March (1963). Our analysis abstracts from the time dimension. If delaying decision-making is costly, this affects the duration of the search for information. Implicitly, we assume that the cost of information acquisition also includes the cost of delay. Relatedly, if information is collected over time, a relevant question is *when* to communicate. Recently, Grenadier et al (2016) and Orlov et al (2018) consider timing of communication in dynamic frameworks, analyzing how the release of information depends on the alignment of the preferences of the sender and receiver. Furthermore, we do not explicitly model how and by whom information is collected. In practice, many people in firms are involved in gathering, recording, and processing information. Team theory, starting with Marschak and Radner (1972), analyzes how firms handle information when processing is costly. Crucially, team theory assumes everyone shares the same objective. Bolton and Dewatripont (1994) show that to handle flows of information most effectively, firms create networks of individuals that resemble classic forms of organizations. Sah and Stiglitz (1986) take the network as given, and analyze the effects of alternative decision processes if individuals can make mistakes. Our paper is influenced by team theory in placing information at the heart of the analysis of organizations. However,

[9] Using the Bayesian Persuasion framework, Michaeli (2017) shows that the sender may acquire more information if only a subset rather than all receivers obtain the information acquired.

[10] In Dur and Swank (2005), the sender exerts costly effort that increases the quality of her signal about the state of the world. They show that the receiver benefits from a sender whose preferences deviate from his own preferences, as this increases the incentive to exert effort. Che and Kartik (2009) derive a similar result for the case that the sender and receiver have different priors.

we take the network, or organizational form, as given. Instead, we focus on how divergence of individuals' objectives affects how much information is collected, what information is conveyed and to whom it is conveyed.

The next section describes the model. In Section 3, we present the analysis and results. Section 4 discusses the effects of the forward induction refinement. In Section 5, we extend our model to show the effect of introducing private reports. We discuss the implications of our findings for understanding firms' information systems in the final section.

2 The Model

We consider a firm that faces an investment opportunity, called the project. The profitability of the project depends on the value of the firm's ongoing activities, represented by random variable v, and on the idiosyncratic characteristics of the project, represented by the random variable z. The incremental value of the project to the firm is $\gamma(v - z)$, where γ measures the importance of the project relative to the ongoing activities. Both v and z are independently and uniformly distributed on the interval $[0, 1]$.

Our model is designed to investigate the incentives of an Executive, X, who has three motives for acquiring and conveying information about v. The first motive we consider is a persuasion motive. We model this motive by assuming that exploration of the project, through which z is learnt, requires the approval of an Outsider, E. Second, we model an informational motive. We assume that the final decision on the project is made by an Insider, I, whose preferences are perfectly aligned with those of X. Finally, to model the impression motive we assume that X is concerned with the Public's, P, perception of firm value.

At the beginning of the game, X acquires information about v. The accuracy of information is reflected by $a \in \mathbb{N}$. For any given a, the interval $[0, 1]$ is split into a subintervals $[\bar{v}_{k-1}, \bar{v}_k]$ of equal length, where $\bar{v}_k \equiv \frac{k}{a}$ and $k \in \{1, .., a\}$. X observes to which subinterval v belongs. We refer to k as X's type. Clearly, the higher is a, the more accurate is X's information about v. The cost of acquiring information is

$(a-1)\,c$. After learning her type k, X's expectation of v is denoted by v_k:

$$v_k \equiv \mathrm{E}\left[v|k\right] = \frac{2k-1}{2a} \tag{1}$$

X's choice of a is publicly observed.[11] Her type, however, is private information. After X has learnt her type, she sends a public cheap-talk report, r, to E, I, and P.[12] This report can take values from any sufficiently large report space.

As mentioned above, exploration and implementation of the project requires E's approval. We denote E's approval decision by $d^E \in \{0,1\}$, where $d^E = 1$ denotes approval and $d^E = 0$ denotes rejection. If $d^E = 0$, the game ends. If $d^E = 1$, I explores the project, observes z, and makes the implementation decision, $d^I \in \{0,1\}$, where $d^I = 1$ denotes that the project is implemented, and $d^I = 0$ denotes that it is not.

We denote firm value by w

$$w = v + \gamma\left(v - z\right) d^E d^I - \left(a-1\right) c \tag{2}$$

Following Stein (1989), we assume that X is concerned with w and with the firm's stock price, s, after E has made his approval decision, but before the implementation decision is made[13]. Hence X's payoff is equal to

$$u^X = \left(1-\lambda\right) w + \lambda s \tag{3}$$

with

$$s = \mathrm{E}\left[w|a, r, d^E\right] \tag{4}$$

Note that s equals P's perception of w, conditional on r and d^E. The parameter $\lambda \in [0,1]$ denotes the relative weight on the stock price in X's utility, which can be interpreted as the strength of stock-based remuneration. In our model, it reflects

[11] This is a strong, but not unrealistic assumption. Reporting regulation requires firms to specify their investments in software (IAS 38) and hardware (IAS 16) in their (public) year reports. These investments in information technology can be used to infer about the extent of information collection.

[12] In Section 5, we analyze the case where X sends private messages to I, E and P.

[13] This could, for instance, reflect short-term financial incentives. Stein (1989) discusses several other reasons why executives may care about current stock prices, as implied by (3).

the strength of the impression motive.

E's payoff is

$$u^E = \left[\gamma\left(v - z\right) d^I - h\right] d^E \tag{5}$$

Threshold h is the cost, borne by E, of allowing X to explore the project. Equation (5) captures that E approves with exploration if the option value of the project exceeds threshold h. Through h we model the persuasion motive.

Lastly, I's payoff is equal to X's payoff, $u^I = u^X$. We abstract from agency problems between X and I to model the informational motive. X wants to share her information with I.[14]

Our model is stylized, but captures in a natural way the trade-offs faced by an executive who wants to persuade, impress, and inform.[15] By sending r, X wants E to approve, s to be high, and I to make the proper decision on the project.

We solve the model for Sequential Equilibria (SEQ). In the main text below, we offer a relatively informal analysis and discussion. In the Appendix, we provide formal results and proofs. We use the following notation regarding players' strategies and beliefs. A SEQ consists of a collection $(\alpha, \rho(k, a))$ of behavioral strategies of X, an approval strategy $\delta^E(r, a)$ of E, a decision strategy $\delta^I(z, r, a)$ of I, and beliefs $G(k|r, a)$ of I, E, and P about X's type such that:

1. For any a, z, and r, decision $d^I = \delta^I(z, r, a)$ maximizes I's expectation of (3) given belief $G(k|r, a)$;

2. For any a and r, approval decision $d^E = \delta^E(r, a)$ maximizes E's expectation of (5) given belief $G(k|r, a)$;

3. For any a and type k, report $r = \rho(k, a)$ maximizes X's expectation of (3);

[14] Identical payoffs of X and I is a straightforward way of creating an informational motive, but not the only way. For instance, as s is determined before I makes a decision, λ is irrelevant for I´s decision. Hence, none of our results change if the level of λ differs between X and I. Similarly, I could maximize firm value (2) or project value $\gamma(v - z) d^I$. None of our results changes if X cares about the stock price that realizes after P observes decision d^I, instead of before.

[15] There are many firm decisions where these three motives for communication play a role. One could think of the development of a new drug by pharma companies that requires the approval of the FDA to start a clinical trial. Or the development of a new real estate project by building companies that requires the approval of the municipality. Both external parties only approve the new projects if the option value of the project is sufficiently high.

4. Information accuracy, $a = \alpha$ maximizes X's expectation of (3).

5. Beliefs $G(k|r, a)$ follow Bayes' rule on all information sets.

By $\Gamma(a)$, we denote the continuation game that is played after a is chosen and observed. In the remainder, for brevity we omit variable a from argument lists of functions and expectations whenever it does not lead to confusion.

As is usual in cheap-talk models, the 'language' of the reporting strategy $\rho(k)$ is defined only in equilibrium. Multiple reporting strategies can lead to the same beliefs and, hence, to the same equilibrium outcome. We ignore this type of equilibrium multiplicity. Therefore, we construct equilibrium sets by placing all equilibria with outcome-equivalent reporting strategies into one set. We refer to such an equilibrium set as an 'equilibrium'.

Cheap-talk games are also plagued by non-outcome-equivalent equilibrium multiplicity. The babbling equilibrium always exists. Hence, any equilibrium with influential communication is never the unique equilibrium. In Section 4, we show that if an equilibrium with influential communication exists, the forward induction refinement eliminates the babbling equilibrium. Furthermore, for some range of parameter values, forward induction yields a unique equilibrium outcome, in which influential communication does take place.

Before turning to the analysis of the game, we first determine the accuracy of information if X were to choose a in the absence of a persuasion and reputation motive ($h = 0$ and $\lambda = 0$), and X reveals her type to I, $r = k$. For ease of exposition, we present the optimal a as a continuous variable. X anticipates that I implements whenever she learns that $z < E(v|k)$, which happens with probability $\frac{2k-1}{2a}$. The expected value of the project, conditional on k and $d^E = 1$, equals

$$E\left[\gamma(v-z)\,d^I d^E|k, d^E = 1\right] = \gamma\left(\frac{2k-1}{2a} - \frac{2k-1}{4a}\right)\frac{2k-1}{2a} = \gamma\frac{(2k-1)^2}{8a^2}$$

When choosing a, X's expectation of project value equals

$$\frac{1}{a}\sum_{k=1}^{a}\gamma\frac{(2k-1)^2}{8a^2} = \frac{\gamma}{24}\left(4 - \frac{1}{a^2}\right) \tag{6}$$

The marginal benefit of a to project value is given by the derivative of (6). Equating this to marginal cost c yields the optimal accuracy

$$a^{opt} \equiv \sqrt[3]{\frac{\gamma}{12c}} \tag{7}$$

The value a^{opt} measures the accuracy of information that maximizes firm value in the absence of persuasion and reputational motives provided the implementation decision is optimal for every type X. We refer to underinvestment (overinvestment) in information acquisition if X chooses $a < a^{opt}$ $(a > a^{opt})$.

3 Analysis

We begin the analysis by considering a continuation game $\Gamma(a)$ that follows a choice of a. As I's and X's preferences are perfectly aligned, maximizing (3) yields that I chooses $d^I = 1$ if the expected value of the project is positive, *i.e.*, if $\mathrm{E}[v|r] > z$, and $d^I = 0$ otherwise. When making the approval decision, E anticipates I's strategy. Maximizing (5) yields that E approves if the option value of the project given r exceeds threshold h. Hence, $d^E = 1$ if $\gamma\,\mathrm{E}\left[(v-z)\,d^I|r\right] > h$ and $d^E = 0$ otherwise.

Lemma 1 characterizes X's communication strategy in $\Gamma(a)$ and presents two immediate consequences.

Lemma 1 *Consider a continuation game $\Gamma(a)$. In any equilibrium of $\Gamma(a)$:*

(i) there is a number of distinct reports $N \in \{1,\ldots,a\}$ and a set of N marginal types $\{k_n\}$, $k_{n-1} < k_n$, $k_0 = 0$ and $k_N = a$, so that for all $n = \{1,\ldots,N\}$, all types $k \in \{(k_{n-1}+1),\ldots,k_n\}$ send report r_n;

(ii) if $\lambda \geq \frac{1}{2}$, then $N = 1$, and the pooling equilibrium is the unique equilibrium of $\Gamma(a)$;

(iii) only report r_1 may lead to E's disapproval, i.e., *$\delta^E(r_1) = \{0,1\}$ and $\delta^E(r_n) = 1$ for $n \geq 2$.*

The communication strategy presented in item (i) of Lemma 1 is equivalent to the communication strategy in a cheap-talk game à la Crawford and Sobel (1982). There is a difference though. If $a \to \infty$, as in Crawford and Sobel (1982), marginal type k_n is indifferent between sending reports r_n and r_{n+1}. For finite a, however, marginal types are generally not indifferent, so that k_n strictly prefers report r_n over report r_{n+1}.[16]

In the communication stage, the informational motive meets the persuasion and reputational motives. Consequently, X faces a dilemma. On the one hand, she wants to inform I about v in order to maximize project value. On the other hand, she has an incentive to overstate v for the two mentioned reasons. First, overstating v may persuade E to approve. Second, overstating v increases P's expectation of firm value and, hence, increases stock price s. The relative strength of the impression motive depends on how much X cares about the firm's stock price, λ. This drives item (ii) in Lemma 1. If X cares too much about s, she cannot credibly communicate her value. Anticipating this, she does not invest in acquiring information, and chooses $a = 1$. If λ is sufficiently small, X's communication can be informative. This requires that the incentive to overstate v is offset by the distortion it induces in I´s decision. The presence of the informational motive makes communication possible, because it leads to a cost of overstating. This cost is a distorted implementation decision.

To see why only report r_1 may lead to E's disapproval in item (iii), suppose that E does not approve after receiving either of two distinct reports which lead to different stock prices. Then there is no project to implement, thus X faces no cost of overstating. X would therefore always send the report that leads to a higher stock price. Hence, in equilibrium, at most one report, send by the lowest subset of types, can lead to rejection by E.

It is useful to distinguish between two factors that together determine the effectiveness of communication in equilibrium. The first factor is the number of reports N. Let $\overline{N}$ denote the *maximum* number of reports over all equilibria of all continuation games.

$$\overline{N}(\gamma, \lambda, h) \equiv \max \{N \in \mathbb{N} : \exists\ a \in \mathbb{N} \text{ such that } \Gamma(a) \text{ has an equilibrium with } N \text{ reports}\}$$

Lemma 2 shows that the informational motive facilitates communication, whereas the persuasion and impression motives hinder communication.

[16] This feature is also present in Argenziano *et al.* (2016) and is caused by the information collection technology used.

Lemma 2 *The maximum number of reports* $\overline{N}(\gamma, \lambda, h)$ *over all equilibria of all continuation games is weakly increasing in* γ *and weakly decreasing in* λ *and in* h.

The second factor that determines the effectiveness of communication is the *relative* precision of the reports. Given N, communication is most effective if all reports are equally precise, i.e. if each report is sent by the same number of types (or, equivalently, if each report is equally likely to be used in equilibrium). Typically, in applications of cheap-talk models without information acquisition à la Crawford and Sobel (1982), the number of reports sent in equilibrium is a sufficient measure of the effectiveness of information. In contrast, in our model, a higher number of reports can lead to less effective communication, if it comes at the expense of the relative precision of reports. As shown below, this implies that X can prefer an equilibrium with $N < \overline{N}$ reports over all equilibria with $\bar{N}$ reports.

3.1 Informing versus Impressing

In this section, we discuss equilibria where E always approves exploration of the project, $d^E = 1$, so that the persuasion motive plays no role. This requires h to be sufficiently small. One of the novel insights of our paper is that if E´s approval strategy imposes no constraints, X's incentive to overstate v for reputational reasons hardly leads to distortions in her reports. Instead, it weakens her incentives to acquire information. We discuss this result in two steps. First, we show that acquiring finer information, *i.e.*, a higher level of a, narrows the range of λ for which a separating equilibrium of $\Gamma(a)$ exists. Second, we show that X prefers to avoid pooling in the communication stage. This implies that X's incentive to overstate v induces her to acquire more coarse information.[17]

Lemma 3 *Consider a continuation game* $\Gamma(a)$. *A separating equilibrium of* $\Gamma(a)$ *in which all types receive approval of* E *exists if and only if*

$$a \leq \bar{a} \equiv \left\lfloor \frac{\gamma}{1+\gamma} \frac{1+\lambda}{2\lambda} \right\rfloor \tag{8}$$

and

$$h \leq \overline{h}(a) \equiv \frac{\gamma}{8a^2} \tag{9}$$

[17] In Lemma 3, $\lfloor x \rfloor$ denotes the floor function, which gives the largest integer not exceeding x: $\lfloor x \rfloor = \max\{n \in \mathbb{Z} | n \leq x\}$. This ensures that $\bar{a}$ is an integer.

A separating equilibrium requires that for all types k, the net benefit of overstating v by sending report r_{k+1} ("my type is $k+1$") instead of r_k should be negative. If all reports lead to E's approval, as ensured by (9), the cost of overstating (distorting I's decision) is constant in type k, whereas the benefit (higher stock price s) is increasing in k. Consequently, type $k = a-1$ has the largest incentive to deviate from truth-telling. Crucially, as long as the informational motive is sufficiently important, X prefers exaggerating to a limited extent over exaggerating a lot. Fine information (*i.e.*, high a) enables X to overstate v by a limited amount, whereas coarse information forces X to overstate heavily. Hence, the maximum level of accuracy at which X can credibly communicate her type is limited, as given by (8). This maximum level is decreasing in λ and increasing in γ.

In a separating equilibrium, the number of distinct reports N equals a. Thus, $\overline{a}$ in equation (9) can be interpreted as an upper bound on the number of reports in all separating equilibria. Lemma 4 gives the conditions for which $\overline{a}$ is the actual upper bound of the number of reports in *all* equilibria, for $\gamma \leq 1$.[18].

Lemma 4 *Suppose $\gamma \leq 1$. Consider the maximum number of reports $\overline{N}$ over all equilibria of all continuation games:*

(i) if $\lambda \leq \frac{\gamma}{4+3\gamma}$ so that $\overline{a} \geq 2$, then $\overline{N}(\gamma, \lambda, h) = \overline{a}$;

(ii) if $\lambda \in \left(\frac{\gamma}{4+3\gamma}, \frac{1}{2}\right)$ so that $\overline{a} = 1$, then $\overline{N}(\gamma, \lambda, h) \leq 2$.

To understand the intuition behind case (i), suppose that $a > \bar{a} \geq 2$.[19] According to Lemma 3, the separating equilibrium of $\Gamma(a)$ does not exist because type $k = a-1$ wants to deviate. Consequently, at least the top two types pool and send the same report. However, as types pool, sending N messages requires $a > N$. A higher a further strengthens incentives to exaggerate, leading to more pooling and less informed decisions. To put it differently, for $a > \bar{a}$ the maximum number of messages (or partitions) in the communication stage does not increase with a. Increasing a beyond $\bar{a}$ leads to more pooling, not to more messages.

Using Lemma's 1 to 4, we can characterize the information acquisition and communication strategy of X in equilibrium. When h is small such that E approves

[18] We discuss the case $\gamma > 1$ after Corollary 1

[19] We discuss case (ii) in Corollary 1 below.

after receiving any message (in equilibrium), the communication continuation game is akin to a cheap talk game as in Crawford and Sobel (1982). Typically, such games are characterized by multiplicity of equilibria. Ours is no exception. In Proposition 1, we describe the equilibrium path of the sender-optimal equilibrium, *i.e.*, the equilibrium that is optimal for X.[20] In Section 4 we show that this equilibrium path is the unique forward induction outcome.

Proposition 1 *Let a^* be*

$$a^* \equiv \min\left\{\overline{a}, a^{opt}\right\} \tag{10}$$

Suppose that $h \leq \overline{h}(a^*)$, $\gamma \leq 1$ and $\lambda \leq \frac{\gamma}{4+3\gamma}$ so that $\overline{a} \geq 2$. The unique equilibrium outcome that maximizes the ex ante *expected utility of X consists of accuracy a^* followed by the separating equilibrium of the continuation game.*

Proposition 1 presents two results. First, X's impression motive to exaggerate v in the hope of influencing the firm's stock price does not lead to pooling of information. Instead, it leads to less information acquisition. The number of distinct reports that X can credibly send is limited. These messages are most efficiently used when each message is equally likely to be sent in equilibrium. The separating equilibrium achieves this, at lowest cost. Second, X's choice of accuracy is either driven by the cost of information c or by the relative weight X attributes to the firm's stock price λ. If information is costly and the impression motive relatively weak, X chooses the level of accuracy that maximizes firm value, a^{opt}. If, instead, information is cheap and the impression motive strong, such that $\overline{a} < a^{opt}$, X's incentive to overstate v leads to underinvestment in information.

Proposition 1 assumes $h \leq \overline{h}(a^*)$, $\gamma \leq 1$ and $\bar{a} \geq 2$. We discuss these assumptions in reversed order. First, if $\bar{a} = 1$, the impression motive is so strong that no separating equilibrium exists for any a (Lemma 3) and that any non-pooling equilibrium of the continuation game is a semi-pooling equilibrium with $N = 2$ messages, which requires $a \geq 3$ (Lemma 4). Corollary 1 immediately follows.[21]

[20] Characterizing the full equilibrium instead of the equilibrium path requires the addition of the sender-optimal communication strategy following any (sub-optimal) choice of a. This adds little, in particular since for $a > \bar{a}$, it cannot be expressed in closed-form (but can be computed numerically).

[21] We assume $h = 0$ in Corollary 1 to ensure that E always approves, to allow for the proper comparison with Proposition 1.

Corollary 1 *Let $\gamma \leq 1$, $h = 0$, and $\lambda \in \left(\frac{\gamma}{4+3\gamma}, \frac{1}{2}\right)$ so that $\bar{a} = 1$. The unique equilibrium outcome that maximizes* ex ante *expected utility of X:*

(i) *either has accuracy $a = 1$ followed by the pooling equilibrium of the continuation game, or*

(ii) *has accuracy $a \geq 3$ followed by a semi-pooling equilibrium of the continuation game with $N = 2$ reports. Moreover, $a \to \infty$ if and only if $\lambda \nearrow \frac{\gamma}{2(2+\gamma)}$ and $c \to 0$.*

If c is sufficiently small, X optimally chooses $a > \bar{a}$ to enable some information transmission. The optimal a converges to infinity only if this is necessary for having two distinct reports in equilibrium, *i.e.*, if λ converges to $\frac{\gamma}{2(2+\gamma)}$ from below. Note that if the impression motive induces some pooling in equilibrium, pooling takes place at the top. This is typical in cheap-talk games.

Now, consider the case where $\gamma > 1$. The value of γ can be interpreted as the importance of the informational motive relative to the impression motive. Above we have shown that if the impression motive is relatively important ($\gamma \leq 1$), acquiring precise information leads to a semi-pooling equilibrium where reports are used too inefficiently. By acquiring less information, X reduces the incentive to pool. For higher values of γ, the incentive to overstate project value is weaker. This means that if in equilibrium some pooling occurs, reports are still used relatively efficiently, so that the cost of pooling is small. This allows for continuation game equilibria with a higher number of reports used efficiently enough to outperform the best-possible separating equilibrium. We cannot fully characterize the sender-optimal equilibria for $\gamma > 1$. However, numerical simulations show that the differences with the equilibrium described in Proposition 1 are small. We find that sender-optimal equilibria with $a > \bar{a} \geq 2$ exist only if $\bar{a}$ is sufficiently small. In these equilibria, only the top two types pool, so that $a = \bar{a} + 2$ and $N = \bar{a} + 1$. This holds even if $c = 0$. Hence, there can be a limited amount of distorted communication if $\gamma > 1$, but X's incentive to overstate v is still pre-dominantly reflected by less information acquisition.

Figure 1 depicts numerically computed sender-optimal a^{OP} as a function of c for various levels of γ under the assumptions $\lambda = 0.1$ and $h = 0$.[22] The graph for

[22] Details on the numerical simulations can be found in Appendix XXX.

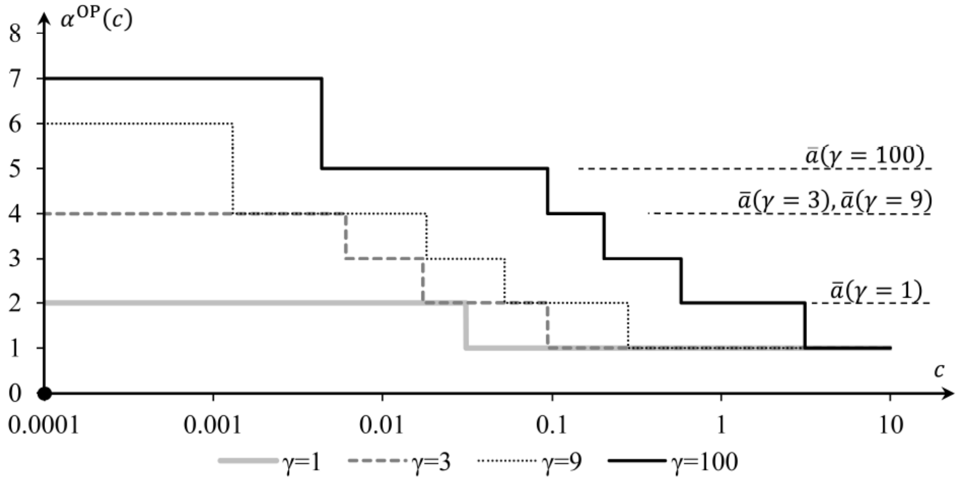

Figure 1: Sender-optimal a^{OP} as a function of c for $\gamma \in \{1, 3, 9, 100\}$, $\lambda = 0.1$, and $h = 0$.

$\gamma = 1$ gives a^{OP} as described in Proposition 1. If $\gamma = 1$ and c is sufficiently small, X optimally chooses $a^* = \bar{a} = 2$. The graph also shows that for $\gamma = 3$, the optimal a for low values of c is equal to $a^{\mathrm{OP}}(\gamma = 3) = 4$. The optimal a for these values is hence equal to the threshold value $\bar{a}$ from Lemma 3. For higher values of γ, for example $\gamma = 100$, the graph shows that for low values of c the optimal a is only 2 parameter values higher than $\bar{a}$, $a^{\mathrm{OP}}(\gamma = 100) = 7$ whilst $\bar{a}(\gamma = 100) = 5$. Hence $a^{\mathrm{OP}} = \bar{a} + 2$, but only when c is sufficiently small. The graph also shows that a^{OP} is decreasing in c.

Figure 2 also depicts a^{OP}, but now for $\lambda = 0.01$. As compared to Figure 1, $\bar{a}$ is higher for each value of γ. Now, the only effect of an increase in γ is an increase in $\bar{a}$. In any sender-optimal equilibrium, $a^{\mathrm{OP}} = a^*$, as in case of $\gamma \leq 1$ (Proposition 1).

Lastly, Figure 3 shows that the sender-optimal a^{OP} is non-monotonic in γ. Starting from $a^{\mathrm{OP}} = N = \bar{a}$ for small values of γ, an increase in γ can sustain a semi-pooling equilibrium with $N = \bar{a} + 1$, which requires $a = \bar{a} + 2$. If γ increases further, $\bar{a}$ increases by 1, which renders this separating equilibrium optimal.

Figure 3 also shows that as the information motive becomes more important, the maximum number of reports in any equilibrium, $\overline{N}$, increases. However, when γ is sufficiently high, it is not optimal for X to opt for an equilibrium with $\bar{N}$ reports. In these semi-pooling equilibria, reports are used inefficiently. X is better off acquiring less information (even when $c = 0$) which either prevents pooling altogether, or allows for very limited pooling (only two highest types pool, and $a^{\mathrm{OP}} = \bar{a} + 2$).

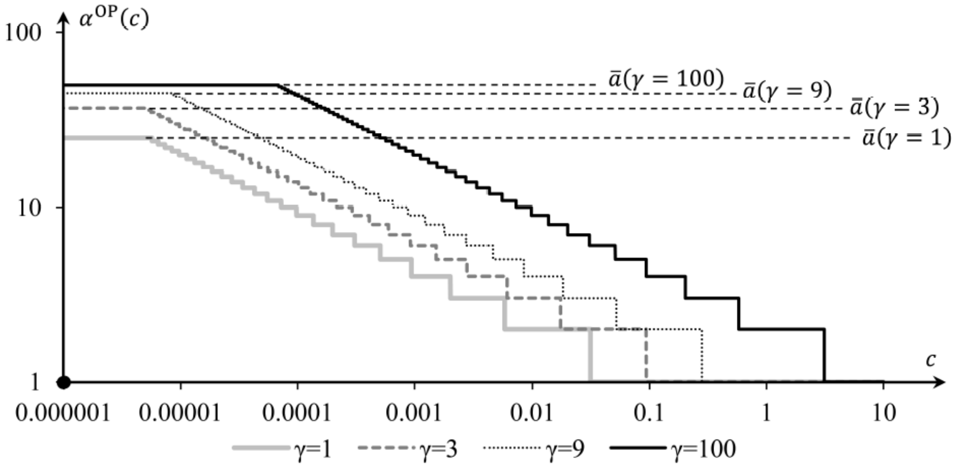

Figure 2: Sender-optimal a^{OP} as a function of c for $\gamma \in \{1, 3, 9, 100\}$, $\lambda = 0.01$, and $h = 0$.

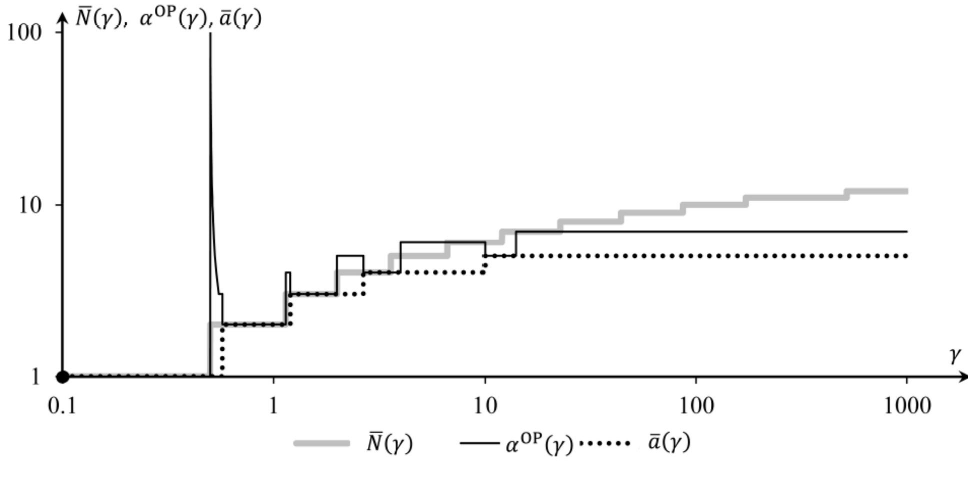

Figure 3: Functions $a^{\mathrm{OP}}\left(\gamma\right)$, $\overline{N}\left(\gamma\right)$, and $\overline{a}\left(\gamma\right)$, for $c = h = 0$.

Despite fewer reports, communication is more effective as (almost) all reports are equally precise.

The condition $h \leq \overline{h}(a^*)$ in Proposition 1 ensures E's approval in equilibrium, even if X sends r_1. If h slightly exceeds $\overline{h}(a^*)$, X can induce E to always approve the project by choosing $a < a^*$. The cost of reducing a is, as before, an implementation decision based on less information. Through this channel, the impression motive may lead to a further underinvestment in information collection. Alternatively, X accepts that E may reject the project, as analyzed in the next section.

3.2 Informing versus Persuading (and Impressing)

Now suppose that h is sufficiently large, such that it is not possible or not optimal for X to choose an a that ensures E's unconditional approval of exploration of the project. Hence, a persuasion motive is present. Then, the optimal equilibrium outcome for X differs in two ways from the outcome stated in Proposition 1. First, at least one type does not receive E's approval in equilibrium. This hurts X, who prefers to minimize this probability. Second, it is possible that some reports are sent by multiple types and, therefore, the equilibrium of the continuation game is a semi-pooling equilibrium. As a result, the optimal value of a for X is affected by all parameters (c, h, γ, λ).

To highlight the effects of X's need to persuade E to get approval, we consider the case where c is infinitely small. Furthermore, we first assume that λ is also infinitely small. This eliminates the impression motive. Then, in the absence of the need to persuade ($h = 0$), the optimal outcome would be a choice of $a \to \infty$ followed by the separating equilibrium. The key result of this section is that $h > 0$ limits credible communication, which, in turn, induces X to acquire only a limited amount of information. We also show that if pooling occurs in equilibrium, we have pooling by the lowest types.

Lemma 5 shows the upper bound of the maximum number of reports in equilibrium.

Lemma 5 *Consider the maximum number of reports $\overline{N}$ over all equilibria of all* *continuation games. In the limit when $\lambda \to 0$, $\overline{N}$ has the following upper-bound:*

$$\overline{N} \leq \max\left\{\left(\frac{\gamma}{4h}+\sqrt{\frac{\gamma}{8h}}+\frac{1}{2}\right), \left(6\sqrt{\frac{\gamma}{8h}}-2\right), 2\right\}$$

Lemma 5 implies that h limits communication, akin to the role of λ in Lemma 4. Given $\lambda \to 0$, each type who receives approval in equilibrium would like to reveal its type. However, for types that do not receive approval, the incentive to misreport is very strong, as revealing one's type implies losing the option value of the project. As only the first report receives no approval (Lemma 1), the incentive to misreport v is largest for type k_1, the highest type in the first partition that reports r_1 and gets no approval. To prevent this type from misreporting, sending r_2 must lead to a negative expected project value for type k_1. This requires that the second partition (*i.e.*, the number of types that send r_2) must be sufficiently wide, which, in turn, requires that the third partition is also sufficiently wide, and so on. As an increase in a brings $\mathrm{E}[v|k_1]$ closer to the border between the first and second partition, the width of the second partition must also increase in a.

In choosing a, X faces the following trade-off: a higher a leads to better project decisions when k is sufficiently large, but to worse decisions when k is small. Furthermore, X prefers to choose an a such that the first partition is small. The effect of a on the length of the first partition can be erratic, especially for small values of a.[23] This prevents us from making precise analytical statements concerning the optimal value of a, except that the persuasion motive induces X to choose a finite value of a even though she could observe v for free by taking $a \to \infty$.

Proposition 2 *Consider the case where $\lambda \to 0$ and $c \to 0$. An equilibrium that maximizes the* ex ante *expected utility of X always exists and is generically unique. There exists a finite number $\widehat{a}$ such that in this equilibrium, X chooses $a \leq \widehat{a}$.*

Proposition 1 states that for small values of h, X chooses a finite a, even if $c \to 0$. Proposition 2 states that this also holds for high values of h. The incentive for X to persuade E to approve limits communication, in particular after acquiring precise information. Acquiring less information improves the effectiveness of communication.

[23] For example, it is possible that for $a = 2$, $v \in \left[0, \frac{1}{2}\right]$ leads to $d^E = 0$, and that $a = 3$, $v \in \left[0, \frac{1}{3}\right]$ leads to $d^E = 0$, and for $a = 5$, $v \in \left[0, \frac{2}{5}\right]$ leads to $d^E = 0$.

Proposition 2 does not exclude partially pooling equilibria, in contrast to Proposition 1. To understand why, consider a separating equilibrium in which $d^E(r_1) = 0$. As discussed above, a should be sufficiently small to prevent X from sending r_2 if her type $k = 1$. The benefit of coarse information acquisition only realizes when $k = 1$. However, the cost of coarse information is poor decision-making by I, which realizes for many types, $k > 1$. Hence, X may prefer to choose a value of a such that some pooling occurs. If so, pooling occurs pre-dominantly for low types. In the absence of the impression motive, the width of partitions is decreasing beyond the second partition. Table 1 lists the sender-optimal equilibria for various levels of h. This shows how the persuasion motive can yield pooling at the bottom.[24]

h	a^{OP}	N	Communication
0.25	2	2	$\{1, 1\}$
0.1	12	3	$\{3, 5, 4\}$
0.09	8	4	$\{2, 3, 2, 1\}$
0.08	5	4	$\{1, 2, 1, 1\}$
0.019	28	16	$\{3, 5, 4, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
0.018	18	14	$\{2, 3, 2, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$
0.006	50	38	$\{3, 5, 4, 3, 2, 1, 1, \ldots, 1\}$
0.005	35	31	$\{2, 3, 2, 1, 1, \ldots, 1\}$

Table 1: Sender-optimal equilibria for various levels of h. The column 'Communication' shows the number of types sending the same report, for consecutive reports.

This highlights another difference between the effects of the impression motive and the persuasion motive. If the impression motive leads to distortions in communication (*e.g.*, if $\gamma > 1$), we obtain pooling at the top, as is typical in cheap-talk games. The persuasion motive, however, leads to pooling at the bottom, as it induces an incentive to overstate v that is stronger for low types than for high types. This also implies that adding an impression motive to the persuasion motive, hence allowing $\lambda > 0$, further restricts communication. Figure 4 depicts the sender-optimal a^{OP} as a function of λ for various levels of h, given $c = 0$ and $\gamma = 1$, obtained numerically.

[24] Table 1 also shows that the sender-optimal a is non-monotone in h, as a result of the discrete nature of a.

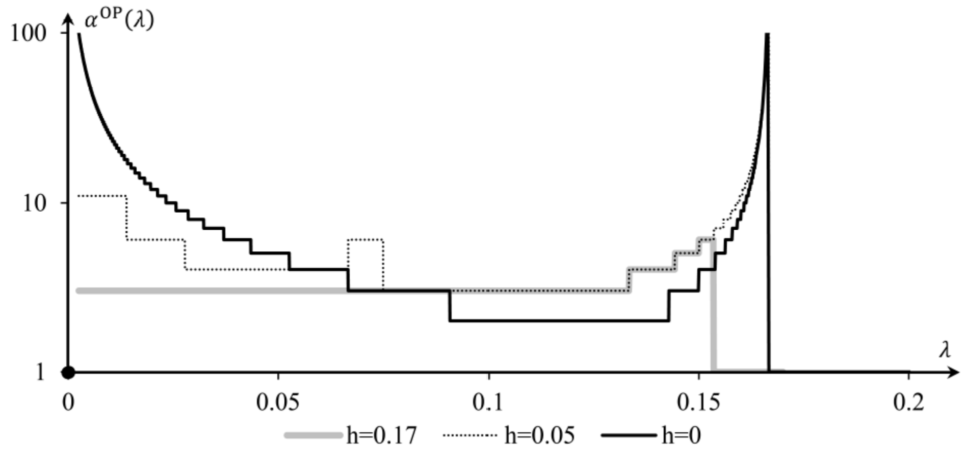

Figure 4: Function $a^{\mathrm{OP}}(\lambda)$ for $c = 0$, $\gamma = 1$, and $h \in \{0, 0.05, 0.17\}$.

If $h = 0$, Proposition 1 implies that $a^{\mathrm{OP}} = \bar{a}$, which is decreasing in λ provided $\bar{a} \geq 2$. For $\lambda > \frac{1}{7}$, $\bar{a} = 1$ and $N = 2$, and Corollary 1 states that $a \geq 2$ can be optimal. Figure 4 illustrates this graphically this: a^{OP} keeps increasing in λ up to the spike at $\lambda = \frac{1}{6}$. A positive value of h generally reduces a^{OP} if λ is small, as illustrated by graphs for $h = 0.05$ and $h = 0.17$.[25] This shows that the persuasion motive and impression motive are imperfect substitutes in hindering communication and, consequently, in reducing optimal information acquisition.

For $h = 0.17$, E does not approve after report r_1. Hence, X needs to take into account the positive probability of being denied approval when determining a. If λ gets larger, the impression motive interferes with the motive to persuade E. For small values of λ, X responds by increasing a, but as λ becomes larger, X can no longer credibly send a report that secures approval. As a consequence, $a^{\mathrm{OP}} = 1$ for large values of λ, and only the babbling equilibrium remains.

4 Forward Induction Refinement

Proposition 1 gives the sender-optimal equilibrium when h is small. It is well-known that cheap-talk models à la Crawford and Sobel (1982) generally have multiple equilibria. In particular, if any equilibrium with influential communication exists, the equilibrium without influential communication (so called 'babbling' equilibrium)

[25] Around $\lambda = 0.07$, the effect of h on a^{OP} is not monotone. Here, choosing $a > \bar{a}$ allows X to increase the probability of approval if $h = 0.05$, but not if $h = 0.17$.

also exists. In our model, the communication game is preceded by an information acquisition stage. This enables us to apply a forward induction refinement. Loosely speaking, forward induction assumes that in equilibrium past actions have been rational. Proposition 3 shows that the sender-optimal equilibrium is the *unique* equilibrium selected by the forward induction refinement, given $c > 0$ and the conditions under which Proposition 1 holds.

Proposition 3 *Suppose that $c > 0$, $h \leq \overline{h}(a^*)$, $\gamma \leq 1$ and $\lambda \leq \frac{\gamma}{4+3\gamma}$ so that $\overline{a} \geq 2$. Then, Proposition 1 characterizes the unique forward induction equilibrium outcome.*

To illustrate how forward induction selects a unique equilibrium, suppose $a^* \geq 2$ and $h \leq \overline{h}(a^*)$. If X has chosen $a = 1$, no information can be conveyed in the communication game. Now suppose $a = 2$. In the continuation game $\Gamma(2)$, a pooling equilibrium as well as a separating equilibrium exist. However, for $c > 0$, the pooling equilibrium does not satisfy forward induction: choosing $a = 1$ yields a higher payoff to X than choosing $a = 2$ followed by pooling. Hence, after observing $a = 2$, forward induction implies that neither E nor I expect themselves to play the pooling equilibrium in $\Gamma(2)$. Therefore, $a = 2$ followed by pooling does not satisfy the forward induction refinement.

This line of reasoning extends to higher levels of information acquisition too. Suppose that $a^* \geq 3$ and that X has chosen $a = 3$. Again, pooling and semi-pooling equilibria exist in $\Gamma(3)$, but $a = 2$ followed by the separating equilibrium would yield a higher payoff to X than $a = 3$ followed by pooling or partially pooling. Consequently, for $a = 3$, none of the (partially) pooling equilibria satisfy forward induction. This process of eliminating equilibria ends when $a = a^* = \min\left\{\overline{a}, \sqrt[3]{\frac{\gamma}{12c}}\right\}$. Choosing $a > a^*$ reduces X's payoff either because acquiring more information is too expensive ($a > \sqrt[3]{\frac{\gamma}{12c}}$) or because more information does not lead to more informative communication ($a > \overline{a}$). Hence, the forward induction refinement selects the sender-optimal equilibrium as the unique equilibrium outcome.[26]

[26] If $h > \overline{h}(a^*)$, forward induction may not select a unique equilibrium. If message r_1 does not lead to approval, an increase in a affects the probability of receiving approval as well as the possible equilibria of $\Gamma(a)$. As a result, after choosing some $a > 2$, there can be more than one equilibrium of $\Gamma(a)$ that leads to a higher payoff for X than the highest possible payoff after choosing $a - 1$. Forward induction then selects all these equilibria of $\Gamma(a)$, implying that there can be no unique forward induction equilibrium outcome.

The same argument implies that forward induction excludes the babbling equilibrium outcome whenever there exists an a and an equilibrium of $\Gamma(a)$ that yields a higher payoff to X than choosing $a = 1$.

Corollary 2 *If for some $a = \hat{a} > 1$, the continuation game $\Gamma(\hat{a})$ has an equilibrium that yields a higher payoff to X than $\Gamma(1)$, then $a \leq \hat{a}$ followed by the pooling equilibrium of $\Gamma(a)$ is not a forward induction equilibrium outcome.*

5 Transparency

This section discusses implications and extensions of our analysis. We first discuss the role of transparency in our model. In the previous sections we assumed that all reports were public, implying that I, E, and P received the same report. Firms can also choose to be less transparent. Proposition 4 states how much information X collects and what she communicates, if she can send a private report to I and a private report to E and P.

Proposition 4 *Suppose X sends private report r^I to I and private report r^E to E and P. An equilibrium in which X truthfully reports its type to I always exists. In this equilibrium:*

(i) *I chooses $d^I = 1$ if $z < \mathrm{E}[v|k] = \frac{2k-1}{2a}$ and $d^I = 0$ otherwise;*

(ii) *the report of X to E and P is uninformative;*

(iii) *E chooses $d^E = 1$ if $h < \frac{\gamma}{24}\left(4 - \frac{1}{a^2}\right)$ and $d^E = 0$ otherwise;*

(iv) *X chooses $a = \alpha^{pr}$, where*

$$\alpha^{pr} = \begin{cases} a^{opt} \equiv \sqrt[3]{\frac{\gamma}{12c}}, & \text{if } h < \frac{\gamma}{8} \\ \max\left\{a^{opt} \equiv \sqrt[3]{\frac{\gamma}{12c}}, \frac{1}{2}\sqrt{\frac{\gamma}{\gamma - 6h}}\right\}, & \text{if } \frac{\gamma}{8} < h < \frac{\gamma}{6} \text{ and } c \leq \bar{c} = \frac{2h}{\sqrt{\frac{\gamma}{\gamma-6h}} - 2} \\ 1, & \text{if } h > \frac{\gamma}{6} \text{ or if } \frac{\gamma}{8} < h < \frac{\gamma}{6} \text{ and } c > \bar{c} \end{cases}$$

The strategy of I, item (i), is the same as before. As r^I is only received by I, only the informational motive affects the content of r^I. The preference alignment of X and I allows for sharing all information. Item (ii) in Proposition 4 is a direct consequence of the misalignment of preferences between X on the one hand and E and P on the other. As the decision by I is not affected by r^E, the informational motive is absent in determining r^E. In isolation, the persuasion motive and the impression motive obstruct influential communication.

Anticipating I's strategy, E infers that the expected project value is given by (6). Lacking any further information, item (iii) implies that E approves if the expected project value is greater than the threshold h. Clearly, E is more willing to approve if accuracy a is higher. A more accurate information system implies that I makes a better decision, leading to higher expected project value. The condition in item (iii) shows that, independent of a, E never approves if $h > \frac{\gamma}{6}$, whereas E always approves if $h < \frac{\gamma}{8}$.

Item (iv) in Proposition 4 shows that if X is not constrained by E's approval decision, she chooses the level of accuracy that maximizes firm value, a^{opt}. If $\frac{\gamma}{8} < h < \frac{\gamma}{6}$, however, X may need to increase a to meet E's approval constraint given in item (iii). In other words, X increases the level of accuracy to persuade E to approve. Of course, increasing a is optimal for X only if the cost of information is sufficiently small, $c \leq \bar{c}$. Finally, if E never approves ($h > \frac{\gamma}{6}$), X sets $a = 1$, as information acquisition would be a pure waste.[27] Remarkably, Proposition 4 shows that information acquisition is not affected by the impression motive, λ. The ability to send private reports to insiders and outsiders decouples the informational and reputational motives, rendering the latter irrelevant for internal decision-making.

The main take-away is that lack of transparency works well for internal decision-making, but poorly for external decision-making. In the absence of a need to persuade, information acquisition and the implementation decision maximize firm value. If the impression motive is sufficiently strong, adopting transparent reporting would reduce firm value. At the same time, the outsider's approval decision is sub-optimal, as it cannot be based on the information on v that is available inside the firm. E may approve projects that he would have rejected if informed fully. Hence, it is not surprising that often firms are reluctant to give in to calls for more transparency.

Yet, if maximizing firm value does not lead to outsider's approval, the firm is forced to alter its information system to persuade E. One way is to over-invest

[27] Other equilibria exist, but given the preference alignment between X and I, this is a natural equilibrium to consider. Furthermore, similar to Proposition 3, we can show that the equilibrium described in Proposition 4 is selected by the forward induction refinement if $c > 0$ and $h < \frac{\gamma}{8}$.

in information acquisition. If that is not sufficient, the firm needs to commit to transparent communication, despite the negative consequences for internal decision-making. Hence, from the firm's perspective, transparency is a necessary evil.[28]

6 Discussion

We view this paper as a first step towards an integral theory of the design and use of (formal and informal) information systems in firms. Various subsequent steps are required to capture all aspects of an information system highlighted by Cyert and March (1963). We have focused on how the main motives of informing, persuading, and impressing affect information acquisition and communication. To do so, we have assumed a given, static decision-making procedure, a given set of stakeholders, a given, simplified incentive structure, and observable expenditures on information acquisition. The effects of each of these assumptions deserve to be explored. Below, we discuss several implications and extensions of our results.

Information often gets stored or processed in categories, even if the underlying data is continuous. Employee performance evaluation forms often have only three or five distinct categories of performance. Credit rating agencies assign firms to one of a limited number of categories based on their assessment of the (continuous) probability of default. Our model provides a rationale. If information is to be stored or processed in a too precise manner, this strengthens potential incentives to manipulate information. By using broad categories, more reliable information can be obtained and retained.

We have shown that if firms need to persuade outsiders, voluntary commitment to transparent reporting can be optimal. If such commitment is difficult to sustain, regulation that requires transparency can be beneficial for companies. At the same time, such regulation hurts companies that do not need to persuade outsiders and prefer private reporting to avoid the negative consequences of the impression motive. Hence, regulation that imposes transparency can have differential effects on firms. Relatedly, we have shown that the possibility to manipulate information can backfire

[28] Durnev and Kim (2005) find that in countries where investor protection is low, so that investors need more persuasion to supply funds, the relation between firms' valuation and transparent reporting is stronger. Similarly, Lang et al. (2012) show that firm-level transparency matters more for firms' valuation if investor uncertainty is higher.

if firms need to persuade outsiders. If transparent reporting does not suffice, firms can attempt to make their reports verifiable, for instance through hiring external auditors. However, this increases the cost of information acquisition, which also reduces the amount of information available.

Our model can also be used to illustrate the interaction between the decision-making process and the information system. In our model, the final decision on the project is made by I. At first glance, there is no reason in our model for X to delegate the final decision. Since the preferences of X and I are aligned, I would be willing to share his information about z with X. Note, however, that if we had assumed that X instead of I would make the final decision, X would lose the possibility of sending influential messages to E. Delegation of this decision, in combination with transparency in communication, thus creates the possibility to persuade outsiders.

Similarly, our model can illustrate how incentive pay affects the use of a firm's information system. Above, we have shown that strong stock-based incentive pay for executives hinders communication to outsiders. Now suppose that X, but not I, receives a long-term incentive plan (LTIP), which makes that X cares about the stock price that arises *after* public P has observed I's decision d^I. Under private reporting, a positive implementation decision is interpreted by P as a positive signal regarding firm value. Thus the LTIP gives X an incentive to overstate v in communication with I. By reducing the alignment of interests between X and I, the LTIP hinders internal communication, which in turn negatively affects information acquisition.[29]

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7 Appendix

Proof of Lemma 1.

Consider an equilibrium of the continuation game $\Gamma(a)$. Using (3) and noting that s is not affected by d^I, the equilibrium strategy of I is:

$$\delta^I(r) = 1 \text{ if } \mathrm{E}[v|r] > z \text{ and } \delta^I(r) = 0 \text{ otherwise} \tag{11}$$

Hence, $\mathrm{E}\left[d^I|r\right] = \mathrm{E}\left[v|r\right]$ and $\mathrm{E}\left[z|r, d^I = 1\right] = \frac{1}{2}\mathrm{E}\left[v|r\right]$. Consider type k. Substituting (1), (2), and (4) into (3) and using (11), we write the expected utility of X of type k from reporting r as follows:

$$\begin{aligned} \mathrm{E}\left[u^X|k, r\right] &= (1-\lambda)\left(v_k + \gamma\,\mathrm{E}\left[v|r\right]\left(v_k - \tfrac{1}{2}\mathrm{E}\left[v|r\right]\right)\delta^E\right) \\ &\quad +\lambda\theta\left(1+\tfrac{\gamma}{2}\mathrm{E}\left[v|r\right]\delta^E\right) - c\left(a-1\right) \end{aligned} \tag{12}$$

Suppose two reports r_1 and r_2 with $\mathrm{E}\left[v|r_2\right] \geq \mathrm{E}\left[v|r_1\right]$ are used in an equilibrium. Then, it must be one of the four cases below:

1. If $\delta^E\left(r_1\right) = \delta^E\left(r_2\right) = 0$ then it must be that $\mathrm{E}\left[v|r_2\right] > \mathrm{E}\left[v|r_1\right]$ (otherwise the reports must be identical) so that

$$\mathrm{E}\left[u^X|k, r_2\right] - \mathrm{E}\left[u^X|k, r_1\right] = \lambda\left(\mathrm{E}\left[v|r_2\right] - \mathrm{E}\left[v|r_1\right]\right) > 0$$

for all k. Then, no type uses r_1, a contradiction.

2. If $\delta^E\left(r_1\right) = \delta^E\left(r_2\right) = 1$, then the difference

$$\begin{aligned} \mathrm{E}\left[u^X|k, r_2\right] - \mathrm{E}\left[u^X|k, r_1\right] &= \left(\gamma\left(1-\lambda\right)v_k - \tfrac{(1-2\lambda)\gamma}{2}\left(\mathrm{E}\left[v|r_2\right] + \mathrm{E}\left[v|r_1\right]\right) + \lambda\right) \times \\ &\quad \times \left(\mathrm{E}\left[v|r_2\right] - \mathrm{E}\left[v|r_1\right]\right) \end{aligned}$$

is strictly increasing in v_k and, hence, in k as well (the so-called 'single-crossing' property). As a result, if some type k_2 prefers reporting r_1 to reporting r_2, all types $k > k_2$ do so as well. Thus, the set of types reporting r_n is necessarily a set of consecutive types $\{(k_{n-1}+1), \ldots, k_n\}$.

3. If $\delta^E\left(r_2\right) = 1$ and $\delta^E\left(r_1\right) = 0$, then then the difference

$$\mathrm{E}\left[u^X|k, r_2\right] - \mathrm{E}\left[u^X|k, r_1\right] = \tfrac{\gamma}{2}\mathrm{E}\left[v|r_2\right]\left(2\left(1-\lambda\right)v_k - \left(1-2\lambda\right)\mathrm{E}\left[v|r_2\right]\right) + \lambda\left(\mathrm{E}\left[v|r_2\right] - \mathrm{E}\left[v|r_1\right]\right)$$

is strictly increasing in k. The single-crossing property holds and combining it with the result from case 1, we conclude that the set of types reporting r_1 is a set of lowest types $\{1, \ldots, k_1\}$.

4. If $\delta^E\left(r_2\right) = 0$ and $\delta^E\left(r_1\right) = 1$, then $\mathrm{E}\left[u^X|k, r_2\right] - \mathrm{E}\left[u^X|k, r_1\right]$ is decreasing

in k, types reporting r_2 are lower than types reporting r_1 so that $\mathrm{E}[v|r_2] < \mathrm{E}[v|r_1]$, a contradiction.

We conclude that each report r_n is sent by a subset of types $\{(k_{n-1}+1),\ldots,k_n\}$, which is fully characterized by the set of marginal types $\{k_n\}$, $n=1,\ldots,N$, with $k_n \geq k_{n-1}$ and $k_{n-1} \equiv 0$. In this case,

$$\mathrm{E}[v|r_n] = \tfrac{1}{2}\left(v_{k_{n-1}+1} + v_{k_n}\right) = \tfrac{1}{2a}\left(k_{n-1}+k_n\right) \tag{13}$$

Moreover, the single-crossing property implies that if type $k=k_n$ prefers reporting r_n to reporting r_{n+1}, all types $k \leq k_n$ also prefer r_n to $r_{n+\tau}$ for any $\tau \geq 1$, and if type $k = k_{n-1}+1$ prefers reporting r_n to reporting r_{n-1}, all types $k \geq k_{n-1}+1$ also prefer r_n to $r_{n+\tau}$ for any $\tau \leq -1$ (the absence of local deviations implies the absence of global deviations). Thus, the necessary incentive compatibility constraints

$$\mathrm{E}\left[u^X|k_n, r_n\right] - \mathrm{E}\left[u^X|k_n, r_{n+1}\right] \geq 0 \tag{14}$$

$$\mathrm{E}\left[u^X|k_n+1, r_{n+1}\right] - \mathrm{E}\left[u^X|k_n+1, r_n\right] \geq 0 \tag{15}$$

are also the sufficient equilibrium conditions. This proves item (i) of the proposition.

It follows from cases 1 and 3 above that only one report, namely r_1, may lead to no approval, so that $\delta^E(r_n) = 1$ for all $k \geq 2$. This proves item (iii) of the proposition.

Next, consider the approval decision d^E by E. Using (5) and I's strategy, the optimal strategy for E is to choose $d^E = 1$ if and only if $\mathrm{E}\left[u^E|r\right] > 0$. Hence, $\delta^E(r) = 1$ if $\frac{\gamma}{2}\mathrm{E}\left[(v_k)^2|r\right] > h$, and $\delta^E(r) = 0$ otherwise, which can be written as follows:

$$\delta^E(r_n) = 1 \text{ if } \tfrac{\gamma}{8a^2}\left(k_{n-1}+k_n\right)^2 > h, \text{ and } \delta^E(r_n) = 0 \text{ otherwise} \tag{16}$$

To prove item (ii), we rewrite the ICC in (14) for $\delta^E(r_n) = 1$ and $\delta^E(r_1) = 0$ correspondingly

$$\gamma\left(\tfrac{1}{2}(1-2\lambda)\left(\mathrm{E}[v|r_n] + \mathrm{E}[v|r_{n+1}]\right) - (1-\lambda)v_k\right) - \lambda \geq 0, \text{ and}$$
$$\tfrac{\gamma}{2}(1-2\lambda)\left(\mathrm{E}[v|r_{n+1}]\right)^2 - \tfrac{\lambda}{2a}\left(k_{n+1}-k_{n-1}\right) - (1-\lambda)\gamma\,\mathrm{E}[v|r_{n+1}]\,v_k \geq 0$$

These inequalities never hold when $\lambda \geq \frac{1}{2}$. Hence, if $\lambda \geq \frac{1}{2}$, the pooling equilibrium is the unique equilibrium of the continuation game, which ends the proof. ■

Proof of Lemma 2.

To determine the properties of $\overline{N}(\gamma, \lambda, h)$, it is helpful to consider an auxiliary function $\underline{a}(N, \lambda, \gamma, h)$, defined as the lowest value of a for which the continuation game $\Gamma(a)$ has an equilibrium with N reports:

$$\underline{a}(N, \lambda, \gamma, h) = \min\{a \in \mathbb{N} : \Gamma(a) \text{ has an equilibrium with } N \text{ reports}\}$$

If, for some N, all equilibria for all a have less than N reports, we set $\underline{a} = \infty$. Function $\underline{a}(N, \lambda, \gamma, h)$ is monotonically increasing in N (as is shown below), so that $\bar{N}$ can be written as

$$\overline{N}(\lambda, \gamma, h) = \max\{N \in \mathbb{N} : \underline{a}(N, \lambda, \gamma, h) < \infty\} \tag{17}$$

We show below that $\underline{a}(N, \lambda, \gamma, h)$ is monotonically increasing in λ and h, and is decreasing in γ, which yields the desired monotonicity properties of $\overline{N}$. This can be shown as follows (for brevity, we omit unnecessary arguments form argument lists). Take γ and γ' such that $\gamma' < \gamma$. Due to the monotonicity of $\underline{a}$, $\underline{a}\left(\overline{N}(\gamma), \gamma\right) \leq \underline{a}\left(\overline{N}(\gamma), \gamma'\right)$ and $\underline{a}\left(\overline{N}(\gamma) + 1, \gamma\right) \leq \underline{a}\left(\overline{N}(\gamma) + 1, \gamma'\right)$. By definition of $\overline{N}(\gamma)$, $\underline{a}\left(\overline{N}(\gamma), \gamma\right) < \underline{a}\left(\overline{N}(\gamma) + 1, \gamma\right) = \infty$. Therefore,

$$\underline{a}\left(\overline{N}(\gamma), \gamma\right) \leq \underline{a}\left(\overline{N}(\gamma), \gamma'\right) \leq \underline{a}\left(\overline{N}(\gamma) + 1, \gamma\right) = \underline{a}\left(\overline{N}(\gamma) + 1, \gamma'\right) = \infty$$

Then, if $\underline{a}\left(\overline{N}(\gamma), \gamma'\right) < \infty$ then $\underline{a}\left(\overline{N}(\gamma), \gamma'\right) \leq \underline{a}\left(\overline{N}(\gamma) + 1, \gamma'\right) = \infty$ implies $\overline{N}(\gamma') = \overline{N}(\gamma)$, and if $\underline{a}\left(\overline{N}(\gamma), \gamma'\right) = \infty$ then it implies $\overline{N}(\gamma') \leq \overline{N}(\gamma) - 1$. The proof of monotonicity of $\overline{N}$ w.r.t. λ and h is similar and is, therefore, omitted.

The remaining part of the proof shows the monotonicity of $\underline{a}$. We define l_n as the number of types sending report r_n:

$$l_n \equiv k_n - k_{n-1} \tag{18}$$

Using (13) we express the ICCs (14) and (15) as follows, respectively:

$$(l_{n+1} - l_n) \geq G(k_n, a) + \left(1 - \delta^E(r_n)\right) \underline{H} \tag{19}$$

$$(l_{n+1} - l_n) \leq G(k_n, a) + \tfrac{4(1-\lambda)}{(1-2\lambda)} + \left(1 - \delta^E(r_n)\right) \overline{H} \tag{20}$$

where

$$G(k, a) = \frac{4\lambda(\gamma k + a) - 2\gamma(1-\lambda)}{\gamma(1-2\lambda)} \tag{21}$$

$$\begin{aligned} \underline{H} &= \tfrac{(2k_n - l_n)}{(l_{n+1}+l_n)}\left(2\tfrac{k_n-(1-\lambda)}{(1-2\lambda)} + l_n\right) > 0 \\ \overline{H} &= \tfrac{(2k_n - l_n)}{(l_{n+1}+l_n)}\left(2\tfrac{k_n+(1-\lambda)}{(1-2\lambda)} + l_n\right) > \underline{H} > 0 \end{aligned}$$

Using (16), we write the equilibrium condition $\delta^E(r_2) = 1$ as

$$k_1 + k_2 > 2a\sqrt{\tfrac{2h}{\gamma}} \tag{22}$$

Consequently, conditions (19), (20), and (22) constitute all the necessary and sufficient conditions that any arbitrary sequence $\{l_n\}$ of size N must satisfy to represent an equilibrium for $a = k_N$ with N reports. Conditions (19) and (20) can jointly be written as the following double inequality

$$l_L(k_n, l_n, a, n) \leq l_{n+1} \leq l_H(k_n, l_n, a, n) \tag{23}$$

In the following Lemma, we establish properties of l_L and l_H that we use in the rest of the proof.[30]

Lemma 6 *Functions $l_L(k, l, a, n)$ and $l_H(k, l, a, n)$ have the following properties:*

(i) For $n \geq 2$, for $n = 1$ and $l < 2\sqrt{\frac{2h}{\gamma}}a$, and $n = 1$ and $l > 2\sqrt{\frac{2h}{\gamma}}a$, l_L and l_H are increasing in (k, l, a, λ), decreasing in γ, and independent of h;

(ii) For for $n = 1$ and $l = 2\sqrt{\frac{2h}{\gamma}}a$ $l_L(l, l, a, 1)$ and $l_H(l, l, a, 1)$ are discontinuous, increasing in (h, a), and decreasing in (l, γ);

(iii) For $n \geq 2$, $l_H - l_L > 4$, and $l_H(l, l, a, 1) - l_L(l, l, a, 1) > 4$;

[30] The proof of this Lemma uses standard extensive algebraic transformations, and is available upon request.

(iv) $l_H(l,l,a,1) - l_L(l+1,l+1,a,1) > 2$*;*

(v) Let, for some integers a*,* $l_1 \geq 1$*, and* $l_2 > l_1$*, it holds that* $l_H(l_1,l_1,a,1) < l_2$*. Then, there exists an integer* $x \geq 1$ *so that* $l_L(l_1+x,l_1+x,a,1) \leq l_2 - x \leq l_H(l_1+x,l_1+x,a,1)$*.*

We also define $\underline{l}$ as the smallest integer for l_{n+1} that satisfies (23):

$$\underline{l} \equiv \lceil l_L \rceil = \min\{z \in \mathbb{Z} : z \geq l_L\}$$

To prove that $\underline{a}(N)$ increases in N we take an arbitrary N such that $\underline{a}(N) < \infty$ and assume, to the contrary, that $\underline{a}(N+1) < \underline{a}(N)$. Consider an equilibrium of the continuation game $\Gamma(a)$ for $a = \underline{a}(N+1)$ with $N+1$ reports. We will show that there exists an $a' < \underline{a}(N+1) < \underline{a}(N)$ and there exists an equilibrium of the continuation game $\Gamma(a')$ with N reports. This will contradict the definition of $\underline{a}(N)$. We construct this equilibrium iteratively in the following steps. We use the iteration number (t) as a superscript.

Step 1. We begin with the set $\{l_n\}$, $n \in \{1,\ldots,N+1\}$, from the equilibrium for $a = \underline{a}(N+1)$ with $N+1$ reports. At first iteration, we set $l_n^{(1)} = l_n$ for $n \in \{1,\ldots,N\}$ (we just truncate $\{l_n\}$ at N) to obtain $\left\{l_n^{(1)}\right\}$, and proceed to Step 2.

Step 2. Using $k_{n+1}^{(t)} = l_{n+1}^{(t)} + k_n^{(t)}$, we obtain $\left\{k_n^{(t)}\right\}$ and $a^{(t)} = k_N^{(t)}$. The set $\left\{k_n^{(t)}\right\}$ may represent no equilibrium because (23) may fail. However, condition (22) holds by its monotonicity. We proceed to Step 3.

Step 3. If the sufficient equilibrium condition (23) holds for all $n \in \{1,\ldots,N\}$, then we have constructed an equilibrium with N reports for $a < \underline{a}(N)$. Otherwise, by the monotonicity of l_L and l_H, it must be the condition $l_{n+1}^{(t)} \leq l_H\left(k_n^{(t)}, l_n^{(t)}, a^{(t)}, n\right)$ that fails (the other condition $l_L\left(k_n^{(t)}, l_n^{(t)}, a^{(t)}, n\right) \leq l_{n+1}^{(t)}$ may only become weaker). We proceed to Step 4 to adjust $\left\{l_n^{(k)}\right\}$.

Step 4. If (23) fails for $n = 2$, we proceed to step 5. Otherwise, let $n^* \geq 3$ be the lowest n for which (23) fails. We set $l_n^{(t+1)} = l_n^{(t)}$ for $n < n^*$ and $l_n^{(t+1)} = \underline{l}\left(k_n^{(t+1)}, l_n^{(t+1)}, a^{(t)}, n\right)$ for $n \geq n^*$ and proceed to the next iteration

$(t+1)$ in Step 2. By the monotonicity of $\underline{l}$, $l_n^{(t+1)} \leq l_n^{(t)}$ for all n, and condition (22) holds.

Step 5. If $l_2^{(t)} > l_H\left(l_1^{(t)}, l_1^{(t)}, a^{(t)}, 1\right)$, then, according to item (v) of the Lemma), there exists an integer $x \geq 1$ such that $l_L\left(l_1^{(t)} + x, l_1^{(t)} + x, a^{(t)}, 1\right) \leq l_2^{(t)} - x \leq l_H\left(l_1^{(t)}, l_1^{(t)}, a^{(t)}, 1\right)$. We set $l_1^{(t+1)} = l_1^{(t)} + x$ and $l_2^{(t+1)} = l_2^{(t)} - x$. As a result, $k_2^{(t+1)} = k_2^{(t)}$, $k_1^{(t+1)} > k_1^{(t)}$ so that condition (22) holds by its monotonicity. Then, we set $l_n^{(t+1)} = \underline{l}\left(k_n^{(t+1)}, l_n^{(t+1)}, a^{(t)}, n\right)$ for $n \geq 3$ and proceed to the next iteration $(t+1)$ in Step 2. By the monotonicity of $\underline{l}$, $l_n^{(t+1)} \leq l_n^{(t)}$ for all $n \geq 2$.

After each iteration, either $a^{(t+1)} < a^{(t)}$ or $l_1^{(t+1)} > l_1^{(t)}$, so that after a finite number of iterations, we obtain an equilibrium with N reports following a choice of $a < \underline{a}(N)$, a contradiction.

To prove that $\underline{a}$ increases h, we take arbitrary h, $h' > h$, and N such that $\underline{a}(N, h') < \infty$. Consider an equilibrium of $\Gamma(a)$ for $a = \underline{a}(N, h')$. We will show that there exists an $a' < a = \underline{a}(N, h')$ and the corresponding equilibrium of $\Gamma(a')$ with N reports. As the first iteration, we take $l_n^{(1)} = l_n$ from the equilibrium for $a = \underline{a}(N, h')$ and proceed to Step 2 above (we use h in all the steps). Since $h < h'$, due to the monotonicity, only the condition $l_{n+1}^{(t)} \leq l_H$ may fail in Step 3 so that the iterative procedure necessarily results in an equilibrium for h and $a' \leq a = \underline{a}(N, h')$ with N reports. This implies that $\underline{a}(N, h') \leq \underline{a}(N, h)$. Similarly, the monotonicity of $\underline{a}$ in γ (by taking $\underline{a}(N, \gamma') < \infty$ and $\gamma > \gamma'$) and in λ (by taking $\underline{a}(N, \lambda') < \infty$ and $\lambda' > \lambda$) can be shown. This ends the proof. ■

Proof of Lemma 3.

For $\lambda \geq \frac{1}{2}$, no separating equilibrium exists according to Lemma 1. Since $\overline{a} \leq 1$ in this case, the Lemma holds for $\lambda \geq \frac{1}{2}$. In the reminder of the proof, $\lambda < \frac{1}{2}$ is assumed.

In any equilibrium, the strategy of E is given by (16). In a separating equilibrium, $\mathrm{E}[v|r_k] = v_k$. Hence, $\delta^E(r_k) = 1$ if

$$h < \tfrac{\gamma}{2}\,\mathrm{E}\left[(v_k)^2 | r_k\right] = \tfrac{\gamma}{2}(v_k)^2$$

and $\delta^E(r_k) = 1$ for all k when $h < \frac{\gamma}{2}(v_1)^2 = \overline{h}(a)$. In this case, the sufficient equilibrium conditions (19) and (20) become:

$$2\lambda k \leq (1-\lambda) - 2a\tfrac{\lambda}{\gamma} \text{ and } 2\lambda k \geq -(1-\lambda) - 2a\tfrac{\lambda}{\gamma}$$

The second ICC always holds whereas the first ICC holds for all $k = 1, \ldots, (a-1)$ if and only if it holds for $k = a-1$, *i.e.*, if $a \leq \frac{\gamma}{1+\gamma}\frac{(1+\lambda)}{2\lambda}$. For integer a it is equivalent to $a \leq \bar{a}$, which ends the proof. ■

Proof of Lemma 4.

Consider an equilibrium of the continuation game $\Gamma(a)$. Since $N \leq a$, when $a \leq \overline{a}$ the proof is straightforward. Suppose that $a > \overline{a}$ and suppose an equilibrium exists with $N > \bar{a}$ reports. For expositional clarity, we introduce the following notation. Let integers x and q be

$$x \equiv a - \bar{a} \text{ and } q \equiv N - \bar{a}$$

According to (8), $\bar{a} + 1 > \frac{\gamma}{1+\gamma}\frac{1+\lambda}{2\lambda}$, and we define δ to be

$$\delta \equiv 2\lambda\left((1+\gamma)\,\bar{a} + 1\right) > \gamma\,(1-\lambda) \tag{24}$$

We also use l_n as the number of types sending report r_n, as given by (18).

The proof is conducted as follows. In Part 1, we show that l_n weakly increases in n. Part 2 is by induction. We show that if $(l_{n+1} - l_n) \geq y$ for some $y \geq 0$ and all $n \leq N-1$, then there is a lower-bound on a, $a \geq a^{LB}$. Using this lower bound, we show that ICC (19) implies $(l_{n+1} - l_n) \geq y + 1$ for all all $n \leq N-1$. By induction, it follows that $(l_{n+1} - l_n)$ is unbounded, a contradiction. This result holds when $N > \overline{a} \geq 2$, and when $\overline{a} = 1$ and $N \geq 3$. Items (i) and (ii) of the lemma then follow.

Part 1. Consider the ICC (19). Since $G(k,a)$ increases in k and $\underline{H} > 0$, it follows that

$$(l_{n+1} - l_n) \geq G(1,a) > \tfrac{\lambda}{\gamma(1-2\lambda)}\left(3\gamma + 2\,(1-\gamma)\,\bar{a} + 2\,(2x-1)\right) - 1 > -1$$

due to $\gamma \leq 1$. Since $(l_{n+1} - l_n)$ is integer, it must be that $(l_{n+1} - l_n) \geq 0$ for all $n \leq N-1$.

Part 2. Suppose (induction assumption) that $(l_{n+1} - l_n) \geq y$ for all $n \leq N-1$ and some $y \in \mathbb{Z}_+$. This assumption holds for $y = 0$. We obtain the following lower-bounds on a and k_{N-1}:

$$\begin{aligned}
a &= \sum_{n=1,\ldots,N} l_n \geq \sum_{n=1,\ldots,N} \left(l_1 + (n-1)\,y\right) \geq N\left(1 + \frac{1}{2}y\,(N-1)\right) \equiv a^{LB} \\
k_{N-1} &= \sum_{n=1,\ldots,N-1} l_n \geq (N-1)\left(1 + \frac{1}{2}y\,(N-2)\right) \equiv k^{LB}
\end{aligned}$$

Using these lower bounds and (24), we evaluate $G\,(k_{N-1}, a) - y$:

$$G\,(k_{N-1}, a) - y > \tfrac{1}{\gamma(1-2\lambda)}\left(\left(4\left(\gamma k^{LB} + a^{LB}\right) + y\gamma\right)\lambda - (2+y)\,\delta\right) > 0$$

Hence, $G\,(k_{N-1}, a) > y$ and, therefore, $(l_N - l_{N-1}) \geq (y+1)$. As a result, $a \geq a^{LB} + 1$. We consider cases $\overline{a} \leq 2$ and $\overline{a} \geq 3$ separately.

1. Let $\overline{a} \leq 2$ and $N \geq 3$. Then,

   $$a = \sum_{n=1,\ldots,N} l_n \geq l_1 + 2\,(l_1 + y) + (y+1) = 3l_1 + 3y + 1 \geq 4 + 3y$$

   which, together with (21) and (24), implies

   $$G\,(1, a) - y \geq \tfrac{1}{\gamma(1-2\lambda)}\left(\left(4\,(\gamma + 4 + 3y) + y\gamma\right)\lambda - (2+y)\,\delta\right) > 0$$

   Thus, $(l_{n+1} - l_n) \geq (y+1)$ for all $n \leq N-1$.

2. Let $\overline{a} \geq 3$ and $N \geq 4$. In this case, we use another induction argument. Suppose $(l_{N-j+1} - l_{N-j}) \geq (y+1)$ for all $j \in \{1, \ldots, r\}$. This assumption holds for $r = 1$. We define (new) lower bounds on a and on k_{N-r-1} as follows:

   $$\begin{aligned}
   a &= \sum_{n=1,\ldots,N} l_n \geq \sum_{n=1,\ldots,N} \left(l_1 + (n-1)\,y\right) + \sum_{n=N-r+1,\ldots,N} \left(n - (N-r)\right) \\
   &\geq N\left(1 + \tfrac{1}{2}y\,(N-1)\right) + \tfrac{1}{2}r\,(r+1) \equiv a^{NLB} \\
   k_{N-r-1} &= \sum_{n=1,\ldots,N-r-1} l_n \geq (N-r-1)\left(1 + \tfrac{1}{2}y\,(N-r-2)\right) \equiv k^{NLB}
   \end{aligned}$$

   Using (21) we show that $G\,(k_{N-r-1}, a) > y$. We consider cases $y = 0$ and $y \geq 1$ separately.

(a) When $y=0$, we define $J_1(r)$ as the following lower-bound on $(\gamma k_{N-r-1}+a)$:

$$(\gamma k_{N-r-1}+a) \geq \left(\gamma k^{NLB}+a^{NLB}\right) \geq (\gamma+1)\,N+\left(\tfrac{1}{2}r-\gamma\right)(r+1) \equiv J_1(r)$$

Since $J_1(r)$ is a second degree convex polynomial, which increases for $r>\left(\gamma-\frac{1}{2}\right)$, it follows that:

$$J_1(r) > J_1(1) = (\gamma+1)\,N+(1-2\gamma)$$

and using (24), we get

$$G(k_{N-r-1},a) > \tfrac{1}{\gamma(1-2\lambda)}\left(4\lambda J_1(r)-2\delta\right) \geq 0 = y$$

(b) When $y \geq 1$, we define $J_2(r)$ as the following lower-bound on $(\gamma k_{N-r-1}+a)$:

$$(\gamma k_{N-r-1}+a) \geq \gamma+\tfrac{1}{2}\gamma y\,(N-r-2)^2+N+\tfrac{1}{2}N\,(N-1)\,y+\tfrac{1}{2}\left(r^2+1\right) \equiv J_2(r)$$

Since $J_2(r)$ is a second degree convex polynomial, it attains its minimum at $r=r^*$, where $r^*=\frac{\gamma y}{\gamma y+1}(N-2)$. Therefore,

$$J_2(r) \geq J_2(r^*) = \gamma+\tfrac{\gamma y}{2(\gamma y+1)}(N-2)^2+N\left(1+\tfrac{1}{2}(N-1)\,y\right)+\tfrac{1}{2}$$

Using $(\gamma k_{N-r-1}+a) \geq J_2(r^*)$, we evaluate $G(k_{N-r-1},a)-y$:

$$G(k_{N-r-1},a)-y \geq \tfrac{1}{\gamma(1-2\lambda)}\left((4J_2(r^*)+y\gamma)\,\lambda-(2+y)\,\delta\right) > 0$$

Hence, finally, $G(k_{N-r-1},a) > y$.

Cases (a) and (b) conclude $G(k_{N-r-1},a) > y$, which implies that $(l_{N-j+1}-l_{N-j}) \geq (y+1)$ holds for $j=(r+1)$. By induction, it holds for all $j \leq N-1$. That is, $(l_{n+1}-l_n) \geq (y+1)$ for all $n \leq N-1$.

In both cases 1 and 2, $(l_{n+1}-l_n) \geq (y+1)$ for all $n \leq N-1$. By induction, $(l_{n+1}-l_n) \geq y$ for any $y \in \mathbb{N}$, a contradiction. Therefore, $N \leq \bar{a}$ if $\bar{a} \geq 3$ and $N \leq 2$ if $\bar{a} \leq 2$. Since for $\bar{a} \geq 2$, the separating subgame equilibrium always exists, it follows that $\overline{N}=\bar{a}$ if $\bar{a} \geq 2$. This occurs when $\frac{\gamma}{1+\gamma}\frac{(1+\lambda)}{2\lambda} \geq 2$, *i.e.*, when $\lambda \leq \frac{\gamma}{4+3\gamma}$.

When $\overline{a} = 1$, $N \leq 2$ so that $\overline{N} \leq 2$ as well. This ends the proof. ■

Proof of Proposition 1.

Consider an equilibrium of the continuation game $\Gamma(a)$. Let $U^X(a)$ be the *ex ante* expected utility of X. Taking expectations of (12) yields:

$$U^X(a) \equiv \mathrm{E}\left[\mathrm{E}\left[u^X|k, r_n\right]\right] = \tfrac{1}{2} + \mathrm{E}\left[\gamma(v - z)\,\delta^I\delta^E\right] - c(a-1) \tag{25}$$

First, we show that over all equilibria with N reports of all continuation games, the separating equilibrium of $\Gamma(N)$ maximizes $U^X(a)$. Second, we maximize (25) with respect to a over $a \in \{2, \ldots, \overline{a}\}$, according to Lemma 4.

Fix the number of reports $N \in \{2, \ldots, \overline{a}\}$. Using (11) and (13), we write the *ex ante* expected project value as follows:

$$\mathrm{E}\left[\gamma(v-z)\,\delta^I\delta^E\right] = \frac{\gamma}{8a^3}\sum_{n=1,\ldots N}(k_n - k_{n-1})(k_n + k_{n-1})^2 - \frac{\gamma}{8a^3}\left(1 - \delta^E(r_1)\right)(k_1)^3 \tag{26}$$

We maximize (26) w.r.t. $\{k_n\}$ assuming k_n are real numbers. For $\delta^E(r_1) = 1$, the first-order conditions are:

$$\tfrac{\gamma}{4a^3}\left(\tfrac{1}{2}(k_{n-1} + k_{n+1}) - k_n\right)(k_{n+1} - k_{n-1}) = 0$$

Hence, (26) attains its global maximum over $\{k_n\}$, $k_n \in \mathbb{R}$, when $k_n = \frac{1}{2}(k_{n-1} + k_{n+1})$, *i.e.*, when intervals $(k_n - k_{n-1})$ are of equal length for all n. It follows that (26) is maximized over a set of natural numbers $\{k_n\}$, $k_n \in \mathbb{N}$, when $k_n = nt$ for some $t \in \mathbb{N}$. In this case, $a = Nt$ and

$$\mathrm{E}\left[\gamma(v-z)\,\delta^I\delta^E\right] = \frac{\gamma}{8N^3}\sum_{n=1}^{N}(2n-1)^2 = \frac{\gamma}{24}\left(4 - \frac{1}{N^2}\right) \tag{27}$$

When $t = 1$, we have a separating equilibrium of $\Gamma(N)$. Hence,

$$U^X(N) = \tfrac{1}{2} + \tfrac{\gamma}{24}\left(4 - \tfrac{1}{N^2}\right) - (N-1)\,c \tag{28}$$

Maximizing (28) yields $N = a^{opt}$ given by (7). Moreover, E's strategy (16), $h \leq \overline{h}(a^*)$ assures E approves after receiving r_1, $\delta^E(r_1) = 1$.

The above arguments are based on the assumption $\delta^E(r_1) = 1$ for all a. However, if $\mathrm{E}\left[\gamma(v-z)\,\delta^I|r_1\right] < h$ for some $a > a^*$, so that $\delta^E(r_1) = 0$ for these a, then the *ex ante* expected utility $U^X(a)$ of X will be lower than in the above derivations. Therefore, $a = N = a^*$ maximizes $U^X(a)$ for all $a \in \mathbb{N}$, including $a > a^*$. This ends the proof. ■

Proof of Corollary 1.

According to Lemma 4, when $\overline{a} = 1$ then either $N = 1$ so that $a = 1$ is optimal, or $N = 2$ so that $a \geq 3$. The limiting property of optimal a can be shown as follows. For $N = 2$, $l_2 = a - l_1$, and $\delta^E(r_1) = 1$, the ICC (19) is

$$l_1 \leq \left(\tfrac{1}{2} - \tfrac{2+\gamma}{\gamma}\lambda\right)a + (1-\lambda) \equiv \overline{l_1}(a) < \tfrac{1}{2}a$$

According to (25) and (26), the expected utility of X in equilibrium, as a function of (l_1, a) is

$$U^X(l_1, a) = \tfrac{1}{2} + \tfrac{\gamma}{32}\left(5 - \left(1 - \tfrac{2l_1}{a}\right)^2\right)$$

which is monotonically increases in l_1 over $\left[0, \overline{l_1}(a)\right]$. Therefore, taking the highest value $l_1 = \left\lfloor \overline{l_1}(a) \right\rfloor$ is optimal for X. The necessary (and sufficient) condition for this is $\overline{l_1}(a) \geq 1$, *i.e.*, $\left(\tfrac{1}{2} - \tfrac{2+\gamma}{\gamma}\lambda\right)a \geq \lambda$. When $\lambda \geq \frac{\gamma}{2(2+\gamma)}$, this condition does not hold for any a, and $a = 1$ is optimal. If, on the other hand, $\lambda < \frac{\gamma}{2(2+\gamma)}$, this condition only holds if $a \geq \frac{2\lambda\gamma}{(\gamma - 2(2+\gamma)\lambda)}$. Hence, optimal a unboundedly increases if $\lambda \to \frac{\gamma}{2(2+\gamma)}$. This ends the proof. ■

Proof of Lemma 5.

In the limit $\lambda \searrow 0$, the ICCs (19) and (20) can be written as follows

$$\left(2 - \delta^E(r_n)\right)l_n - 1 \leq l_{n+1} \leq \left(2 - \delta^E(r_n)\right)l_n + 2 \qquad (29)$$

Let there be $N \geq 2$ reports in an equilibrium of the continuation game $\Gamma(a)$ following a choice of $a \geq N$. Suppose, first, that $\delta^E(r_1) = 1$. In this case, $l_{n+1} = l_n = 1$ satisfies (29) and the largest N is achieved in the separating equilibrium when $N = a$. Using Lemma 3, the largest N among the equilibria where $\delta^E(r_1) = 1$ is achieved when $a = a_0$ where $\bar{h}(a_0 + 1) < h \leq \bar{h}(a_0)$.

When a increases by 1, $a = (a_0 + 1)$, then $\delta^E(r_1) = 0$ and $\delta^E(r_2) = 1$ even when $l_2 = l_1 = 1$. The latter follows from E's strategy (16). As $l_2 = l_1 = 1$ still satisfies (29), N also increases by 1, $N = (a_0 + 1)$. Hence, the largest N arises when $\delta^E(r_1) = 0$.

Now, we compute lower-bounds for N when $\delta^E(r_1) = 0$. We consider two cases.

1. Suppose $l_2 \geq (N-1)$. Then $l_{n+1} \geq l_n - 1$ for $n \in \{2, \ldots, N\}$, so that

$$a = l_1 + \sum_{n=2,\ldots,N} l_n \geq l_1 + \sum_{n=2,\ldots,N} (l_2 - (n-2)) > l_1 + \frac{1}{2} N l_2$$

   Using $\delta^E(r_2) = 1$ (see Lemma 1) and (16) we write $h \leq \frac{\gamma}{8a^2}(2l_1 + l_2)^2$ so that $\sqrt{\frac{\gamma}{8h}}(2l_1 + l_2) \geq a > l_1 + \frac{1}{2} N l_2$. This can be rewritten as the following upper-bound on N:

$$N < 2\left(2\sqrt{\tfrac{\gamma}{8h}} - 1\right)\frac{l_1}{l_2} + 2\sqrt{\tfrac{\gamma}{8h}} \leq 6\sqrt{\tfrac{\gamma}{8h}} - 2$$

2. Suppose $l_2 \leq (N-2)$. Then $l_{n+1} \geq l_n - 1$ for $n \in \{2, \ldots, l_2 + 1\}$ and $l_n \geq 1$ for $n \in \{l_2 + 2, \ldots, N\}$, so that

$$a \geq l_1 + \sum_{n=2}^{l_2+1} (l_2 - (n-2)) + \sum_{n=l_2+2}^{N} 1 = l_1 + \frac{1}{2}(l_2 - 1) l_2 + N - 1$$

   Using $h \leq \frac{\gamma}{8a^2}(2l_1 + l_2)^2$ and $l_1 \leq \frac{1}{2}(l_2 + 1)$ from (29) yields:

$$N \leq \frac{1}{2}\left(4\sqrt{\tfrac{\gamma}{8h}} - l_2\right) l_2 + \sqrt{\tfrac{\gamma}{8h}} + \frac{1}{2}$$

   The right-hand side of this inequality is a second-degree polynomial in l_2, which attains its maximum at $l_2 = 2\sqrt{\frac{\gamma}{8h}}$. Hence,

$$N \leq \tfrac{\gamma}{4h} + \sqrt{\tfrac{\gamma}{8h}} + \frac{1}{2}$$

Combining the two cases above results in $N \leq \overline{N} \equiv \max\left\{\left(\frac{\gamma}{4h} + \sqrt{\frac{\gamma}{8h}} + \frac{1}{2}\right), \left(6\sqrt{\frac{\gamma}{8h}} - 2\right), 2\right\}$, which ends the proof. ■

Proof of Proposition 2.

Let, for each $N \leq \overline{N}$ consider a set A_N of values of a such that for any $a \in A_N$, the corresponding continuation game $\Gamma(a)$ has an equilibrium (separating or partially separating) with N reports. First, we show for each $N \leq \overline{N}$, there is an $a^*(N) \in A_N$ such that choosing $a > a^*(N)$ is not optimal for X for any c. Second, we define

$$\widehat{a} \equiv \max_{N \leq \overline{N}} a^*(N)$$

This maximum always exists and, by construction, $a > \widehat{a}$ is not optimal for X for any N.

If A_N is finite, $a^*(N)$ is also finite. Let A_N be infinite. For any $a \in A_N$, we consider an equilibrium Ω_a of $\Gamma(a)$ with N reports (if there are multiple equilibria, we choose Ω_a arbitrarily). We define

$$x_n \equiv \frac{k_n}{a} \text{ and } y_n \equiv \frac{l_n}{a} = x_n - x_{n-1}$$

The *ex ante* expected utility $U^X(a)$ of X in equilibrium Ω_a is given by (25). It follows that a only affects $U^X(a)$ through the *ex ante* expected project value, which can be written as follows:

$$\mathrm{E}\left[\gamma(v-z)\delta^I\delta^E\right] = \frac{\gamma}{8}\sum_{n=1,\ldots,N}(x_n - x_{n-1})(x_n + x_{n-1})^2 - \frac{\gamma}{8}\left(1 - \delta_a^E(r_1)\right)x_1^3 \quad (30)$$

where δ_a^E is the approval strategy of E in Ω_a. Next, we write the equilibrium condition (29) for Ω_a as follows:

$$y_{n+1} - \left(2 - \delta_a^E(r_n)\right)y_n \in \left[-\tfrac{1}{a}, \tfrac{2}{a}\right] \quad (31)$$

Consider a limit $a \to \infty$ in which a takes on increasing values from A_N. The interval $\left[-\frac{1}{a}, \frac{2}{a}\right]$ converges to a point $\{0\}$. As $\delta_a^E(r_1) \in \{0,1\}$, the sequence $\left\{\delta_a^E(r_1)\right\}$ either has a limit of 0 or 1, or it oscillates between these two values. The following three cases are possible.

1. Let $\left\{\delta_a^E(r_1)\right\}$ converge to 1. Then, $\delta_a^E(r_1) = 1$ and (31) implies $y_{n+1} = y_n$ for all n in the limit. Hence, $x_n \to \frac{n}{N}$ and (30) converges to:

$$\lim_{a\to\infty} \mathrm{E}\left[\gamma(v-z)\delta^I\delta^E\right] = \frac{\gamma}{8N^3}\sum_{n=1,\ldots,N}(2n-1)^2 = \frac{\gamma}{24}\left(4 - \frac{1}{N^2}\right)$$

which is identical to the project value in the separating equilibrium with $a = N$, as given by (27). This implies that by choosing unboundedly large a, X cannot get higher project value than by choosing $a = N$. Hence, (30) attains its maximum at some finite value of a, $a^*(N)$.

2. Let $\{\delta_a^E(r_1)\}$ converge to 0. Then, $\delta_a^E(r_1) = 0$ and (31) implies $y_{n+1} = y_n$ for $n \geq 2$ and $y_2 = 2y_1$ in the limit. Hence, $x_n \to \frac{2n-1}{2N-1}$ and (30) converges to:

$$\lim_{a\to\infty} \mathrm{E}\left[\gamma (v-z)\, \delta^I \delta^E\right] = \frac{\gamma}{(2N-1)^3} \sum_{n=1,\dots,N} (2n-2)^2 = \frac{2}{3}\gamma \frac{N(N-1)}{(2N-1)^2}$$

which is identical to the project value obtained in the equilibrium where $a = 2N-1$ and $k_n = 2n-1$. Hence, again, by choosing unboundedly large a, X cannot get higher project value than by choosing $a = 2N-1$, implying that (30) attains its maximum at some finite value of a, $a^*(N)$.

3. Let $\{\delta_a^E(r_1)\}$ have no limit. The maximum of $a^*(N)$ from the two previous cases becomes the upper-bound on optimal a.

Summarizing, choosing $a > \widehat{a}$ is not optimal to X for any N. Consequently, there exists an optimal value of N, $N \leq \overline{N}$, an optimal $a \leq a^*(N)$, and an optimal equilibrium Ω_a of $\Gamma(a)$ that maximizes $U^X(a)$, so that the sender-optimal equilibrium exists. The uniqueness is generic and follows from continuous dependence of $U^X(a)$ on parameters γ and c. This ends the proof. ■

Proof of Proposition 3.

When $h \leq \overline{h}(a^*)$, E's strategy (16) implies $d^E = 1$ in all continuation games $\Gamma(a)$ for all $a \leq a^*$. The *ex ante* expected utility $U_{sep}^X(a)$ of X from choosing a and playing the separating equilibrium in $\Gamma(a)$ is given by (28) $N = a$:

$$U_{sep}^X(a) \equiv \frac{1}{2} + \frac{\gamma}{24}\left(4 - \frac{1}{a^2}\right) - (a-1)\,c \tag{32}$$

When $a = 1$ only the pooling equilibrium of $\Gamma(a)$ exists. When $a = 2$, $\Gamma(a)$ has two equilibria: the pooling equilibrium and the separating equilibrium. When $c > 0$, the pooling equilibrium following $a = 2$ does not satisfy the forward induction refinement: choosing $a = 1$ yields a higher payoff than choosing $a = 2$ followed by the pooling equilibrium. When $a^* \geq 2$ so that $U_{sep}^X(2) > U_{sep}^X(1)$, the unique forward induction equilibrium of $\Gamma(2)$ is the separating equilibrium.

Suppose, as an induction assumption, that for some $t \leq a^* - 1$, for each $a \in \{2, \ldots, t\}$ the unique forward induction equilibrium of $\Gamma(a)$ is the separating equilibrium yielding $U_{sep}^X(a)$. Then, the separating equilibrium of $\Gamma(t+1)$ is the unique forward induction equilibrium. Indeed, (i) all semi-pooling equilibria have $N \leq t$ reports and, therefore, yield a strictly lower payoff to X than $U_{sep}^X(t)$ (as is shown in the proof of Proposition 1), and (ii) as $U_{sep}^X(a)$ increases in a over $a \in \{2, \ldots, a^*\}$ (by the definition of a^*). Hence, for $a = t+1$, $U^X(a) > U_{sep}^X(t)$ holds only when the separating equilibrium is played in $\Gamma(a)$. It follows that choosing $a = a^*$ dominates choosing any $a < a^*$. What remains to be shown is that choosing $a = a^*$ also dominates choosing any $a > a^*$. If $a^* = \overline{a}$, then for any $a > a^*$ only semi-pooling equilibria with $N \leq a^*$ reports exist, and they all yield lower utility to X than $U_{sep}^X(a^*)$. If, on the other hand, $a^* < \overline{a}$, then $a^* = \sqrt[3]{\frac{\gamma}{12c}}$. Since $U_{sep}^X(a)$ decreases in a for all $a > \sqrt[3]{\frac{\gamma}{12c}}$, choosing $a > a^*$ is dominated by choosing $a = a^*$. Thus, choosing $a = a^*$ is the unique optimal choice of X. This ends the proof. ■

Proof of Proposition 4.

Consider an equilibrium the game with private communication in which X sends truthful internal reports, *i.e.*, in which $\rho^I(k)$ is one-to-one. Let r_k^I be the internal message send by type k. Accordingly, $\mathrm{E}\left[v|r_k^I\right] = v_k$, as defined in (1). Using backward induction, consider I. It follows from (3) that I chooses $d^I = 1$ if $z < v_k$ and $d^I = 0$ otherwise (hereinafter, without loss of generality, we use strict inequalities in the constraints). This proves item (i) of the proposition. As a result, given k, the expected project value equals $\mathrm{E}\left[\gamma(v-z)\, d^I\right] = \frac{\gamma}{2}(v_k)^2$.

Next, consider the approval decision d^E by E. Using (5) and I's strategy, the optimal strategy for E is to choose $d^E = 1$ if and only if $\mathrm{E}\left[u^E|r^E\right] > 0$. Hence ,

$$\delta^E\left(r^E\right) = 1 \text{ if } \tfrac{\gamma}{2}\,\mathrm{E}\left[(v_k)^2\,|r^E\right] > h, \text{ and } \delta^E\left(r^E\right) = 0 \text{ otherwise} \tag{33}$$

Next, consider communication between X and E. Stock price (4) becomes

$$s\left(r^E\right) = \mathrm{E}\left[v|r^E\right] + \tfrac{\gamma}{2}\,\mathrm{E}\left[(v_k)^2\,|r^E\right]\delta^E\left(r^E\right)$$

and the expected utility $\mathrm{E}\left[u^X|k, r^E\right]$ of X of type k when he reports r^E to E is:

$$\mathrm{E}\left[u^X|k, r^E\right] = (1-\lambda)\left(v_k + \tfrac{\gamma}{2}(v_k)^2 \delta^E\left(r^E\right)\right) + \lambda s\left(r^E\right) - c(a-1) \tag{34}$$

We proof item (ii) by contradiction. Suppose that in equilibrium, two reports r_1^E and r_2^E are used such that $s\left(r_1^E\right) \neq s\left(r_2^E\right)$ or $\delta^E\left(r_1^E\right) \neq \delta^E\left(r_2^E\right)$ (or both). If $\delta^E\left(r_2^E\right) = 1$ and $\delta^E\left(r_1^E\right) = 0$, then the difference

$$\mathrm{E}\left[u^X|k, r_2^E\right] - \mathrm{E}\left[u^X|k, r_1^E\right] = (1-\lambda)\tfrac{\gamma}{2}(v_k)^2 + \lambda\left(s\left(r_2^E\right) - s\left(r_1^E\right)\right)$$

is increasing in v_k. This implies that if a type $\widetilde{k}$ prefers reporting r_2^E to reporting r_1^E, all types $k > \widetilde{k}$ do so as well, and, therefore, $s\left(r_2^E\right) > s\left(r_1^E\right)$ so that $\mathrm{E}\left[u^X|k, r_2^E\right] > \mathrm{E}\left[u^X|k, r_1^E\right]$ for all k. Hence, no types report r_1^E. If, on the other hand, $\delta^E\left(r_1^E\right) = \delta^E\left(r_2^E\right)$ and $s\left(r_2^E\right) > s\left(r_1^E\right)$, then $\mathrm{E}\left[u^X|k, r_2^E\right] > \mathrm{E}\left[u^X|k, r_1^E\right]$ for all k, and no types report r_1^E. Thus, in any equilibrium, only one report r^E can be used, which proves item (ii) of the proposition.

Since r^E is independent of k, the *ex ante* expected value of the project conditional on approval is:

$$\tfrac{\gamma}{2}\mathrm{E}\left[(v_k)^2|r^E\right] = \tfrac{\gamma}{2}\mathrm{E}\left[(v_k)^2\right] = \tfrac{\gamma}{8a^3}\sum_{k=1,\dots,a}(2k-1)^2 = \tfrac{\gamma}{24}\left(4 - \tfrac{1}{a^2}\right) \tag{35}$$

Substituting (35) into E's strategy (33) yields item (iii) of the proposition.

Lastly, we consider the choice of a by X. Let $U^X(a)$ denote the *ex ante* expected utility of X. Taking expectations of (34) over k and using (35) and $\mathrm{E}\left[v|r^E\right] = \mathrm{E}\left[v_k\right] = \frac{1}{2}$ yields

$$U^X(a) = \tfrac{1}{2} + \tfrac{\gamma}{24}\left(4 - \tfrac{1}{a^2}\right)d^E - (a-1)c \tag{36}$$

According to item (iii), if $h \geq \frac{\gamma}{6}$, then E chooses $d^E = 0$ for any a. Choosing $a = 1$ is optimal in this case. If $h < \frac{\gamma}{8}$ then E approves for any a. Maximizing (36) yields

$$a^{pr} = a^{opt} = \sqrt[3]{\tfrac{\gamma}{12c}}$$

When $h \in \left(\frac{\gamma}{8}, \frac{\gamma}{6}\right)$, E only approves if $a > \underline{a}$, where

$$\underline{a} \equiv \frac{1}{2}\sqrt{\frac{\gamma}{\gamma - 6h}}$$

If $a^* \geq \underline{a}$, choosing $a = a^*$ is optimal, as this maximizes (36). Suppose $a^* < \underline{a}$. This occurs when $c > \underline{c}$, where

$$\underline{c} \equiv \frac{2}{3}\gamma\left(1 - 6\frac{h}{\gamma}\right)^{\frac{3}{2}}$$

In this case, X either chooses $a = \underline{a} > a^*$or chooses $a = 1$. Using (36), the first option yields a higher payoff if and only if $c < \overline{c}$, where

$$\overline{c} \equiv \frac{h}{\underline{a} - 1}$$

This ends the proof of item (iv) of the proposition.

Finally, consider X's internal reporting strategy $\rho^I(k)$. It follows from (34) that r^I only affects X's payoff through the expected project value $\mathrm{E}\left[\gamma(v - z)\right]$. Given I's strategy and using (1), type k prefers reporting r^I_k to $r^I_{k+\tau}$ if

$$\frac{\gamma}{2}(v_k)^2 - \gamma\left(v_{k+\tau}\left(v_k - \frac{1}{2}v_{k+\tau}\right)\right) = \frac{1}{2a^2}\tau^2\gamma \geq 0$$

which holds for any τ. Hence, $\rho^I(k) = r^I_k$ maximizes X's payoff for any type k. ■