Vertical Contracting with Endogenous Market Structure*

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Abstract

A manufacturer chooses the optimal retail market structure and bilaterally and secretly contracts with each (homogeneous) retailer. In a classic framework without asymmetric information, the manufacturer sells through a single exclusive retailer in order to eliminate the *opportunism problem*. When retailers are privately informed about their (common) marginal cost, however, the number of competing retailers also affects their information rents and the manufacturer may prefer an oligopolistic market structure. We characterize how the manufacturer's production technology, the elasticity of final demand, and the size of the market affect the optimal number of retailers. Our results arise both with price and quantity competition, and also when retailers' costs are imperfectly correlated.

Keywords: asymmetric information, distribution network, opportunism, retail market structure, vertical contracting.

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1 Introduction

The retail market structure chosen by a manufacturer and the contractual arrangements within the distribution network affect retail competition and thus determine firms' profit and social welfare. Therefore, the determinants of an optimal distribution network have been extensively analyzed in the theoretical industrial organization literature. Moreover, vertical foreclosure practices are often under the scrutiny of antitrust authorities, worried by the risk of monopolization. In fact, several antitrust cases consider whether manufacturers restrict intrabrand competition and harm consumers through the choice of their distribution networks. For example, in two recent cases, the distribution systems of the cosmetics manufacturer Pierre Fabre and of the sport shoe producer Asics were ruled to violate competition law by the European Commission and the German Federal Cartel Office, respectively, because they limited downstream competition by prohibiting retailers from selling products on third-party websites.[1]

The seminal papers analyzing vertical contracting—Hart and Tirole (1990), McAfee and Schwartz (1994), Segal (1999) and Segal and Whinston (2003)—show that, if retailers are undifferentiated, the *opportunism problem* induces a manufacturer to distribute through a monopolistic retailer, to the detriment of final consumers.[2] The reason is that, when she secretly contracts with multiple retailers,[3] the manufacturer has an incentive to lower the wholesale price in each bilateral negotiation, thus reducing her aggregate profits.

These papers and most of the subsequent literature, however, do not take into account the presence of information asymmetry between manufacturers and retailers, even though this is a prevalent feature of distribution networks. In fact, retailers are typically better informed than manufacturers about demand and/or cost characteristics. For example, they are likely to obtain better information about demand by interacting directly with final consumers, and thus observing their idiosyncratic tastes. Similarly, retailers may also have superior information about their production technology, because downstream costs may depend on price shocks to local input that are not directly observable by manufacturers.

In this paper, we analyze the interplay between asymmetric information and the opportunism problem, and the implications of this interaction for the optimal retail market structure.

[1] See European Court of Justice, judgment of 13 October 2011, Case C - 439/09, *Pierre Fabre Dermo-Cosmétique*, and German Federal Cartel Office (Bundeskartellamt), 13 January 2016, Case Summary, "Unlawful Restrictions of Online Sales of ASICS Running Shoes", B2-98/11, judgment of 26 August 2015. Less recently, the electronic products manufacturer AEG-Telefunken was fined by the European Commission because it discriminated distributors in order to reduce competition between them. See European Court of Justice, judgment of 25 October 1983, Case 107/82, *AEG-Telefunken*.

[2] See Rey and Tirole (2007) for a comprehensive summary of the literature.

[3] We refer to the manufacturer by "she" and to a retailer by "he."

When retailers have private information, does a manufacturer still prefer to distribute through a monopolistic retailer? If not, what is the optimal number of retailers? How is this number affected by the characteristics of the downstream market?

To address these issues, we consider a game in which a manufacturer chooses the retail market structure—i.e., the number of undifferentiated retailers through which she distributes her product—and retailers have private information about their common marginal distribution cost.[4] Subsequently, the manufacturer bilaterally contracts with each retailer by secretly offering a menu consisting of a quantity sold by the manufacturer and a transfer paid by the retailer, both dependent on the retailer's report about her cost. Retailers choose whether to accept the manufacturer's contract and compete in the final-consumer market.

Bilateral and secret contracting typically occurs due to institutional constraints (McAfee and Schwartz, 1994). In fact, it is usually too costly for a manufacturer to write a complete multilateral contract with all retailers, since this requires to foresee and verify a large number of contingencies. In addition, antitrust laws often preclude public multilateral agreements in which the quantity sold to a retailer depends on trades made with his competitors. Because of these reasons, we assume that contracts cannot be contingent on elements external to the bilateral relationship between the manufacturer and each retailer like: (*i*) the quantity sold by other retailers, or (*ii*) the reports that the manufacturer receives from other retailers, since it is too costly to credibly disclose private communications (see, e.g., Dequiedt and Martimort, 2015). This assumption prevents the manufacturer from obtaining monopoly profits. First, the manufacturer cannot use contracts based on aggregate performances to eliminate the opportunism problem when distributing through multiple retailers. Second, the manufacturer cannot exploit yardstick competition to eliminate the retailers' information rent,[5] nor can she select retailers by auctioning the right to distribute her product.

We show that, in contrast to a standard framework with the opportunism problem (e.g., Rey and Tirole, 2007), in the presence of asymmetric information the manufacturer may obtain a higher profit by using a larger distribution network. Hence, monopolization through exclusive distribution is less likely in industries with strong uncertainty about, for example, retail costs or downstream demand. This result arises because of a novel trade-off between

[4] The assumption of undifferentiated retailers allows to describe our results in the simplest way and makes them directly comparable with the seminal work by Hart and Tirole (1990). However, the effects that we highlight also arise with differentiated retailers (see, e.g., Section 6.1). Similarly, a common cost is a simplifying assumption, but our results do not hinge on it (see Section 6.2).

[5] It is a well-established result in the mechanism design literature that, with correlated types, yardstick competition allows full surplus extraction by the manufacturer, but this requires a stochastic multilateral mechanism in the spirit of Crémer and Mclean (1985). If these mechanisms are allowed, it is obvious that the manufacturer has a strong incentive to use more than one retailer.

the opportunism problem and information asymmetries in vertical contracting. On the one hand, as is well known, with a monopolistic retailer the manufacturer solves the opportunism problem. On the other hand, however, competition among multiple retailers has a disciplining effect on their incentive to misreport their cost, and hence reduces their information rent—a *competing-contracts effect* in the spirit of Martimort (1996).

To see this, notice that, when a retailer deviates from a truthful equilibrium and overstates his cost in order to pay a lower transfer to the manufacturer, (due to the common cost component) the retailer knows that his rivals will sell a relatively large quantity, leading to a low market price. However, the manufacturer requests a transfer under the presumption that all retailers sell a lower quantity, which would result in a higher market price and profit for the retailer. This reduces a retailer's incentive to misreport his cost compared to a situation in which he has fewer or no competitors in the downstream market. Hence, other things being equal, stronger competition in the downstream market reduces the information rent that the manufacturer pays to elicit truthful information. This result holds both with quantity and price competition between retailers and also with imperfectly correlated retailers' costs.[6]

We examine the trade-off between the opportunism problem and the competing-contracts effect with a general demand function and show that—ignoring the integer constraint on the number of retailers—the manufacturer *never* prefers a monopolistic retail market structure. Therefore, at the monopoly benchmark obtained without asymmetric information, the incentive to reduce information rents dominates. Moreover, we also show that the optimal number of retailers is always finite.

To determine the exact size of the optimal retail market, we consider a specification with linear demand, quadratic costs for the manufacturer, and a beta distribution of retail costs. We find that the optimal number of retailers is often relatively large. Moreover, the optimal number of retailers increases when: (*i*) the manufacturer's cost function becomes more convex, (*ii*) the elasticity of inverse demand increases, (*iii*), the market size decreases, and (*iii*) the retailers' expected cost decreases. The intuition is that, when the convexity of the manufacturer's cost function increases, it becomes more costly to increase a retailer's production, which reduces the opportunism problem. In this case, the disciplining effect of competition on information rents dominates. Similarly, the opportunism problem is less relevant when the elasticity of demand with respect to prices increases or the size of the market decreases, because in these cases reducing the number of retailers has a weaker effect on aggregate profit

[6] Obviously, when retailers' costs are imperfectly correlated, the strength of the competing-contracts effect depends on the degree of correlation.

in the absence of asymmetric information. Finally, when the retailers' expected cost decreases, information rents increase and the competing-contracts effect becomes stronger.

In sum, our analysis unveils a novel effect arising because of the presence of information asymmetries in a canonical vertical-contracting framework. Our results suggest that exclusive distribution via a single retailer may not be the optimal market structure for a manufacturer who deals with privately informed retailers. Therefore, asymmetric information tends to increase the size of optimal distribution networks and mitigate concerns of low intrabrand competition. This insight may help to explain why, in practice, different structures of distribution networks are observed in different industries. For example, in the automobile industry, in which demand and cost conditions are usually relatively stable over time and hence asymmetric information is less relevant, manufacturers often choose a single retailer in a region. By contrast, manufacturers of electronic products typically sell through multiple retailers. More fluctuating demand and costs in this industry enhances asymmetric information, which may induce a manufacturer to use multiple retailers to reduce information rents.[7]

Of course, there are many other factors that influence the size of a manufacturer's distribution network. For example, a manufacturer may prefer multiple retailers when final consumers perceive retailers' products as differentiated (see, e.g., Motta, 2004, Ch. 6), or because the manufacturer wants to sell in geographically differentiated areas, that cannot be served by a single retailer (e.g., Rey and Stiglitz, 1995). Moreover, a manufacturer may use multiple retailers: (i) in order to sample their ability and quality (see Hansen and Motta, 2012, which is discussed below), (ii) when she has relatively low bargaining power (see Marx and Shaffer, 2007), or (iii) when the hold-up problem distorts upstream and downstream investments (see, e.g., Bolton and Whinston, 1993; and Hart and Tirole, 1990). All these explanations are complementary to the effect of asymmetric information that we highlight.

The rest of the paper is organized as follows. After discussing the existing literature, Section 2 describes the main model and Section 3 considers a simple model with two types. We analyze the optimal retail market structure in Section 4, first in the complete information benchmark and then with asymmetric information. Section 5 considers an example with linear demand and quadratic costs. In Section 6, we discuss various extensions of our analysis. Section 7 concludes. All proofs are in the Appendix.

Related Literature. Hart and Tirole (1990) were the first to highlight the opportunism problem of a manufacturer dealing with multiple retailers. Building on their framework, many

[7] Dyer *et al.* (2014) classify uncertainty in different industries and find that the automobile and truck industry is exposed to significantly less uncertainty than the electronic and electrical equipment industry.

subsequent papers further analyzed this issue. O'Brien and Shaffer (1992) prove that exclusive territories,[8] or appropriate forms of resale price control such as a market-wide price floor, can solve the opportunism problem in a contract equilibrium.[9] McAfee and Schwartz (1994) and Rey and Vergé (2004) explore how the problem depends on different types of off-equilibrium beliefs by retailers. Segal and Whinston (2003) prove that, regardless of the choice of off-equilibrium beliefs, a manufacturer can considerably weaken the opportunism problem by using menus of two-part tariffs that internalize any bilateral attempt to reduce prices.[10] In contrast to this literature, we show that with asymmetric information exclusive distribution through a single retailer may not be desirable for a manufacturer, since downstream competition erodes the retailers' information rents and may offset the loss caused by the opportunism problem.

Our work is also related to the strand of literature analyzing asymmetric information in manufacturer-retailer relationships. These papers usually examine common agency games (e.g., Calzolari and Denicolò, 2013, 2015; Martimort, 1996; Martimort and Stole, 2009a, 2009b) or games played by competing organizations (e.g., Caillaud *et al.*, 1995; Gal-Or, 1996; Kastl *et al.*, 2011; and Pagnozzi *et al.*, 2016). None of these papers, however, jointly considers the opportunism problem and asymmetric information in vertical contracting.

To the best of our knowledge, only Dequiedt and Martimort (2015) examine the link between opportunism and asymmetric information. They consider a framework with public contracting in which the manufacturer can condition contracts with retailers who are privately informed about their correlated costs on the information obtained from other retailers. Dequiedt and Martimort (2015) show that this creates a new form of informational opportunism, even when retailers do not impose production externalities on each other, which prevents the manufacturer from achieving the monopoly outcome.[11] By contrast, we focus on how the optimal retail market structure is shaped by asymmetric information and classical opportunism, in the absence of informational opportunism.

Hansen and Motta (2012) also analyze a model in which a manufacturer deals with privately informed retailers. In contrast to our model, they consider public contracts and independently

[8] See Rey and Stiglitz (1995) for a detailed analysis of exclusive territories in a different framework.

[9] In a contract equilibrium, the manufacturer behaves optimally in each bilateral relationship, given the contracts with other retailers. This equilibrium concept, however, does not consider multi-lateral deviations.

[10] More recently, Montez (2015) finds that commitment of the manufacturer to buy back units of unsold stock from retailers may restore the monopoly outcome in a contract equilibrium, whereas Nocke and Rey (2018) consider the opportunism problem in a framework with multiple differentiated manufacturers and show that exclusive dealing or vertical integration can increase profits.

[11] As in our model, their analysis excludes yardstick competition *à la* Crémer and Mclean (1985) because it requires forms of multilateral contracts that may be difficult to implement in practice.

distributed costs. They show that if retailers are sufficiently risk averse, the manufacturer uses a single retailer to avoid the uncertainty imposed by competing retailers.

2 The Model

Players and Environment. Consider a vertical contracting model with externalities *à la* Segal and Whinston (2003). A manufacturer M chooses the number of retailers $N \geq 1$ to which to sell her product. Each retailer R_i, $i = 1, .., N$, then sells in the downstream market by converting each unit of M's product into one unit of the final good. We denote by x_i the quantity sold by R_i and let $X \triangleq \sum_{i=1}^{N} x_i$. The downstream demand function is $P(X)$, with $P'(\cdot) < 0$.

Following Segal and Whinston (2003), we assume that M's cost function $c(X)$ is increasing and weakly convex because of decreasing returns to scale, with $c(0) = 0$ (see also O'Brien and Shaffer, 1992, and Dequiedt and Martimort, 2015).[12] In order to make our game equivalent to Hart and Tirole (1990) in the case of complete information, we assume that retailers are symmetric and have a constant common marginal cost θ—i.e., retail costs are the same for all retailers (e.g., because they depend on a common input price shock which affects all retailers symmetrically). Hence, our results only hinge on the presence of asymmetric information, and not on asymmetry among retailers. The assumption of a common marginal cost, however, is not crucial for the results—in Section 6.2, we show that the effects that we identify arise with any positive correlation among costs.

Retailers are privately informed about θ, which is drawn from a common knowledge, non-negative, twice continuously differentiable, bounded, and atomless density function $f(\theta)$ on the compact support $\Theta \triangleq [\underline{\theta}, \overline{\theta}]$.[13] We assume that the associated distribution function $F(\theta)$ satisfies the (inverse) Monotone Hazard Rate Property—i.e., $h(\theta) \triangleq F(\theta)/f(\theta)$ is increasing.

Contracts. Following previous literature, the manufacturer contracts with all retailers simultaneously (see, e.g., Segal, 1999). Contracts are secret—i.e., a retailer does not observe the contracts that M offers to other retailers. M offers a quantity-forcing contract to R_i, which is a menu

$$\{T_i(m_i), x_i(m_i)\}_{m_i \in \Theta}$$

[12] The assumption of convex cost is not necessary for our results because they arise even if marginal costs are constant. However, the assumption allows us to provide comparative-static results on the curvature of the manufacturer's cost function. See Section 5 for details.

[13] Similar results arise when retailers are privately informed about demand rather than costs.

specifying the quantity $x_i(m_i)$ that M supplies to R_i and that R_i sells in the downstream market, and the tariff $T_i(m_i)$ that R_i pays to M, contingent on the R_i's report $m_i \in \Theta$ about the cost θ. At the end of Section 4 we show that there is no loss of generality in considering quantity-forcing contracts because, in equilibrium, a retailer has no incentive to sell a quantity lower than the one acquired from the manufacturer. We restrict attention to differentiable equilibria such that, for every i, the functions $x_i(m_i)$ and $T_i(m_i)$ are continuously differentiable. A retailer's outside option is normalized to zero.

If contracts are accepted by retailers, M's total profit is

$$\sum_{i=1}^{N} T_i(m_i) - c\left(\sum_{i=1}^{N} x_i(m_i)\right),$$

while R_i's profit is

$$\left[P\left(\sum_{j=1}^{N} x_j(m_j)\right) - \theta\right] x_i(m_i) - T_i(m_i).$$

Notice that we consider simple bilateral contracts that are fully determined by a retailer's report about the (common) cost. The assumption that the contract offered to R_i does not depend on other retailers' reports or quantities is in line with the vertical contracting literature, in which the tariff offered to a retailer is independent of trades with competing retailers because competition law forbids such dependency. An additional reason to rule out contracts that are conditioned on the reports of all retailers is that contracting and communication is typically secret, and it may be too costly for the manufacturer to credibly disclose to a retailer the reports of other retailers.[14] Notice that this restriction to the contract space *de facto* prevents the manufacturer from selecting retailers through auctions, where the probability of a retailer winning depends on the bids by other retailers (see also Section 6.4).

By the Taxation Principle, the direct mechanisms that we consider are equivalent to non-linear tariffs of form $T_i(x_i)$, which are usually observed in practice (see, e.g., Laffont and Martimort, 2002).

Timing and Equilibrium Concept. The timing of the game is as follows:

1. *Retail Market Structure.* M chooses the number of retailers N.

[14] Dequiedt and Martimort (2015) consider public contracts and examine how limits to communication shape vertical contracting when a retailer's quantity may depend on the manufacturer's communication about the reports made by rival retailers.

2. *Contracting.* Retailers observe their cost θ and M simultaneously offers contracts. If R_i accepts his contract, he reports m_i, obtains the quantity $x_i(m_i)$ and pays the tariff $T_i(m_i)$ accordingly.

3. *Downstream Competition.* Retailers sell their quantities in the final market and profits realize.

Hence, retailers play a Cournot game in the downstream market. In Section 6.1, we show that our results arise even with price rather than quantity competition. Moreover, the equilibrium that we characterize is equivalent to the one of a game in which retailers set prices but are capacity constrained in the downstream market, because the manufacturer produces to order before prices are set and final demand realizes (Rey and Tirole, 2007).[15] Essentially, price competition with capacity constraints as in Kreps and Scheinkman (1983) leads to a Cournot outcome.

We consider a Perfect Bayesian Nash Equilibrium in direct revelation mechanisms such that retailers truthfully report their cost—i.e., $m_i = \theta$ for every $i = 1, .., N$—with the standard 'passive beliefs' refinement (Hart and Tirole, 1990; McAfee and Schwartz, 1994; Rey and Tirole, 2007). With passive beliefs and multiple retailers, a retailer's conjecture about the contracts offered to other retailers is not influenced by an out-of-equilibrium offer he receives. This is a natural refinement for games with secret contracting and production to order because, from the perspective of the manufacturer, each retailer forms a separate market (Rey and Tirole, 2007). In addition, as shown by McAfee and Schwartz (1994), passive beliefs correspond to wary beliefs in our game.[16]

Assumptions. We first treat N as a continuous variable and ignore the integer constraint on the number of retailers (see, e.g., Mankiw and Whinston, 1986). In Section 5, we analyze a closed-form example of our model and explicitly consider the effects of the integer constraint.

We also impose the following technical assumptions.

Assumption 1. $P\left(0\right) - c'\left(0\right) > \overline{\theta} + h\left(\overline{\theta}\right)$.

This assumption guarantees that production is always positive.

[15] Alternatively, one could imagine that the transformation activity is sufficiently time consuming that a downstream firm cannot instantaneously reorder the manufacturer's product and satisfy customers when their demand is larger than expected, or reduce it when demand is unexpectedly low.

[16] With quantity competition, equilibria with passive beliefs are equivalent to contract equilibria (Crémer and Riordan, 1987)—i.e., each equilibrium with passive beliefs is also a contract equilibrium and vice versa. By contrast, with price competition the two equilibrium concepts are not equivalent, as we discuss in Section 6.1.

Assumption 2. The inverse demand function satisfies the following conditions:

(*i*) $P'(X) + P''(X)X < 0$;

(*ii*) $\lim_{X\to+\infty} P(X) = 0$ and $|P'(X)| < +\infty$ for every X;

(*iii*) $P'''(X)$ is not too large—i.e., $P'''(X) < -2P''(X)/x_i$, $\forall x_i$.

Part (*i*) of Assumption 2 implies that all profit functions are strictly concave and that quantities are strategic substitutes.[17] Part (*ii*) ensures that the equilibrium market price is zero as the quantity gets unbounded, and that the equilibrium quantity is positive. Part (*iii*) is a natural extension of the single-crossing property imposed in the context of competing hierarchies, and is analogous to the 'aggregation property' in Martimort (1996).

3 A Binary Example

In order to gain intuition on the main trade-off and insights of the paper, we first analyze a stylized model with a binary type space where the manufacturer can only choose between one and two retailers. Accordingly, we assume that $\Theta \triangleq \{0, \overline{\theta}\}$, with $\Pr[\theta = 0] = \Pr[\theta = \overline{\theta}] = \frac{1}{2}$ and $\overline{\theta} < \frac{1}{2}$ (to guarantee positive quantities in equilibrium). We also assume that the manufacturer's cost function is quadratic,

$$c(X) \triangleq \beta \frac{X^2}{2},$$

and that demand is linear,

$$P(X) \triangleq \max\{1 - X, 0\}.$$

With a monopolistic retailer, the manufacturer faces a standard monopoly screening problem in which the participation constraint requires that the utility of the high-cost retailer is $\overline{u} \geq 0$,[18] and the incentive compatibility constraint for the utility of the low-cost retailer is

$$\underline{u} \geq \overline{u} + \overline{\theta}\overline{x}.$$

[17] This is a standard assumption in games with quantity competition (see e.g., Vives, 2001).

[18] Using standard notation, an underline denotes a variable referring to a low-cost retailer while an overline denotes a variable referring to a high-cost retailer—i.e., for example, $\overline{x}$ is the equilibrium quantity sold to a retailer with cost $\theta = \overline{\theta}$ and $\underline{u}$ is the equilibrium utility of a retailer with cost $\theta = 0$.

This constraint ensures that a low-cost retailer does not report a high cost in order to pay a lower tariff. (As standard, the remaining incentive-compatibility and participation constraints are redundant—see the Appendix for the details of the analysis.)

At the optimal contract, these constraints are binding. Hence, the manufacturer chooses the quantities that solve

$$\max_{(\overline{x},\underline{x})\geq 0}\left\{\tfrac{1}{2}\left[P\left(\underline{x}\right)\underline{x}-\overline{\theta}\overline{x}-c\left(\underline{x}\right)\right]+\tfrac{1}{2}\left[\left(P\left(\overline{x}\right)-\overline{\theta}\right)\overline{x}-c\left(\overline{x}\right)\right]\right\},$$

yielding solutions

$$\underline{x}^{*}\left(1\right)\triangleq\frac{1}{2+\beta}>\overline{x}^{*}\left(1\right)\triangleq\frac{1-2\overline{\theta}}{2+\beta},$$

which feature no distortion for a low-cost retailer and a downward distortion for a high-cost retailer.[19] Of course, an increase in β reduces the manufacturer profit with a single retailer.

Suppose now that the manufacturer uses two retailers. In a symmetric equilibrium where retailers with the same cost produce the same quantity, it can be shown that the incentive compatibility constraint of a low-cost retailer is

$$\begin{aligned}\underline{u}_i &\geq \overline{u}_i+\overline{\theta}\overline{x}_i-\left[P\left(\overline{x}_i+\overline{x}^{*}\left(2\right)\right)-P\left(\overline{x}_i+\underline{x}^{*}\left(2\right)\right)\right]\overline{x}_i\\ &= \overline{u}_i+\underbrace{\overline{\theta}\overline{x}_i}_{\text{Standard rent}}-\underbrace{\Delta x^{*}\overline{x}_i}_{\text{Competing contracts}},\end{aligned}\tag{1}$$

where $\Delta x^{*}\triangleq\underline{x}^{*}\left(2\right)-\overline{x}^{*}\left(2\right)>0$ represents the difference between the equilibrium quantities of a low-cost and a high-cost retailer. Expression (1) embeds two contrasting effects. First, as observed above, R_i has an incentive to over-report the cost in order to pay a lower tariff, which allows him to obtain a standard monopoly information rent that is increasing in the quantity sold by a high-cost retailer—see, e.g., Baron and Myerson (1982), Maskin and Riley (1985) and Mussa and Rosen (1978).

Second, there is a *competing-contracts effect* (see, e.g., Gal-Or, 1999, and Martimort, 1996). When R_i over-reports his cost, he knows that the other retailer acquires and sells a larger quantity than he does, because he has the same cost and truthfully reports it in equilibrium. Hence, the price in the downstream market is relatively low. However, the tariff requested by the manufacturer does not take this into account because she assumes that both retailers have a high cost, according to R_i's report, so that the downstream price is relatively high. As a consequence, R_i's utility is lower and, other things being equal, R_i's incentive to

[19] With complete information on θ, the profit maximizing quantity is $\left(1-\theta\right)/\left(2+\beta\right)$.

overstate his cost is weaker than without competition in the downstream market. Therefore, compared to the monopoly case, a duopolistic retail market structure reduces each retailer's information rent.

The bilateral contract offered by M to R_i solves

$$\max_{(\underline{x}_i,\overline{x}_i)\geq 0} \left\{ \tfrac{1}{2}\left[P\left(\underline{x}_i+\underline{x}^*(2)\right)\underline{x}_i-\left(\overline{\theta}-\Delta x^*\right)\overline{x}_i-c\left(\underline{x}_i+\underline{x}^*(2)\right)\right]+ \right.$$
$$\left. +\tfrac{1}{2}\left[\left(P\left(\overline{x}_i+\overline{x}^*(2)\right)-\overline{\theta}\right)\overline{x}_i-c\left(\overline{x}_i+\overline{x}^*(2)\right)\right]\right\}.$$

Differentiating with respect to $\underline{x}_i$ and $\overline{x}_i$, we obtain that, in equilibrium,

$$\underline{x}^*(2) \triangleq \frac{1}{3+2\beta} > \overline{x}^*(2) \triangleq \frac{1-\overline{\theta}}{3+2\beta} - \frac{(1+\beta)\,\overline{\theta}}{(2+\beta)(3+2\beta)}. \quad (2)$$

Again, there is no distortion at the top (i.e., for the low-cost retailer) but a downward distortion for the high-cost retailer, represented by the second term of $\overline{x}^*(2)$ in expression (2). However, the aggregate quantity in both states of the world is larger than with a monopolistic retailer due to the opportunism problem (i.e., $2\overline{x}^*(2) > \overline{x}^*(1)$ and $2\underline{x}^*(2) > \underline{x}^*(1)$). As β increases the opportunism problem becomes weaker and the difference between the aggregate quantity produced with two retailers and the quantity produced with one retailer decreases.

Comparing the manufacturer's expected profit with one retailer $\pi^*(1)$ and her expected profit with two retailers $\pi^*(2)$, we obtain the following result.

Proposition 1 *If* $\overline{\theta}=0$, *then* $\pi^*(1) > \pi^*(2)$. *For any* $\overline{\theta}\in(0,\frac{1}{2})$, *there exists a threshold* $\hat{\beta}\geq 0$ *such that* $\pi^*(2) > \pi^*(1)$ *if and only if* $\beta > \hat{\beta}$.

Clearly, when there is no asymmetry of information—i.e., $\overline{\theta}=0$—the model converges to the standard Hart and Tirole (1990) framework. In this case, a market structure with two retailers can only harm the manufacturer (compared to the monopoly case) due to the opportunism problem. However, with asymmetric information—i.e., $\overline{\theta}>0$—a market structure with two retailers reduces their information rents because of the competing-contracts effect and (other things being equal) increases the manufacturer's profit. When increasing production is sufficiently costly for the manufacturer—i.e., β is large—this effect dominates the opportunism problem and induces the manufacturer to distribute via two retailers.[20]

[20] As shown in the proof of Proposition 1, $\hat{\beta}=0$ when $\overline{\theta}=1/3$, which implies that two retailers are more profitable than one even if the manufacturer's cost is (close to) zero.

4 Optimal Retail Market Structure

In this section we analyze our more general model with a continuum of types, a generic demand and cost function, and $N \geq 1$.

4.1 Benchmark with Complete Information

As a benchmark case, first assume that the manufacturer knows retailers' costs. The manufacturer optimally offers a single contract to the N retailers and fully extracts their surplus.

Let $x_i(\theta)$ be the quantity distributed by retailer R_i. Consider a symmetric equilibrium in which each retailer sells the same quantity $x_N^{CI}(\theta)$. With secret contracts and passive beliefs, (since retailers' expectations are correct in equilibrium) the tariff requested by M to ensure contract acceptance of R_i must be

$$T_i(\theta) \leq \left[P(x_i(\theta) + (N-1)x_N^{CI}(\theta)) - \theta\right] x_i(\theta).$$

Since this constraint is binding at the optimal contract, M's maximization problem can be split into N bilateral contracting problems of the form

$$\max_{x_i(\theta)\geq 0} \left[P(x_i(\theta) + (N-1)\, x_N^{CI}(\theta)) - \theta\right] x_i(\theta) - c(x_i(\theta) + (N-1)\, x_N^{CI}(\theta)).$$

Differentiating with respect to x_i and imposing symmetry, the first-order condition yields

$$P(X_N^{CI}(\theta)) + P'(X_N^{CI}(\theta))x_N^{CI}(\theta) = \theta + c'(X_N^{CI}(\theta)), \tag{3}$$

where $X_N^{CI}(\theta) \triangleq N x_N^{CI}(\theta)$.

Condition (3) shows that, in equilibrium, each retailer sells a quantity such that the retailer's marginal revenue equals the total marginal cost, which is the sum of the manufacturer's and the retailer's cost. The manufacturer maximizes the bilateral profit with each retailer, which implies that she does not internalize the effect of selling an additional unit to a retailer on the profit of the other $N-1$ retailers. Hence, retailers only accept contracts with the Cournot quantity (since otherwise each retailer would expect the manufacturer to secretly sell a larger quantity to his rivals). This prevents the manufacturer from achieving the monopoly profit—the *opportunism problem*.

Lemma 1 *With complete information, the equilibrium quantity* $x_N^{CI}(\theta)$ *is decreasing in* θ *and* N*, and the aggregate equilibrium quantity* $X_N^{CI}(\theta)$ *is increasing in* N*. Moreover,* $\lim_{N\to+\infty} x_N^{CI}(\theta) = 0$ *and* $\lim_{N\to+\infty} X_N^{CI}(\theta)$ *is equal to the perfectly competitive quantity.*

These are standard properties: the equilibrium quantity sold by each retailer is decreasing in the marginal cost and in the number of active retailers, while aggregate production is increasing in the number of retailers. Moreover, there is competitive convergence because the equilibrium quantity of each retailer converges to zero as the downstream market approaches the perfectly competitive limit.

For a given N, in the symmetric equilibrium the manufacturer's aggregate expected profit is

$$\pi^{CI}(N) \triangleq \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(P\left(X_N^{CI}(\theta)\right) - \theta\right) X_N^{CI}(\theta) - c\left(X_N^{CI}(\theta)\right)\right] dF(\theta).$$

Maximizing this profit with respect to N yields the following result.

Proposition 2 *With complete information,* M *prefers to distribute through a single monopolistic retailer.*

With complete information, the manufacturer's optimal choice is to use a single retailer in order to avoid the opportunism problem. This exclusive retailer monopolizes the downstream market, and the manufacturer obtains the monopoly profit.

4.2 Asymmetric Information

Assume now that retailers have private information about their costs. We first characterize the optimal contract offered by M for a given a number of retailers, and then analyze the optimal retail market structure.

Consider a (differentiable) symmetric equilibrium where each retailer sells the same quantity $x_N^*(\theta)$. Let

$$u_i(m_i,\theta) \triangleq \left(P\left(x_i(m_i) + (N-1)\, x_N^*(\theta)\right) - \theta\right) x_i(m_i) - T_i(m_i)$$

be R_i's utility when M offers the contract $\{T_i(m_i), x_i(m_i)\}$, he reports m_i and the cost is θ; and let $u_i(\theta) \triangleq u_i(m_i = \theta, \theta)$ be R_i's information rent. Following standard techniques, the necessary (local) first-order condition for R_i to truthfully report his type is

$$\dot{x}_i(\theta)\, P'\left(x_i(\theta) + (N-1)\, x_N^*(\theta)\right) x_i(\theta) + \left(P\left(x_i(\theta) + (N-1)\, x_N^*(\theta)\right) - \theta\right) \dot{x}_i(\theta) - \dot{T}_i(\theta) = 0,$$

which yields the derivative of R_i's information rent

$$\dot{u}_i(\theta) = -x_i(\theta) + (N-1) P'(x_i(\theta) + (N-1) x_N^*(\theta)) \dot{x}_N^*(\theta) x_i(\theta). \tag{4}$$

Hence, R_i's information rent is

$$u_i(\theta) \triangleq u_i(\overline{\theta}) + \int_\theta^{\overline{\theta}} x_i(z) dz - \underbrace{(N-1) \int_\theta^{\overline{\theta}} P'(x_i(z) + (N-1) x_N^*(z)) \dot{x}_N^*(z) x_i(z) dz}_{\text{Competing-contracts effect}}. \tag{5}$$

This expression generalizes equation (1)—the information rent in the two-types example—and reflects the *competing-contracts effect.* When R_i over-reports his cost, he knows that his rivals acquire a larger quantity because they report a lower cost to the manufacturer, which reduces the downstream price. The tariff requested by the manufacturer, however, assumes that all retailers have a cost equal to R_i's report, which reduces R_i's utility.

Other things being equal, R_i's incentive to overstate his cost decreases in the number of competing retailers in the downstream market. The reason is that the competing-contracts effect gets stronger as the downstream market becomes more competitive, while it vanishes when $N \to 1$. In fact, as N increases, each retailer knows that he will face an even lower price when he over-reports his cost, since the aggregate quantity produced by other retailers will be larger.

So far, we focused on the first-order condition to characterize the equilibrium quantity. From expression (4), the local second-order condition for R_i's maximization problem is

$$-\dot{x}_i(\theta) [1 - (N-1) \dot{x}_N^*(\theta) (P''(\cdot) x_i(\theta) + P'(\cdot))] \geq 0. \tag{6}$$

When $N \to 1$, there is no competing-contracts effect and this expression yields the standard result that output is non-increasing in the marginal cost. By contrast, when $N > 1$, the effect of condition (6) becomes less obvious, since it depends on the equilibrium contracts that M offers to the other retailers. We first neglect this condition, and verify it ex-post.[21]

Substituting for $u_i(\theta)$ into M's objective function and integrating by parts, in the bilateral (relaxed) contracting problem with R_i, M solves

$$\max_{x_i(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} [(P(\cdot) - \theta - h(\cdot)(1 - (N-1) P'(\cdot) \dot{x}_N^*(\cdot))) x_i(\cdot) - c(x_i(\cdot) + (N-1) x_N^*(\cdot))] dF(\theta).$$

[21] See the Appendix for details.

Let $X_N^*(\theta) \triangleq N x_N^*(\theta)$. Differentiating pointwisely with respect to $x_i(\cdot)$, imposing symmetry and rearranging, the symmetric equilibrium of the game is characterized by the following differential equation

$$\dot{x}_N^*(\theta) = \frac{\theta + h(\theta) + c'(X_N^*(\theta)) - (P'(X_N^*(\theta))\, x_N^*(\theta) + P(X_N^*(\theta)))}{h(\theta)(N-1)(P'(X_N^*(\theta)) + P''(X_N^*(\theta))\, x_N^*(\theta))}, \tag{7}$$

with boundary condition $x_N^*(\underline{\theta}) = x_N^{CI}(\underline{\theta})$.

The solution of this differential equation has the following properties.

Lemma 2 *With asymmetric information, the equilibrium quantity is $x_N^*(\theta) \leq x_N^{CI}(\theta)$ for every θ, with equality only at $\theta = \underline{\theta}$. Moreover, $\dot{x}_N^*(\theta) < 0$.*

Therefore, in the presence of asymmetric information, the manufacturer sells to retailers a lower quantity than with complete information, in order to limit their information rents. As expected, the equilibrium output is decreasing in the marginal cost, there is no distortion at the top (i.e., for type $\underline{\theta}$) and a downward distortion for all types $\theta > \underline{\theta}$.

The manufacturer chooses the optimal number of retailers N^* to maximize her aggregate expected profit

$$\pi^*(N) \triangleq \int_{\underline{\theta}}^{\overline{\theta}} \left[\left(P(X_N^*(\cdot)) - \theta - h(\cdot)\left(1 - (N-1)\, P'(X_N^*(\cdot))\, \dot{x}_N^*(\cdot)\right)\right) X_N^*(\cdot) - c(X_N^*(\cdot)) \right] dF(\theta). \tag{8}$$

The effect of a change in the number of retailers on this function can be decomposed in two terms:

$$\frac{\partial \pi^*(N)}{\partial N} = \underbrace{\int_{\underline{\theta}}^{\overline{\theta}} \left[P(X_N^*(\cdot)) + P'(X_N^*(\cdot))\, X_N^*(\cdot) - \theta - h(\cdot) - c'(\cdot) \right] \frac{\partial X_N^*(\cdot)}{\partial N} dF(\theta) +}_{\text{Strategic effect}}$$
$$+ \underbrace{\frac{\partial}{\partial N} \int_{\underline{\theta}}^{\overline{\theta}} h(\cdot)(N-1)\, P'(X_N^*(\cdot))\, \dot{x}_N^*(\cdot)\, X_N^*(\cdot)\, dF(\theta).}_{\text{Rent-extraction effect}} \tag{9}$$

The first term of the right-hand side of (9) reflects the strategic effect of a change in N on aggregate profit, excluding the competing-contracts effect. In fact, the term in square parenthesis is the difference between marginal revenue and total marginal cost, minus the retailers' monopoly rent (i.e., the information rent of a retailer who has no competition in the downstream market). The second term, by contrast, only reflects the effect of a change in N on the competing-contracts effect.

The interaction between these two effects determines the optimal retail market structure. When $N \to 1$, the first effect vanishes because the aggregate quantity converges to the second-best monopoly one,[22] while the second effect is positive, as we show in the following theorem.

Theorem 1 *Neglecting the integer constraint, a monopolistic retail market structure is never optimal because*

$$\lim_{N \to 1^+} \frac{\partial \pi^* (N)}{\partial N} > 0.$$

Moreover, the optimal number of retailers N^* is finite because $\pi^* (N) < \pi^* (1)$ for N sufficiently large.

The intuition of this result is as follows. With a single retailer, there is no competition in the downstream market. A marginal increase in competition (ignoring the integer constraint) has only a second-order effect on the manufacturer's profit through the opportunism problem because this problem is relatively weak. By contrast, the competing-contracts effect is of first-order magnitude due to the fact that the retailers' costs are distributed over a non-negligible support. As a consequence, increasing the number of retailers increases the manufacturer's profit. However, the manufacturer never chooses a retail market structure that approaches the perfectly competitive level. In fact, as downstream competition becomes more intense, the opportunism problem strengthens and offsets the competing-contracts effect (in the perfectly competitive limit, when $N \to +\infty$, the manufacturer makes zero profit). Therefore, the manufacturer's choice of the optimal retail market structure is always interior (when neglecting the integer constraint): she chooses neither a monopolistic retailer nor perfectly competitive retailers.

Remark. Focusing on quantity-forcing contracts is without loss of generality. In fact, even if the manufacturer does not control the quantity sold by retailers in the downstream market, retailers have no incentive to sell a quantity that is lower than the one acquired from the manufacturer. The reason is that, as we have shown, each retailer acquires a quantity that is weakly lower than the Cournot quantity. Therefore, no retailer has an incentive to individually reduce the quantity sold in the downstream market because their marginal revenue is higher than the marginal cost at the quantity acquired by the manufacturer.[23]

[22] In fact, when $N \to 1$, equation (7) implies that $P\left(X_N^* (\cdot)\right) + P'\left(X_N^* (\cdot)\right) X_N^* (\cdot) = c' (\cdot) + \theta + h (\cdot)$.

[23] For a formal proof of this point, see Martimort and Piccolo (2007).

5 Linear-Quadratic Framework

The analysis in Section 4 did not allow us to analyze how the retail market structure depends on demand and cost conditions and to clarify the intensity of the effects of asymmetric information—i.e., whether the optimal number of retailers is in fact sizable.

To address these issues and obtain a complete characterization of the optimal retail market structure for the manufacturer, we now specialize the model by assuming that the manufacturer's cost function is quadratic—i.e.,

$$c(X) = \beta \frac{X^2}{2},$$

and that the demand function is linear—i.e., the (inverse) demand function is

$$P(X) \triangleq \max\{a - bX, 0\},$$

where a reflects the size of the market, while b is a measure of how the market price reacts to changes in the quantity sold by retailers.

We also assume that the random variable θ is distributed on $[0,1]$ according to the beta distribution—i.e., $\theta \sim \text{Beta}\left[1, \lambda^{-1}\right]$ such that $F(\theta) = \theta^{\frac{1}{\lambda}}$ and $h(\theta) = \lambda\theta$, with $\lambda \geq 0$ (see, e.g., Miravete, 2002). Since $F(\theta)$ is increasing in λ,[24] beta distributions parametrized by a lower value of λ first-order stochastically dominate those parametrized by higher values of λ. This implies that, as λ increases, retailers' marginal costs are more likely to be low, and therefore distortions are lower too (*ceteris paribus*). When $\lambda = 1$, the beta distribution converges to the uniform distribution. All our assumptions are satisfied if $a > 1 + \lambda$.

Condition (7) yields the following linear differential equation

$$\dot{x}_N^*(\theta) = \frac{a - \theta - h(\theta) - (b(N+1) + \beta N)\, x_N^*(\theta)}{h(\theta)\, b\,(N-1)}, \tag{10}$$

with boundary condition

$$x_N^*(0) = \frac{a}{b(N+1) + \beta N}. \tag{11}$$

In the Appendix, we show that this differential equation has a unique linear solution

$$x_N^*(\theta) = \frac{a}{b(N+1) + \beta N} - \frac{\theta(1+\lambda)}{b(N+1) + \beta N + \lambda b(N-1)}, \tag{12}$$

[24] In fact, $\frac{\partial F(\theta)}{\partial \lambda} = -\frac{\theta^{\frac{1}{\lambda}}}{\lambda^2} \ln\theta > 0$ for $\theta \in [0,1]$.

and that M's expected profit is

$$\pi^*(N) \triangleq \frac{2Nb+\beta N^2}{2} \int_0^1 x_N^*(\theta)^2 \, d\theta^{\frac{1}{\lambda}}.$$

The next proposition compares the manufacturer's expected profit with 1 and 2 retailers, thereby taking into account the integer constraint.

Proposition 3 *There exist two thresholds $\hat{a} > 1+\lambda$ and $\left(\frac{\hat{\beta}}{b}\right) \geq 0$ such that: (i) when $a \in (1+\lambda, \hat{a}]$, $\pi^*(2) \geq \pi^*(1)$; (ii) when $a > \hat{a}$, $\pi^*(2) \geq \pi^*(1)$ if and only if $\frac{\beta}{b} > \left(\frac{\hat{\beta}}{b}\right)$.*

Therefore, the manufacturer prefers a duopolistic retail market structure rather than a monopolistic one when either (i) the market is sufficiently small or (ii) her cost function is sufficiently convex and/or the market price is not too responsive to aggregate quantity.

The intuition for these results is as follows. First, when the size of the market measured by a increases, the manufacturer sells a larger quantity and has a stronger incentive to expand the quantity of each retailer. Because the difference between the monopoly and the total duopoly profit increases, the opportunism problem gets worse and the manufacturer tends to prefer a monopolistic retailer. Second, as the manufacturer's cost becomes more convex, it gets (relatively) more costly for her to increase production. This implies that the opportunism problem gets weaker because expanding the quantity of one retailer is less profitable. In this case, the importance of the disciplining effect of competition on information rents is magnified. Third, if b increases, the market price (and not the production cost) becomes more responsive to changes in quantity. As a consequence, the opportunism problem gets worse because each retailer suffers more from an expansion in the quantity of his rivals, and the manufacturer prefers a monopolistic retailer.

As Proposition 3 shows, when a is small, a market structure with two retailers is more profitable than one with a single retailer even if the manufacturer's cost function is linear—i.e., $\beta = 0$. In general, for any combination of a and λ, there is a sufficiently high β and a sufficiently low b such that the monopolistic retail structure is dominated by a duopolistic one. Moreover, market structures with a much larger number of retailers can be optimal for the manufacturer. For example, if $a = 10$, $b = 1$, and $\lambda = 3$, then the optimal number of retailers is $N^* = 4$ when $\beta = 4$, and $N^* = 7$ when $\beta = 5$.

To provide a full analysis of the comparative statics of the parameters of the model, we consider a uniform distribution of the retailers' cost—i.e., we assume that $\lambda = 1$. In this case, expression (9), which characterizes the effects of a change in N on the manufacturer's profit, is

$$\frac{\partial \pi^*(N)}{\partial N} = \int_0^1 \left[a - 2bX_N^*(\cdot) - \beta X_N^*(\cdot) - 2\theta\right] \frac{\partial X_N^*(\cdot)}{\partial N} d\theta + \\ + \frac{\partial}{\partial N} \int_0^1 \theta (N-1) b |\dot{x}_N^*(\cdot)| X_N^*(\cdot)\, d\theta. \quad (13)$$

Again, the first term is the strategic effect, which captures the opportunism problem, and is strictly negative, while the second term is the rent-extraction effect, which reflects the effect of competing contracts, and is strictly positive (see the Appendix). We then obtain the following result.

Proposition 4 *Assume that $\lambda = 1$. The optimal number of retailers is increasing in β and decreasing in a and b.*

Hence, with a uniform distribution, the effects shaping the comparison between a duopolistic and a monopolistic market structure in Proposition 3 apply more generally: N^* is globally increasing in β and decreasing in a and b.

Although we cannot obtain an analytical solution for the comparative statics with respect to λ, Figure 1 shows by numerical simulations the effect of changes in λ on N^*. For the chosen parameters, the optimal number of retailers is increasing in λ.[25] The intuition is the following. An increase in λ increases the mass of types distributed on the lower tail of the support, so that the marginal cost of retailers is likely to be low. But this implies that a retailer's costs from overstating his type by reporting higher marginal costs to the manufacturer are large (*ceteris paribus*), as the manufacturer expects retailers to have low cost with a high probability and only offers a small information rent. The competing-contracts effect then becomes relatively more important, implying that the manufacturer benefits from using more retailers.

6 Extensions

6.1 Price Competition

In this section, we consider price competition and assume that retailers sell differentiated products.[26] R_i's demand function is $D^i(p_i, p_{-i})$, where $p_{-i} \triangleq \sum_{j=1, j\neq i}^{N} p_j$. We assume that

[25] Specifically, $N^* = 2$ if $\lambda = 0.5$, $N^* = 3$ if $\lambda = 1$, and $N^* = 6$ if $\lambda = 2$.

[26] With homogenous goods, price competition drives downstream profits to zero, making the problem uninteresting.

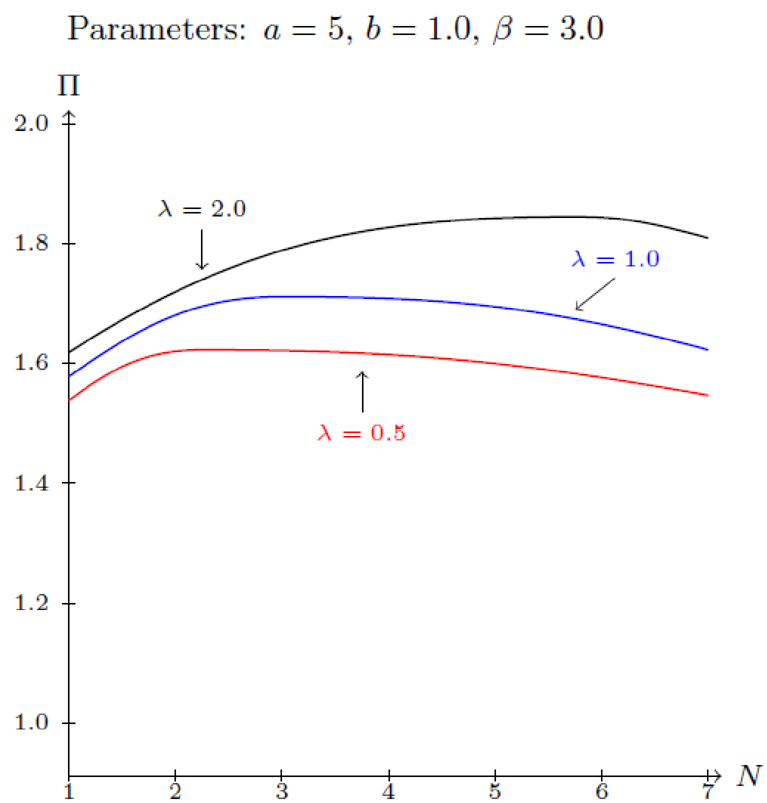

Figure 1: N^* for different values of λ

$D^i_i(\cdot) \triangleq \partial D^i(\cdot)/\partial p_i < 0$, $D^i_{-i}(\cdot) \triangleq \partial D^i(\cdot)/\partial p_{-i} \geq 0$, and $|D^i_i(\cdot)| > |D^i_{-i}(\cdot)|$ (see, e.g., Vives, 2001). Hence, for simplicity, we assume that the demand system is symmetric.

Suppose that M can contract with retailers directly on the retail price—i.e., the contract offered by M to R_i is a menu

$$\{T_i(m_i), p_i(m_i)\}_{m_i \in \Theta},$$

which specifies the price $p_i(m_i)$ that R_i charges in the final market and the tariff $T_i(m_i)$ that R_i pays to M, contingent on the R_i's report m_i about the cost θ.[27] We can focus on this contract space because there is a one-to-one mapping between wholesale and retail prices.[28] We maintain the passive beliefs refinement, which is plausible since retailers pay the tariff to

[27] With private contracts, the resale price control exerted by the manufacturer in our framework is different from the one discussed in the previous literature, where a market-wide price floor allows a manufacturer to achieve the monopoly profit.

[28] In other words, for each set of retail prices offered by the manufacturer in the retailers' contracts, there exists a set of wholesale prices, which yield the same outcome. As we show in the Appendix, the qualitative insights remain the same if we considered two-part tariff contracts, in which M offers a wholesale price and a fixed fee.

the manufacturer before downstream competition takes place, which implies that, from the manufacturer's perspective, each retailer forms a separate market.

Assume that there is a symmetric equilibrium such that $p_i(\theta) = p_N^*(\theta)$ for every $\theta \in \Theta$ and $i = 1, .., N$.[29] Letting

$$u_i(\theta, m_i) \triangleq D\left(p_i(m_i), (N-1)p_N^*(\theta)\right)\left(p_i(m_i) - \theta\right) - T(m_i)$$

be R_i's rent when he reports m_i and his type is θ_i, by the Envelope Theorem

$$\dot{u}_i(\theta) = -D\left(p_i(\theta), (N-1)p_N^*(\theta)\right) + (N-1)D_{-i}\left(p_i(\theta), (N-1)p_N^*(\theta)\right)\left(p_i(\theta) - \theta\right)\dot{p}_N^*(\theta).$$

Integrating and assuming no rent at the bottom—i.e., $u_i(\overline{\theta}) = 0$—$R_i$'s information rent is

$$u_i(\theta) = \underbrace{\int_\theta^{\overline{\theta}} D\left(p_i(z), (N-1)p^*(z)\right) dz +}_{\text{Standard rent}} \\ \underbrace{-(N-1)\int_\theta^{\overline{\theta}} \left(p_i(z) - z\right) D_{-i}\left(p_i(z), (N-1)p_N^*(z)\right)\dot{p}_N^*(z)\, dz.}_{\text{Competing-contracts effect}} \quad (14)$$

Hence, if $\dot{p}_N^*(z) \geq 0$—i.e., the equilibrium price is increasing in the marginal cost—the competing-contracts effect arises also with price competition (since $D_{-i}(\cdot) \geq 0$) so that retailers obtain lower rents in more competitive retail market structures.[30] The intuition is as follows. Suppose that R_i over-reports his cost in order to be charged a lower tariff. The manufacturer, however, incorrectly assumes that R_i's rivals also have the same high cost and hence that R_i's residual demand is relatively high. This increases the tariff charged by M. In reality, because R_i's rivals have a lower cost, his demand and profit are actually lower than what M expects.

Notice that, with product differentiation, M has an additional incentive to implement a market structure with more than one retailer even with symmetric information. This is because, when reducing the number of retailers, the manufacturer also reduces the number of products available on the market, which lowers her profit.

[29] Rey and Vergé (2004) show that with price competition in the retail market, a Perfect Bayesian Nash Equilibrium with passive beliefs and two-part tariffs does not exist if products are sufficiently homogeneous because of multilateral wholesale price deviations by the manufacturer. Due to our contract space, which separates the manufacturer's profit from the outcome in the retail market, this problem does not occur.

[30] Moreover, we can explicitly determine the optimal size of the retail network in the linear-quadratic framework and obtain very similar comparative-statics results as in our main model.

6.2 Imperfect Cost Correlation

In this section we show that our qualitative results hold even when retailers' costs are not perfectly correlated. Following previous literature—see, e.g., Armstrong and Vickers (2010) and Dequiedt and Martimort (2015)—suppose that retailers' costs are either identical or perfectly independent. Specifically, given a realization of a retailer's cost, we assume that with probability ν other retailers have the same cost, while with probability $1-\nu$ each rival retailer's cost is independently distributed according to the original prior distribution.

Consider a symmetric equilibrium in which each retailer produces $x_N^*(\theta_i)$ when his cost is θ_i and let

$$\begin{aligned} u_i(\theta_i, m_i) \triangleq\; & [\nu P(x_i(m_i) + (N-1)x_N^*(\theta_i)) \\ & + (1-\nu)\,\mathbb{E}_{\theta_{-i}}\left[P\left(x_i(m_i) + X_{-i}^*(\theta_{-i})\right)\right] - \theta_i]\, x_i(m_i) - T_i(m_i) \end{aligned}$$

be R_i's rent when he reports m_i and his type is θ_i. By the Envelope Theorem

$$\dot{u}_i(\theta_i) = -x_i(\theta_i) + \nu(N-1)\,P'(x_i(\theta_i) + (N-1)\,x_N^*(\theta_i))\,\dot{x}_N^*(\theta_i)\,x_i(\theta_i)\,.$$

Integrating and assuming no rent at the bottom—i.e., $u_i(\overline{\theta}) = 0$—$R_i$'s information rent is

$$u_i(\theta_i) = \underbrace{\int_{\theta_i}^{\overline{\theta}} x_i(z)\,dz}_{\text{Standard rent}} - \underbrace{\nu(N-1)\int_{\theta_i}^{\overline{\theta}} P'(x_i(z) + (N-1)\,x_N^*(z))\,\dot{x}_N^*(z)\,x_i(z)\,dz.}_{\text{Competing-contracts effect}}$$

If $\dot{x}_N^*(\cdot) \leq 0$, that is, if in equilibrium each retailer produces a lower quantity when his marginal cost increases, this expression shows that the competing-contracts effect continues to be present even with imperfect cost correlation, but becomes weaker as ν decreases. As intuition suggests, the expression converges to the standard Baron-Myerson rent for $\nu \to 0$ (i.e., when costs are uncorrelated) and to expression (5) for $\nu \to 1$ (i.e., when costs are perfectly correlated). To complete the analysis, in the Appendix we consider an example with linear demand and show that M implements a market structure with two retailers rather than one when β and ν are sufficiently large.[31]

Finally, as with differentiated products, with imperfectly correlated types there is an additional sampling reason that may induce M to implement a market structure with more than one retailer. In fact, by increasing the number of retailers, M increases the variance of

[31] It can also be shown that our qualitative results hold with price competition and imperfect cost correlation.

aggregate output, and hence the price variability, which increases the fixed fees collected by M because downstream (indirect) profit functions are convex in prices (see also Hansen and Motta, 2012).

6.3 Unique versus Nash Implementation

In our analysis, we focused on truthful equilibria in the retailers' reporting game (i.e., in the continuation game of the second stage after the contract offers of the manufacturer), and showed that the equilibrium characterized by condition (7) is a Nash equilibrium. One may wonder whether this is the unique (symmetric) equilibrium or whether other non-truthful reporting equilibria exist—i.e., whether our equilibrium can be implemented as the unique one or just as one of multiple equilibria. Specifically, a different equilibrium may exist in which all retailers report a higher cost (say $\theta' > \theta$) in order to weaken the competing-contracts effect and obtain a higher rent. This behavior could be interpreted as "implicit collusion" between retailers, that allows them to coordinate on an equilibrium with a higher (expected) profit.

To show that the truthful-reporting equilibrium is the unique equilibrium in the second stage, given the equilibrium offer of the manufacturer, we show that every retailer has the incentive to report his cost truthfully if all rivals report $\theta' \neq \theta$. This is equivalent to

$$[P\left(x_N^*\left(\theta\right) + \left(N-1\right)x_N^*\left(\theta'\right)\right) - \theta]\, x_N^*\left(\theta\right) - T_N^*\left(\theta\right) \geq [P\left(N x_N^*\left(\theta'\right)\right) - \theta]\, x_N^*\left(\theta'\right) - T_N^*\left(\theta'\right). \quad (15)$$

The condition guarantees that a retailer's profit is weakly larger when reporting truthfully than when misreporting, even if all rivals misreport.

Recalling that, for every θ,

$$T_N^*\left(\theta\right) \triangleq [P\left(X_N^*\left(\theta\right)\right) - \theta]\, x_N^*\left(\theta\right) - \underbrace{\int_{\theta}^{\overline{\theta}} [1 - \left(N-1\right) P'\left(X_N^*\left(z\right)\right) \dot{x}_N^*\left(z\right)]\, x_N^*\left(z\right) dx}_{\triangleq u_N^*(\theta)},$$

and substituting this into equation (7) we can show that (15) is always fulfilled (see the Appendix). We therefore obtain the following result.

Proposition 5 *The truthful-reporting equilibrium characterized in Section 4.2 is the unique equilibrium in the second stage.*

The intuition is straightforward and is closely connected to the opportunism problem. Since a retailer tells the truth when his rivals do the same, he will *a fortiori* do so when they jointly over-report the cost and produce less than in a truthful equilibrium.

6.4 Alternative Mechanisms and Timing

Following the literature, we have assumed that the manufacturer and the retailers contract simultaneously and bilaterally. This is a realistic assumption in many environments. However, while this assumption is innocuous with complete information (as there can always be secret renegotiation), with incomplete information the equilibrium may not be robust to perturbations of the timing of the game—e.g., to the introduction of sequential contracting—and/or to the use of a bidding rather than an offer game (Segal and Whinston, 2003). In this section, we show that, under natural assumptions, our main results survive when these aspects are taken into account.

Sequential Contracting. Since costs are correlated, one may wonder whether sequential rather than simultaneous contracting with retailers could allow the manufacturer to use information obtained in early stages to improve later contracting and if, in that case, the competing contracts effect would still arise. Although sequential contracting can improve M's bargaining position with later retailers, it also introduces an incentive for retailers who contract early to influence contracts offered later on, which tends to increase their information rents. For example, a retailer may want to mis-report the common cost to exclude competitors from the downstream market. Moreover, in contrast to simultaneous contracting, with sequential contracting the manufacturer needs to credibly communicate a retailer's position in the sequence. Therefore, the manufacturer does not necessarily benefit from sequential contracting.

When the manufacturer does not benefit from sequential contracting and, hence, prefers to contract with retailers simultaneously, all results of our analysis hold. By contrast, if the manufacturer prefers to contract with retailers sequentially rather than simultaneously, this necessarily implies that the manufacturer chooses a market structure with more than one retailer. The reason is that the manufacturer's profit when using a monopolistic retailer is not affected by sequential contracting, while she obtains higher profit by approaching multiple retailer sequentially. Therefore, sequential contracting yields qualitative results that are consistent with our analysis.

Bidding Game. In our analysis, we have considered an offer game in which, after deciding how many retailers to use, M contracts with them simultaneously. Could M benefit from committing to implement a monopolistic market structure and allowing potential retailers to bid for the right to be the monopolist in the final market?[32] Consider, for example, a (first-price) auction in which retailers bid to acquire an exclusive licence to distribute the manufacturer product's and the winner pays his bid to the manufacturer.[33] The manufacturer then simply chooses the quantity that the winning monopolistic retailer distributes in the downstream market. In this environment, competition erodes retailers' rents and allows the manufacturer to maximize profit: the auction price paid by the winning retailer is equal to the monopoly profit in the downstream market, and the manufacturer sells the monopoly quantity.[34]

This result, however, hinges on the absence of bidding costs. By contrast, if there is an arbitrarily small bidding cost and retailers sequentially choose whether to enter the auction, then only one retailer participates and pays a price equal to zero to the manufacturer.[35] The reason is that, if there are two or more retailers, their profit from contracting with the manufacturer is equal to zero and hence they have no incentive to participate in the auction organized by the manufacturer. By contrast, in our game retailers obtain a strictly positive profit in expectation due to the information rent. Therefore, with an arbitrarily small bidding cost, using an auction does not allow the manufacturer to obtain the monopoly profit.

Similarly, (even without bidding costs) at most one retailer would have a *strict* incentive to participate in a bidding game that selects the monopolistic distributor to which the manufacturer offers a contract, as in our model. The reason is that, if two or more retailers participate, (in a pure-strategy equilibrium) their bids reveal the common cost and, hence, the winner obtains no rent from the contract offered by the manufacturer.

7 Conclusions

Vertical foreclosure is a major concern for antitrust authorities. This concern is often driven by the logic of the opportunism problem, which induces a manufacturer to eliminate intrabrand

[32] We would like to thank Claire Chambolle for raising this point.

[33] Analogous results would be obtained if retailers bid a unit price at which to acquire the manufacturer's product, rather than a fixed price.

[34] The logic of this bidding mechanism is closely related to the result by Crémer and McLean (1988) that, with correlated types, a principal can achieve full surplus extraction by conditioning the offer made to one agent on the reports of his rivals, which is in fact what an auction does.

[35] With simultaneous entry by retailers, there are multiple pure-strategy equilibria, but in all of them only one retailer participates in the auction.

competition by distributing through a monopolistic retailer. Our analysis suggests that this concern is less relevant when retailers are privately informed about common cost or demand characteristics.

In order to highlight this issue, we have examined a vertical contracting environment *à la* Segal and Whinston (2003) in which a manufacturer chooses the number of retailers who distribute her product. With privately informed retailers and secret contracting, the manufacturer's incentive to foreclose and use a monopolistic retailer depends on the retailers' information rent and its interplay with the opportunism problem. Unlike in standard models, with asymmetric information the manufacturer may prefer a market structure with multiple retailers in order to exploit the disciplining effect of downstream competition on their information rents. This effect is sizable, and is stronger when the manufacturer's marginal cost is increasing, when the downstream market is sufficiently small, and when the market price is not particularly responsive to changes in quantity.

Our results contribute to a better understanding of the forces that shape the retail market structure and may partly explain why, in practice, different structures arise in different industries, depending on the intensity of asymmetric information between manufacturers and retailers.

A Appendix

Two-Type Model. With a single retailer, using the expressions for $\underline{x}^*(1)$ and $\overline{x}^*(1)$, the manufacturer's expected profit is

$$\pi^*(1) \triangleq \frac{1 - 2\overline{\theta}\left(1 - \overline{\theta}\right)}{2(2+\beta)}.$$

Let now $N = 2$ and

$$\underline{u}_i \triangleq \left(P\left(\underline{x}_i + \underline{x}^*(2)\right)\right)\underline{x}_i - \underline{T}_i,$$
$$\overline{u}_i \triangleq \left(P\left(\overline{x}_i + \overline{x}^*(2)\right) - \overline{\theta}\right)\overline{x}_i - \overline{T}_i.$$

Using standard techniques, the incentive compatibility constraints are

$$\underline{u}_i \geq P\left(\overline{x}_i + \underline{x}^*(2)\right)\overline{x}_i - \overline{T}_i \triangleq \overline{u}_i + \overline{\theta}\overline{x}_i - \Delta x^* \overline{x}_i \tag{16}$$

and

$$\overline{u}_i \geq \left(P\left(\underline{x}_i + \overline{x}^*(2)\right) - \overline{\theta}\right)\underline{x}_i - \underline{T}_i \triangleq \underline{u}_i - \overline{\theta}\underline{x}_i + \Delta x^* \underline{x}_i,$$

and the participation constraints require that $\underline{u}_i \geq 0$ and $\overline{u}_i \geq 0$.

Conjecturing that only the incentive compatibility constraint (16) matters, and that $\overline{u}_i = 0$ at the optimal contract, it is easy to obtain expressions (2), where

$$\Delta x^* = \frac{\overline{\theta}}{2+\beta}.$$

Condition (16) yields the equilibrium rents

$$\underline{u}^*(2) = \frac{1+\beta}{2+\beta}\overline{x}^*(2)\,\overline{\theta} > 0$$

and

$$\overline{u}^*(2) = 0 > \frac{1+\beta}{2+\beta}\left(\overline{x}^*(2) - \underline{x}^*(2)\right)\overline{\theta}.$$

Hence, the starting conjecture is correct. Finally, letting $\underline{X}^* \triangleq 2\underline{x}^*(2)$, $\overline{X}^* \triangleq 2\underline{x}^*(2)$ and using (2), the manufacturer's expected profit is

$$\begin{aligned} \pi^*(2) &= \tfrac{1}{2}\left[P\left(\underline{X}^*\right)\underline{X}^* - c\left(\overline{X}^*\right) + P\left(\overline{X}^*\right)\overline{X}^* - c\left(\underline{X}^*\right)\right] - \frac{(1+\beta)\,\overline{\theta}}{2(2+\beta)}\overline{X}^* \\ &= \frac{2(1+\beta)}{(3+2\beta)^2} - \frac{2(1+\beta)\,\overline{\theta}}{(3+2\beta)(2+\beta)} + \frac{(1+\beta)\,\overline{\theta}^2}{(2+\beta)^2}. \end{aligned}$$

Proof of Proposition 1. Let $\Delta\pi \triangleq \pi^*(2) - \pi^*(1)$. Using the expression for M's expected profit we have

$$\Delta\pi = \frac{4\overline{\theta}\left(1-2\overline{\theta}\right)\beta^2 + \beta\left(12\overline{\theta}-1\right)\left(1-2\overline{\theta}\right) - 2\left(1-3\overline{\theta}\right)^2}{2\left(2+\beta\right)^2\left(3+2\beta\right)^2},$$

with

$$\hat{\beta} \triangleq \frac{1+24\overline{\theta}^2 - 14\overline{\theta} + \sqrt{\left(6\overline{\theta}+1\right)\left(1-2\overline{\theta}\right)}}{8\overline{\theta}\left(1-2\overline{\theta}\right)}$$

being the unique positive root of $\Delta\pi = 0$ in the relevant region of parameters—i.e., $\overline{\theta} \in [0, 1/2)$. Because the denominator of $\Delta\pi$ is strictly positive, the sign of $\Delta\pi$ is equal to the sign of the numerator. Taking the derivative of $\Delta\pi$ with respect to β and evaluating the result at $\beta = \hat{\beta}$ yields $\sqrt{(1-2\overline{\theta})(6\overline{\theta}+1)}$, which is strictly positive for $\overline{\theta} \in [0, 1/2)$. Therefore, $\Delta\pi > 0$ if $\beta > \hat{\beta}$. If $\overline{\theta} = 0$, then $\hat{\beta} = \infty$, which implies that $\Delta\pi < 0$ for all β. Instead, for $\overline{\theta} \in [0, 1/2)$, $\hat{\beta} > 0$ and finite. Moreover, $\hat{\beta} = 0$ if $\overline{\theta} = 1/3$. ■

Proof of Lemma 1. Recall that $X_N^{CI}(\theta) \triangleq N x_N^{CI}(\theta)$. From the first-order condition (3), the Implicit Function Theorem yields

$$\dot{x}_N^{CI}(\theta) = \frac{1}{P''(X_N^{CI}(\theta))\,X_N^{CI}(\theta) + P'(X_N^{CI}(\theta))\,(1+N) - c''(X_N^{CI}(\theta))\,N},$$

and

$$\frac{\partial x_N^{CI}(\theta)}{\partial N} = -x_N^{CI}(\theta)\,\frac{P''\left(X_N^{CI}(\theta)\right)x_N^{CI}(\theta) + P'\left(X_N^{CI}(\theta)\right) - c''\left(X_N^{CI}(\theta)\right)}{P''(X_N^{CI}(\theta))\,X_N^{CI}(\theta) + P'(X_N^{CI}(\theta))\,(1+N) - c''(X_N^{CI}(\theta))\,N}.$$

Both expressions are negative since $c''(\cdot) \geq 0$ and $P'(X) + P''(X)\,X < 0$. Hence,

$$\begin{aligned}
\frac{\partial X_N^{CI}(\theta)}{\partial N} &= x_N^{CI}(\theta) + N\frac{\partial x_N^{CI}(\theta)}{\partial N} \\
&= x_N^{CI}(\theta)\left[1 - N\frac{P''\left(X_N^{CI}(\theta)\right)x_N^{CI}(\theta) + P'\left(X_N^{CI}(\theta)\right) - c''\left(X_N^{CI}(\theta)\right)}{P''(X_N^{CI}(\theta))\,X_N^{CI}(\theta) + P'(X_N^{CI}(\theta))\,(1+N) - c''(X_N^{CI}(\theta))\,N}\right] \\
&= x_N^{CI}(\theta)\left[\frac{P'\left(X_N^{CI}(\theta)\right)}{P''(X_N^{CI}(\theta))\,X_N^{CI}(\theta) + P'(X_N^{CI}(\theta))\,(1+N) - c''(X_N^{CI}(\theta))\,N}\right] > 0.
\end{aligned}$$

Let $X^C(\theta)$ be the perfectly competitive aggregate quantity—i.e., the value of X that solves

$$P(X) = \theta + c'(X). \tag{17}$$

Rearranging the first-order condition (3) and the taking the limit for $N \to +\infty$ yields

$$P\left(\lim_{N\to+\infty} X_N^{CI}(\theta)\right) - \theta - c'\left(\lim_{N\to+\infty} X_N^{CI}(\theta)\right) = -\frac{P'\left(\lim_{N\to+\infty} X_N^{CI}(\theta)\right)\lim_{N\to+\infty} X_N^{CI}(\theta)}{\lim_{N\to+\infty} N}.$$

The right-hand-side is equal to zero since, by Assumption 2, $|P'(\cdot)| < +\infty$ and $X_N^{CI}(\theta)$ is bounded (because $X_N^{CI}(\theta) \leq X^C(\theta)$ by (3) and (17)). Hence, $\lim_{N\to+\infty} X_N^{CI}(\theta) = X^C(\theta)$.

Finally, as $N \to +\infty$, the derivative of M's objective function is

$$P(X^C(\theta)) + P'(X^C(\theta)) \lim_{N\to+\infty} x_N^{CI}(\theta) - \left(\theta + c'(X^C(\theta))\right) = P'(X^C(\theta)) \lim_{N\to+\infty} x_N^{CI}(\theta).$$

If $\lim_{N\to+\infty} x_N^{CI}(\theta) > 0$, this derivative is strictly negative, which yields a contradiction because M would choose a smaller quantity to increase profit. Hence, $\lim_{N\to+\infty} x_N^{CI}(\theta) = 0$. ■

Proof of Proposition 2. The quantity produced by a monopolist in the downstream market is

$$X^M(\theta) \triangleq \arg\max_{X\geq 0} \left(P(X) - \theta\right) X - c(X),$$

which is unique since the function $(P(X) - \theta) X - c(X)$ is strictly concave. From condition (3), when $N = 1$, $X^M(\theta) = X_1^{CI}(\theta)$ for every θ. Moreover, since $X_N^{CI}(\theta)$ is decreasing in N by Lemma 1, $X^M(\theta) < N x_N^{CI}(\theta) = X_N^{CI}(\theta)$ for $N > 1$ and every θ.

M's aggregate (state contingent) profit is

$$\pi^{CI}(N, \theta) \triangleq N(P\left(X_N^{CI}(\theta)\right) - \theta) x_N^{CI}(\theta) - c\left(X_N^{CI}(\theta)\right) = (P\left(X_N^{CI}(\theta)\right) - \theta) X_N^{CI}(\theta) - c\left(X_N^{CI}(\theta)\right).$$

Hence,

$$\pi^{CI}(N = 1, \theta) = (P\left(X^M(\theta)\right) - \theta) X^M(\theta) - c\left(X^M(\theta)\right),$$

and $\pi^{CI}(N, \theta) < \pi^{CI}(N = 1, \theta)$ for every θ and $N > 1$. This implies that the manufacturer chooses a single retailer to maximize her profit. ■

Proof of Lemma 2. The equilibrium output $x_N^*(\theta)$ solves the differential equation (7) with boundary condition $x_N^*(\underline{\theta}) = x_N^{CI}(\underline{\theta})$.

We first show that $x_N^*(\theta) \leq x_N^{CI}(\theta)$ $\forall\theta$. To simplify notation, let $P' \triangleq P'(X_N^{CI}(\underline{\theta}))$, $P'' \triangleq P''(X_N^{CI}(\underline{\theta}))$, and $c'' \triangleq c''\left(X_N^{CI}(\underline{\theta})\right)$. Notice that $\lim_{\theta\to\underline{\theta}} \dot{x}_N^*(\theta) = 0/0$. Using L'Hôpital's rule,

$$\dot{x}_N^*(\underline{\theta}) = \frac{2}{2NP' + (2N-1)P'' x_N^{CI}(\underline{\theta}) - Nc''},$$

which is strictly negative under our assumptions. Hence, in a neighborhood of $\underline{\theta}$,

$$x_N^*(\theta) \approx x_N^{CI}(\underline{\theta}) + \dot{x}_N^*(\underline{\theta})(\theta - \underline{\theta}).$$

Similarly,

$$\dot{x}_N^{CI}(\underline{\theta}) = \frac{1}{(N+1)P' + NP''x_N^{CI}(\underline{\theta}) - Nc''},$$

so that in a neighborhood of $\underline{\theta}$

$$x_N^{CI}(\theta) \approx x_N^{CI}(\underline{\theta}) + \dot{x}_N^{CI}(\underline{\theta})(\theta - \underline{\theta}).$$

Therefore,

$$\dot{x}_N^*(\underline{\theta}) - \dot{x}_N^{CI}(\underline{\theta}) = \frac{2P' + P''x_N^{CI}(\underline{\theta}) - Nc''}{((N+1)P' + NP''x_N^{CI}(\underline{\theta}) - Nc'')(2NP' + (2N-1)P''x_N^{CI}(\underline{\theta}) - Nc'')},$$

which is strictly negative under our assumptions on $P(\cdot)$ and $c(\cdot)$. Hence, $x_N^*(\theta) < x_N^{CI}(\theta)$ for $\theta \to \underline{\theta}$.

We now show by contradiction that this property holds globally. Suppose that $x_N^*(\theta) > x_N^{CI}(\theta)$ for some θ. Then, consider the lowest θ (say $\theta_1 > \underline{\theta}$) at which $x_N^*(\theta) = x_N^{CI}(\theta)$. By definition of $x_N^{CI}(\theta)$, equation (3) yields

$$\dot{x}_N^*(\theta_1) = \frac{1}{(N-1)\left(P'\left(X_N^{CI}(\theta_1)\right) + P''\left(X_N^{CI}(\theta_1)\right)x_N^{CI}(\theta_1)\right)},$$

which is negative because $P'(X) + P''(X)X < 0$. Note that

$$\operatorname{sign}\left[\dot{x}_N^*(\theta_1) - \dot{x}_N^{CI}(\theta_1)\right] = \operatorname{sign}\left[2P'\left(X_N^{CI}(\theta_1)\right) + P''\left(X_N^{CI}(\theta_1)\right)x_N^{CI}(\theta_1) - Nc''(X_N^{CI}(\theta_1))\right],$$

which is negative under our assumptions. Hence, for ε positive and small, a first-order Taylor approximation yields

$$\operatorname{sign}\left[x_N^*(\theta_1 - \varepsilon) - x_N^{CI}(\theta_1 - \varepsilon)\right] = \operatorname{sign}\left[\dot{x}_N^{CI}(\theta_1) - \dot{x}_N^*(\theta_1)\right] > 0,$$

which implies the desired contradiction $x_N^*(\theta_1 - \varepsilon) > x_N^{CI}(\theta_1 - \varepsilon)$. Hence, $x_N^*(\theta) < x_N^{CI}(\theta)$ for every θ and N.

We now show that $\dot{x}_N^*(\theta) \leq 0$ for every θ. Let $x_N^S(\theta)$ be the solution of

$$\theta + h(\theta) + c'(Nx) - \left[P'(Nx)x + P(Nx)\right] = 0, \tag{18}$$

that is, if x were equal to $x_N^S(\theta)$, the numerator of (7) would be zero. The left-hand side of

(18) is increasing in x because of Assumption 2 (ii) and $c''(\cdot) \geq 0$. Notice that

$$\dot{x}_N^S(\theta) = \frac{1 + \dot{h}(\theta)}{P'\left(Nx_N^S(\theta)\right)(N+1) + NP''\left(Nx_N^S(\theta)\right)x_N^S(\theta) - c''\left(Nx_N^S(\theta)\right)N},$$

which is strictly negative by assumption. For $\theta \to \underline{\theta}$ it can be shown that

$$\dot{x}_N^S(\underline{\theta}) = \frac{2}{(N+1)P' + NP''x_N^S(\underline{\theta}) - Nc''} < \dot{x}_N^*(\underline{\theta}) = \frac{2}{2NP' + (2N-1)P''x_N^{CI}(\underline{\theta}) - Nc''}.$$

so that $x_N^S(\theta) \leq x_N^*(\theta)$ for $\theta \to \underline{\theta}$. But this implies that the numerator of (7) is positive. As the denominator is strictly negative, $\dot{x}_N^*(\underline{\theta}) < 0$.

Finally, we show by contradiction that $x_N^S(\theta) \leq x_N^*(\theta)$ also holds globally. Suppose that $x_N^S(\theta) > x_N^*(\theta)$ for some θ. Let θ_2 be the lowest value for which $x_N^S(\theta) = x_N^*(\theta)$. By definition $\dot{x}_N^*(\theta_2) = 0 > \dot{x}_N^S(\theta_2)$. Now consider $\varepsilon > 0$ and small enough. By definition of θ_2, $x_N^S(\theta_2 - \varepsilon) < x_N^*(\theta_2 - \varepsilon)$ must hold. But, taking the limit $\varepsilon \to 0$, we have

$$x_N^S(\theta_2 - \varepsilon) - x_N^*(\theta_2 - \varepsilon) \approx -\dot{x}_N^S(\theta_2) > 0,$$

which again yields a contradiction. ■

Incentive-Compatibility Constraints. We now show that the equilibrium satisfies the constraints that we have neglected in the analysis. First, consider the local second-order incentive compatibility constraint (6). Using (7), in a symmetric equilibrium, this constraint requires that

$$\begin{aligned} 0 &\leq -\dot{x}_N^*(\theta)\left[1 - (N-1)\dot{x}_N^*(\theta)\left(P'\left(X_N^*(\theta)\right) + P''\left(X_N^*(\theta)\right)x_N^*(\theta)\right)\right] = \\ &= \frac{\dot{x}_N^*(\theta)}{h(\theta)}\left[\theta + c'\left(X_N^*(\theta)\right) - \left(P'\left(X_N^*(\theta)\right)x_N^*(\theta) + P\left(X_N^*(\theta)\right)\right)\right]. \end{aligned} \tag{19}$$

Since $x_N^*(\theta) \leq x_N^{CI}(\theta)$, the assumptions $P'(X) + P''(X)X < 0$ and $c''(\cdot) \geq 0$ yield

$$P'\left(X_N^*(\theta)\right)x_N^*(\theta) + P\left(X_N^*(\theta)\right) - c'\left(X_N^*(\theta)\right) > P'\left(X_N^{CI}(\theta)\right)x_N^{CI}(\theta) + P\left(X_N^{CI}(\theta)\right) - c'\left(X_N^{CI}(\theta)\right) = \theta.$$

Hence, the constraint is satisfied since $\dot{x}_N^*(\theta) \leq 0$.

Finally, we show that the global incentive compatibility constraint holds, too. R_i's global incentive compatibility constraint holds if and only if, in equilibrium, $u^*(\theta) \geq u^*(m_i, \theta)$ for every $m_i \neq \theta$. Let $m_i \geq \theta$ (without loss of generality), we have

$$u^*(\theta) - u^*(m_i, \theta) = \int_{m_i}^{\theta} \{(P(x_N^*(z) + (N-1)x_N^*(\theta)) - \theta)\dot{x}_N^*(z) + \\ +P'(x_N^*(z) + (N-1)x_N^*(\theta))\dot{x}_N^*(z)x_N^*(z) - \dot{T}_N^*\Big\}(z)\,dz.$$

By definition,

$$\dot{T}_N^*(z) = P'(x_N^*(z) + (N-1)x_N^*(z))\dot{x}_N^*(z)x_N^*(z) + (P(x_N^*(z) + (N-1)x_N^*(z)) - z)\dot{x}_N^*(z).$$

Substituting and using Assumption 2 on $P'''(\cdot)$, we have

$$\begin{aligned} u^*(\theta) - u^*(m_i, \theta) &= \int_{m_i}^{\theta} \dot{x}_N^*(z) \int_z^{\theta} \{-1 + (N-1)\dot{x}_N^*(y)[P'(x_N^*(z) + (N-1)x_N^*(y)) \\ &\quad + P''(x_N^*(z) + (N-1)x_N^*(y))x_N^*(z)]\}\,dydz \\ &\geq -\int_{m_i}^{\theta} \dot{x}_N^*(z) \int_z^{\theta} \{1 - (N-1)\dot{x}_N^*(y)[P'(X_N^*(y)) + P''(X_N^*(y))x_N^*(y)]\}\,dydz, \end{aligned}$$

which is positive by the second-order incentive-compatibility constraint—see condition (19).

Proof of Theorem 1. In a symmetric equilibrium M's expected profit is

$$\pi^*(N) \triangleq \int_{\underline{\theta}}^{\overline{\theta}} \pi_N^*(\theta)dF(\theta),$$

where

$$\pi_N^*(\theta) \triangleq N(P(X_N^*(\cdot)) - \theta - h(\cdot)(1 - (N-1)P'(X_N^*(\cdot))\dot{x}_N^*(\cdot)))x_N^*(\cdot) - c(X_N^*(\cdot)).$$

Differentiating $\pi_N^*(\theta)$ with respect to N, by the Envelope Theorem we have

$$\begin{aligned} \frac{\partial \pi_N^*(\theta)}{\partial N} &= (P(\cdot) - \theta - h(\cdot)(1 - (N-1)P'(\cdot)\dot{x}_N^*(\cdot)))x_N^*(\cdot) + \\ -c'(\cdot)&\left(x_N^*(\cdot) + (N-1)\tfrac{\partial x_N^*(\cdot)}{\partial N}\right) + N\left[P'(\cdot)\left(x_N^*(\theta) + (N-1)\tfrac{\partial x_N^*(\cdot)}{\partial N}\right) + h(\cdot)P'(\cdot)\dot{x}_N^*(\cdot)\right]x_N^*(\cdot) + \\ &+ (N-1)Nh(\cdot)\left[P''(\cdot)\left(x_N^*(\theta) + (N-1)\tfrac{\partial x_N^*(\cdot)}{\partial N}\right)\dot{x}_N^*(\cdot) + P'(\cdot)\tfrac{\partial \dot{x}_N^*(\cdot)}{\partial N}\right]x_N^*(\cdot). \end{aligned}$$

Hence,

$$\lim_{N\to 1^+} \frac{\partial \pi_N^*(\theta)}{\partial N} = \left(P\left(x_1^*(\cdot)\right) - \theta - h(\cdot)\right) x_1^*(\cdot) - c'\left(x_1^*(\cdot)\right) x_1^*(\cdot) +$$
$$+ \left(P'\left(x_1^*(\cdot)\right) x_1^*(\theta) + h(\cdot) P'\left(x_1^*(\cdot)\right) \dot{x}_1^*(\cdot)\right) x_1^*(\cdot)$$
$$= \left(P\left(x_1^*(\cdot)\right) + P'\left(x_1^*(\cdot)\right) x_1^*(\theta) - \theta - h(\cdot) - c'\left(x_1^*(\cdot)\right)\right) x_1^*(\cdot) +$$
$$+ h(\cdot) P'\left(x_1^*(\cdot)\right) \dot{x}_1^*(\cdot) x_1^*(\cdot).$$

By definition of $x_1^*(\theta)$,

$$P\left(x_1^*(\cdot)\right) + P'\left(x_1^*(\cdot)\right) x_1^*(\theta) - \theta - h(\cdot) - c'\left(x_1^*(\cdot)\right) = 0,$$

Therefore,

$$\lim_{N\to 1^+} \frac{\partial \pi^*(N)}{\partial N} = \int_{\underline{\theta}}^{\overline{\theta}} h(\cdot) P'\left(x_1^*(\cdot)\right) \dot{x}_1^*(\cdot) x_1^*(\cdot)\, dF(\theta),$$

which is strictly positive since $\dot{x}_1^*(\cdot) \leq 0$ and $P'(\cdot) < 0$.

Finally, we now show that $\pi^*(N) < \pi^*(1)$ for N sufficiently large. Notice that

$$\pi^*(N) \leq \pi^{CI}(N) = \int_{\underline{\theta}}^{\overline{\theta}} \left[N\left(P\left(X_N^{CI}(\cdot)\right) - \theta\right) x_N^{CI}(\cdot) - c\left(X_N^*(\cdot)\right)\right] dF(\theta).$$

As $\lim_{N\to+\infty} x_N^{CI}(\theta) = 0$, it follows that

$$\lim_{N\to+\infty} \pi^*(N) \leq \lim_{N\to+\infty} \int_{\underline{\theta}}^{\overline{\theta}} \left[N\left(P\left(X_N^{CI}(\cdot)\right) - \theta\right) x_N^{CI}(\cdot) - c\left(X_N^*(\cdot)\right)\right] dF(\theta) = 0.$$

Since $\pi^*(1) > 0$, the result follows immediately. ■

Linear-Quadratic Framework. We solve the differential equation (10) with boundary condition (11). Rearranging terms, we obtain

$$\dot{x}_N^*(\theta) + \frac{b(N+1) + \beta N}{\lambda\theta b(N-1)} x_N^*(\theta) = \frac{a - \theta(1+\lambda)}{\lambda\theta b(N-1)}.$$

Letting $\Gamma \triangleq \frac{b(N+1)+\beta N}{\lambda b(N-1)}$, the solution to the previous equation is

$$x_N^*(\theta) = k e^{-\int_0^\theta \frac{\Gamma}{z_1} dz_1} + \int_0^\theta e^{-\int_{z_2}^\theta \frac{\Gamma}{z_1} dz_1} \frac{a - z_2(1+\lambda)}{z_2 \lambda b(N-1)} dz_2. \tag{20}$$

Notice that $e^{-\int_0^\theta \frac{\Gamma}{z_1} dz_1} = e^{-\Gamma \ln z_1|_0^\theta} = +\infty$ due to the fact that $\Gamma > 0$ and $0 \le \theta \le 1$. Because $x_N^*(\theta)$ is bounded, it follows that $k = 0$ to fulfill (20). Hence,

$$x_N^*(\theta) = \int_0^\theta e^{-\Gamma \ln z_1|_{z_2}^\theta} \frac{a - z_2(1+\lambda)}{z_2 \lambda b (N-1)} dz_2.$$

Since $e^{\Gamma \ln z_1|_{z_2}^\theta} = \frac{z_2^\Gamma}{\theta^\Gamma}$, rearranging yields

$$\begin{aligned}
x_N^*(\theta) &= \frac{1}{\lambda b (N-1)\theta^\Gamma} \int_0^\theta \left(a z_2^{\Gamma-1} - z_2^\Gamma (1+\lambda) \right) dz_2 \\
&= \frac{1}{\lambda b (N-1)\theta^\Gamma} \left(\frac{a\theta^\Gamma}{\Gamma} - \frac{\theta^{\Gamma+1}(1+\lambda)}{\Gamma+1} \right) \\
&= \frac{a}{b(N+1)+\beta N} - \frac{\theta(1+\lambda)}{b(N+1)+\beta N + \lambda b (N-1)}.
\end{aligned}$$

From this, we obtain

$$\dot{x}_N^*(\theta) = -\frac{(1+\lambda)}{b(N+1)+\beta N + \lambda b(N-1)}.$$

The expected profit of the manufacturer (8) can be written as

$$\begin{aligned}
\pi^*(N) &= \int_0^1 \left(N x_N^*(\theta)^2 (b + \beta N) - \frac{\beta}{2} (N x_N^*(\theta))^2 \right) dF(\theta) \\
&= \int_0^1 \left(N x_N^*(\theta)^2 b + \frac{\beta}{2} (N x_N^*(\theta))^2 \right) dF(\theta) \\
&= \frac{2Nb + \beta N^2}{2} \int_0^1 x_N^*(\theta)^2 dF(\theta).
\end{aligned}$$

Since $\theta \sim$ Beta $\left[1, \lambda^{-1}\right]$

$$\mathbb{E}[\theta] = \frac{1}{\lambda} \int_0^1 \theta^{\frac{1}{\lambda}} d\theta = \frac{1}{1+\lambda} \left. \theta^{\frac{1}{\lambda}+1} \right|_0^1 = \frac{1}{1+\lambda}$$

and

$$\mathbb{E}\left[\theta^2\right] = \frac{1}{\lambda} \int_0^1 \theta^{\frac{1}{\lambda}+1} d\theta = \frac{1}{1+2\lambda} \left. \theta^{\frac{1}{\lambda}+2} \right|_0^1 = \frac{1}{1+2\lambda}.$$

Substituting for $x_N^*(\theta)$ and integrating yields

$$\pi^*(N) = \frac{a^2(2Nb+\beta N^2)}{2\left(b(N+1)+\beta N\right)^2} + \frac{\left(2Nb+\beta N^2\right)(1+\lambda)^2}{2\left(1+2\lambda\right)\left(b(N+1)+\beta N+\lambda b\left(N-1\right)\right)^2} + \\ - \frac{a\left(2Nb+\beta N^2\right)}{\left(b\left(N+1\right)+\beta N\right)\left(b(N+1)+\beta N+\lambda b\left(N-1\right)\right)}. \quad (21)$$

When $\lambda = 1$, the two terms of expression (13) are

$$\int_0^1 \left[a - 2bX_N^*(\cdot) - \beta X_N^*(\cdot) - 2\theta\right] \frac{\partial X_N^*(\cdot)}{\partial N} d\theta = -\frac{a^2 b\left(N-1\right)}{\left(b+N\beta+Nb\right)^3} < 0,$$

and

$$\frac{\partial}{\partial N}\int_0^1 \theta \frac{2\left(N-1\right)bX_N^*(\cdot)}{b(N+1)+\beta N+b\left(N-1\right)} dF(\theta).$$

This second expression is strictly positive because $\partial X_N^*(\cdot)/\partial N = (ab)/\left(b+N\beta+Nb\right)^2 > 0$ and

$$\frac{\partial}{\partial N}\left(\frac{2\left(N-1\right)b}{b(N+1)+\beta N+b\left(N-1\right)}\right) = \frac{2b}{N^2\left(2b+\beta\right)} > 0.$$

Proof of Proposition 3. We first divide both the numerator and the denominator of the equilibrium profit (21) by b^2. Doing so and denoting $\beta/b \equiv \psi$ yields

$$\pi^*(N) = \frac{a^2(2N+\psi N^2)}{2b\left((N+1)+\psi N\right)^2} + \frac{\left(2N+\psi N^2\right)(1+\lambda)^2}{2b\left(1+2\lambda\right)\left((N+1)+\psi N+\lambda\left(N-1\right)\right)^2} + \\ - \frac{a\left(2N+\psi N^2\right)}{b\left((N+1)+\psi N\right)\left((N+1)+\psi N+\lambda\left(N-1\right)\right)}. \quad (22)$$

Hence,

$$\pi^*(1) = \frac{1}{b(2+\psi)}\left(\frac{a^2}{2} - a + \frac{(1+\lambda)^2}{2(2\lambda+1)}\right),$$

and

$$\pi^*(2) = \frac{2\left(1+\psi\right)}{b}\left(\frac{a^2}{(3+2\psi)^2} + \frac{(1+\lambda)^2}{(3+2\psi+\lambda)^2(2\lambda+1)} - \frac{2a}{(3+\psi)(3+2\psi+\lambda)}\right).$$

The sign of the difference $\pi^*(2) - \pi^*(1)$ is a polynomial function of third order in ψ, with a leading term of

$$16\lambda\left(a + 2\lambda(a-1) - 1 - \lambda^2\right). \quad (23)$$

Since $a > 1+\lambda$ by assumption, and (23) is equal to $16\lambda^2(1+\lambda) > 0$ for $a = 1+\lambda$ and is increasing in a, the leading term is positive for all a in the admissible range. Moreover, there is no bound on ψ. It follows that for any combination of a and λ, the difference $\pi^*(2) - \pi^*(1)$ is positive if ψ large enough.

We will now show that the difference $\pi^*(2) - \pi^*(1)$ is either positive for all $\psi \geq 0$ or that there exists a unique threshold, denoted by $\hat{\psi}$, such that the difference is positive if and only if $\psi > \hat{\psi}$.

Let us first consider the case $\psi = 0$. The sign of the difference $\pi^*(2) - \pi^*(1)$ is then given by the sign of

$$a(3+\lambda)(2\lambda+1)(a-1)(6+18\lambda-3a-a\lambda) - 9(1+\lambda)^2(1+6\lambda+\lambda^2).$$

Setting this term equal to zero and solving for a yields two solutions,

$$a_1 = \frac{3\left(1+5\lambda+6\lambda^2 - \sqrt{\lambda^2(1+2\lambda)(7+10\lambda-\lambda^2)}\right)}{(3+\lambda)(1+2\lambda)}$$

and

$$a_2 = \frac{3\left(1+5\lambda+6\lambda^2 + \sqrt{\lambda^2(1+2\lambda)(7+10\lambda-\lambda^2)}\right)}{(3+\lambda)(1+2\lambda)}.$$

For all $a \in [a_1, a_2]$, $\pi^*(2) \geq \pi^*(1)$. Since $a_1 < 1+\lambda$ and $a_2 > 1+\lambda$ for all $\lambda > 0$, the profit with two retailers is larger than with one retailer if $a \in (1+\lambda, a_2)$. We denote $\hat{a} \triangleq a_2$.

We now show that, if $a \in (1+\lambda, \hat{a})$, the same result holds for any $\psi > 0$. The derivative of $\pi^*(2) - \pi^*(1)$ with respect to ψ, evaluated at $\psi = 0$, is

$$-4(3+\lambda)^2(1+2\lambda)a^2 + 8(1+2\lambda)(3+14\lambda+3\lambda^2)a - 12(1+\lambda)^2(1+9\lambda+\lambda^2). \tag{24}$$

It is easy to check that (24) is positive for all $a \in [1+\lambda, \hat{a})$. Moreover, setting the derivative of $\pi^*(2) - \pi^*(1)$ with respect to ψ equal to 0, we obtain that the lower one of the two solutions is

$$\frac{(1+2\lambda)a\left(a-2(1+9\lambda+\lambda^2)\right) + (1+\lambda)^2(1+18\lambda+\lambda^2) - \sqrt{\xi}}{12\left(a+2\lambda(a-1)-1-\lambda^2\right)}, \tag{25}$$

with

$$\begin{aligned}\xi \equiv\ & (1+2\lambda)^2a^3\left[a+4(2\lambda^2-1)\right] + (1+2\lambda)^2(1+\lambda)^4 \\ & + 2(1+2\lambda)a\left[a(3+6\lambda(1-\lambda)-14\lambda^3-3\lambda^4+4\lambda^5) - 2(1-\lambda^2+\lambda^4)(1+\lambda)^2\right].\end{aligned}$$

Tedious but routine manipulations yield that (25) is negative for all admissible values of a and

λ. Hence, since $\pi^*(2) - \pi^*(1)$ is a polynomial of third order with a positive leading term, the unique local maximum of this difference occurs at a negative value of ψ. As the difference is positive at $\psi = 0$ for $a \in (1 + \lambda, \hat{a})$ and the derivative of the difference at is also positive at $\psi = 0$, $\pi^*(2) - \pi^*(1)$ is positive for $\psi > 0$.

We now turn to the range $a \geq \hat{a}$, where the difference between $\pi^*(2)$ and $\pi^*(1)$ is negative at $\psi = 0$. As the leading term of the third-order polynomial is positive and there is no bound on ψ, the difference is positive for ψ large enough. Using again the fact that the difference has a unique local maximum at $\psi < 0$, it follows that there must be a unique solution in the region $\psi > 0$, denoted by $\hat{\psi}$, such that the difference is positive if and only if $\psi > \hat{\psi}$. ■

Proof of Proposition 4. Setting $\lambda = 1$ in the profit function (22) and taking the derivative with respect to N, we obtain the first-order condition

$$a - \frac{3a^2N^2(2+\psi)^2(N-1) + 4(N+1+\psi N)^3}{3N^2(2+\psi)^2(N+1+\psi N)^3} = 0. \tag{26}$$

This condition cannot be solved explicitly for N. However, solving (26) for a yields two roots and the only one consistent with Assumption 1 (which requires that $a \geq 2$) is

$$a = \frac{3N(2+\psi) + \sqrt{3}\sqrt{(4+2N+3\psi N)(4+\psi N-2N)}}{6N(N-1)(2+\psi)}. \tag{27}$$

To determine whether the profit function is concave in N, we differentiate (26) with respect to N and substitute a from (27) to obtain

$$-\frac{3\psi^3N^3 + 2\psi^2N^2(4+5N) + 4\psi N(12N-4-N^2) + 8(4N+2N^2-2-N^3)}{6N^3(N-1)^2(2+\psi)^2(N+1+\psi N)^2}$$
$$-\frac{N^2(2+\psi)^2\sqrt{3}\sqrt{(4+2N+3\psi N)(4+\psi N-2N)}}{6N^3(N-1)^2(2+\psi)^2(N+1+\psi N)^2}. \tag{28}$$

Since the denominator of both terms is the same and strictly positive, the sign of (28) is determined by the sign of the numerator of each fraction of (28). As there is a minus sign in front of each fraction, this implies that the numerator of (28) is strictly decreasing in ψ. Inserting $\psi = 0$ into (28) yields

$$-\frac{2(2N-1) + N^2(2-N) + N^2\sqrt{3}\sqrt{(2+N)(2-N)}}{3N^3(N+1)^2(N-1)^2},$$

which is strictly negative due to the fact that, at $\psi = 0$, the optimal N must be lower than or equal to 2 (because otherwise a given by (27) is not a real number and the first-order condition cannot be fulfilled). It follows that the second derivative of $\pi^*(N)$ with respect to N is negative at any N satisfying the first-order condition. Since we know from Theorem 1 that

$\pi^*(N)$ is increasing in N at $N = 1$ but decreasing as N gets large, $\pi^*(N)$ must be globally concave for all $N \geq 1$.

We can now apply the Implicit-Function Theorem to determine how N^* changes in ψ and a. Differentiating (26) with respect to ψ and using (27), we obtain

$$\frac{dN^*}{d\psi} = -\frac{\frac{\partial^2 \pi^*}{\partial \psi \partial N}}{\frac{\partial^2 \pi^*}{\partial N^2}} = \frac{N(N-1)}{2+\psi} > 0.$$

Since $\psi = \beta/b$, the optimal number of retailers is increasing in β and decreasing in b. Following the same procedure for the derivative with respect to a, we obtain

$$\text{sign}\left\{\frac{dN^*}{da}\right\} = \text{sign}\left\{-\frac{\sqrt{(4+2N+3\psi N)(4+\psi N-2N)}}{\sqrt{3}N(2+\psi)(N+1+\psi N)^2}\right\} < 0.$$

Hence, the optimal number of retailers is decreasing in a. ■

Price Competition. Assume that M offers to R_i a menu of two-part tariffs

$$\{T_i(m_i), w_i(m_i)\}_{m_i \in \Theta},$$

where $w_i(m_i)$ is the wholesale price paid by R_i to M for every unit of the product and $T_i(m_i)$ is the fixed fee, both functions of R_i's report. Consider a symmetric equilibrium where $w_i^*(\theta) = w_N^*(\theta)$ for every i and θ, so that every retailer charges $p_N^*(\theta)$ in equilibrium. Then, let

$$u_i(m_i, \theta, p_i, w_i(m_i)) \triangleq D(p_i, (N-1)p_N^*(\theta))(p_i - w_i(m_i) - \theta) - T_i(m_i),$$

and

$$u_i(m_i, \theta) \triangleq \max_{p_i \geq 0} u_i(m_i, \theta, p_i, w_i(m_i)),$$

with $u_i(\theta) \triangleq u_i(m_i = \theta, \theta)$. Letting

$$p_i(\theta) \triangleq \arg\max_{p_i \geq 0} u_i(m_i = \theta, \theta, p_i, w_i(m_i = \theta)),$$

by the Envelope Theorem

$$\begin{aligned}\dot{u}_i(\theta) = &-D(p_i(\theta), (N-1)p^*(\theta)) \\ &+ (N-1)D_{-i}(p_i(\theta), (N-1)p_N^*(\theta))(p_i(\theta) - w_i(\theta) - \theta)\dot{p}_N^*(\theta),\end{aligned}$$

which, after integrating both sides, yields an expression similar to the one in (14). It follows that the competing-contracts effects occurs in this case as well.

Imperfect Correlation: the Linear Example. Consider the linear-quadratic framework developed in Section 5. Assume that $\lambda = 1$ and focus on a symmetric equilibrium in which every retailer i produces $x_N^*(\theta_i)$. Following the same techniques developed above, the bilateral contracting problem between M and R_i is

$$\max_{x_i(\theta)\geq 0} \Bigg\{ \int_{\theta_i} \left\{ \nu P\left(x_i(\theta_i) + (N-1)\, x_N^*(\theta_i)\right) + (1-\nu)\, \mathbb{E}_{\theta_{-i}}\left[P\left(x_i(\theta_i) + X_{N,-i}^*(\theta_{-i})\right)\right] - \theta_i x_i(\theta_i)\right\} dF(\theta_i)$$
$$- \int_{\theta_i} h(\theta_i)\left[1 - \nu(N-1)P'\left(x_i(\theta_i) + (N-1)x_N^*(\theta)\right)\dot{x}_N^*(\theta)\right] dF(\theta_i)$$
$$- \int_{\theta_i} \left[\nu c\left(x_i(\theta_i) + (N-1)\, x_N^*(\theta_i)\right) + (1-\nu)\, \mathbb{E}_{\theta_{-i}}\left[c\left(x_i(\theta_i) + X_{N,-i}^*(\theta_{-i})\right)\right]\right] dF(\theta_i) \Bigg\}.$$

Differentiating with respect to $x_i(\cdot)$, and substituting the linear specification, yields the following differential equation:

$$\dot{x}_N^*(\theta) = \frac{a - \theta - (1-\nu)(N-1)(b+\beta)\,\hat{x}_N - x_N^*(\theta)\left(b + (b+\beta)(1+\nu(N-1))\right) - \theta}{\theta b \nu (N-1)}, \quad (29)$$

with $\hat{x}_N \triangleq \mathbb{E}\left[x_N^*(\theta)\right]$ and boundary condition

$$x_N^*(0) = \frac{a - (1-\nu)(N-1)(b+\beta)\,\hat{x}_N}{b + (b+\beta)(1+\nu(N-1))}.$$

Equation (29) can be rewritten as

$$\dot{x}_N^*(\theta) + x_N^*(\theta)\frac{\Phi}{\theta b \nu (N-1)} = \frac{a - \bar{a}}{\theta b \nu (N-1)} - \frac{2}{b\nu(N-1)},$$

where

$$\Phi \triangleq 2b + b\nu(N-1) + \nu\beta N + (1-\nu)\beta$$

and

$$\bar{a} \triangleq (1-\nu)(N-1)(b+\beta)\hat{x}_N.$$

The solution is

$$x_N^*(\theta) = k e^{-\int_0^\theta \frac{\Phi}{z_1 b\nu(N-1)} dz_1} + \int_0^\theta e^{-\int_{z_2}^\theta \frac{\Phi}{z_1 b\nu(N-1)} dz_1}\left(\frac{a - \bar{a}}{z_2 \nu b (N-1)} - \frac{2}{b\nu(N-1)}\right) dz_2.$$

It can be easily seen that, as before, the constant k must be equal to zero because $\Phi > 0$.

Hence,

$$\begin{aligned} x_N^*(\theta) &= \frac{\theta^{-\frac{\Phi}{\nu b(N-1)}}}{\nu b(N-1)} \int_0^\theta \theta_2^{\frac{\Phi}{b\nu(N-1)}} \left(\frac{a-\bar{a}}{\theta_2} - 2\right) d\theta_2 \\ &= \frac{\theta^{-\frac{\Phi}{\nu b(N-1)}}}{\nu b(N-1)} \int_0^\theta \left(\theta_2^{\frac{\Phi}{b\nu(N-1)}-1}(a-\bar{a}) - 2\theta_2^{\frac{\Phi}{b\nu(N-1)}}\right) d\theta_2 \\ &= \frac{\theta^{-\frac{\Phi}{\nu b(N-1)}}}{\nu b(N-1)} \left| \frac{(a-\bar{a})\, b\nu(N-1)}{\Phi} \theta_2^{\frac{\Phi}{b\nu(N-1)}} - \frac{2}{\frac{\Phi}{b\nu(N-1)}+1} \theta_2^{\frac{\Phi}{b\nu(N-1)}+1} \right|_0^\theta d\theta_2 \\ &= \frac{a-\bar{a}}{\Phi} - \frac{2\theta}{\Phi + b\nu(N-1)}. \end{aligned}$$

Integrating yields

$$\begin{aligned} \int_0^1 x_N^*(\theta)\, d\theta \triangleq \hat{x}_N &= \frac{a-(1-\nu)(N-1)(b+\beta)\,\hat{x}_N}{\Phi} - \frac{1}{\Phi + b\nu(N-1)} \\ \Leftrightarrow \quad \hat{x}_N &= \frac{a}{b+N(b+\beta)} - \frac{b+(b+\beta)(1+\nu(N-1))}{(b+N(b+\beta))(2b+\beta)(1+\nu(N-1))}. \end{aligned}$$

Hence,

$$x_N^*(\theta) = \frac{a}{b+(b+\beta)(1+\nu(N-1))} - \underbrace{\frac{(1-\nu)(N-1)(b+\beta)}{b+(b+\beta)(1+\nu(N-1))}\hat{x}_N}_{\text{Bayes-Nash component}} - \frac{2\theta}{(2b+\beta)(1+\nu(N-1))}.$$

It can be shown that for $\nu \to 1$ the solution converges to (12). Notice that, compared to the baseline model, with imperfectly correlated types there is a new term in the expression of the equilibrium quantity. This term reduces retailers' individual quantity since it captures the uncertainty about the rivals' types—i.e., the extent to which costs are independently distributed.

The expected profit $\pi_\nu^*(N)$ can be obtained by rearranging M's first-order condition with respect to $x_i(\theta)$ and aggregating over the number of retailers:

$$\begin{aligned} \pi_\nu^*(N) \triangleq N \int_0^1 x_N^*(\theta)\left((b+\nu\beta N)\, x_N^*(\theta) + \beta(1-\nu)\left(x_N^*(\theta) + (N-1)\,\hat{x}_N\right)\right) d\theta + \\ - \frac{\nu\beta}{2} N^2 \int_0^1 x_N^*(\theta)^2\, d\theta - \frac{(1-\nu)\beta}{2} \mathbb{E}\left[\left(\sum\nolimits_{i=1}^N x_N^*(\theta_i)\right)^2\right]. \end{aligned}$$

Since with probability $(1-\nu)$ types are i.i.d., it follows that

$$\frac{(1-\nu)\beta}{2}\mathbb{E}\left[\left(\sum_{i=1}^{N} x_N^*(\theta_i)\right)^2\right] = \frac{(1-\nu)\beta}{2}\mathbb{E}\left[\sum_{i=1}^{N}\sum_{j=1}^{N} x_N^*(\theta_i)\, x_N^*(\theta_j)\right] = \frac{(1-\nu)\beta}{2}N^2\hat{x}_N^2.$$

We compare $\pi_\nu^*(2)$ and $\pi_\nu^*(1)$ numerically. For example, if $a=5$ and $b=1$, then

$$\pi_\nu^*(2)-\pi_\nu^*(1) = \frac{4(\nu+2)(1-\nu)^2\beta^3 + 4(11\nu+22\nu^2+3\nu^3+8)\beta^2}{6(2+\beta)^2(3+2\beta)^2(1+\nu)^2} + \\ + \frac{3(39\nu+76\nu^2+3\nu^3-2)\beta+18\nu(5\nu-2)-78}{6(2+\beta)^2(3+2\beta)^2(1+\nu)^2},$$

It is then easy to check that M prefers a duopolistic market structure compared to a monopolistic one when β and ν are sufficiently large. For example, $\pi_\nu^*(2)\geq \pi_\nu^*(1)$ if and only: *(i)* $\beta=0.5$ and $\nu\geq 0.49$; *(ii)* $\beta=1$ and $\nu\geq 0.21$; *(iii)* $\beta=1.5$ and $\nu\geq 0$.

Proof of Proposition 5. Let

$$\Phi(z) \triangleq [1-(N-1)P'(X_N^*(z))\,\dot{x}_N^*(z)]\,x_N^*(z).$$

Substituting $T_N^*(\theta)$ into (15), we obtain

$$\int_\theta^{\theta'}(N-1)P'(x_N^*(\theta)+(N-1)x_N^*(z))\,\dot{x}_N^*(z)\,x_N^*(\theta)\,dz + \int_\theta^{\bar{\theta}}\Phi(z)\,dz \geq x_N^*(\theta')(\theta'-\theta)+\int_{\theta'}^{\bar{\theta}}\Phi(z)\,dz$$
$$\Leftrightarrow \quad \int_\theta^{\theta'}(N-1)P'(x_N^*(\theta)+(N-1)x_N^*(z))\,\dot{x}_N^*(z)\,x_N^*(\theta)\,dz + \int_\theta^{\theta'}\Phi(z)\,dz - x_N^*(\theta')(\theta'-\theta)\geq 0.$$

Using the definition of $\Phi(z)$ we can rewrite this as

$$(N-1)\int_\theta^{\theta'}\dot{x}_N^*(z)\int_\theta^{z}\dot{x}_N^*(y)\left[P'(x_N^*(y)+(N-1)x_N^*(z))+x_N^*(y)P''(x_N^*(y)+(N-1)x_N^*(z))\right]dydz$$
$$\leq -\int_\theta^{\theta'}\int_z^{\theta'}\dot{x}_N^*(y)\,dydz.$$

This inequality is strictly satisfied for all $\theta'>\theta$, since $\dot{x}_N^*(\cdot)<0$ and $P'(\cdot)+xP''(\cdot)<0$ by Assumption 2.

We now turn to the case $\theta'<\theta$. Retailers never want to coordinate on a symmetric equilibrium in which they report θ' lower than the true cost θ if

$$[P(x_N^*(\theta')+(N-1)x_N^*(\theta'))-\theta]\,x_N^*(\theta')-T_N^*(\theta')\geq[P(Nx_N^*(\theta))-\theta]\,x_N^*(\theta)-T_N^*(\theta). \quad (30)$$

Suppose that (30) holds for some $\theta' < \theta$. Substituting $T_N^* (\theta)$ on both sides and rearranging, we obtain

$$\int_{\theta}^{\theta'} (x_N^* (\theta') - x_N^* (z)) \, dz + (N-1) \int_{\theta}^{\theta'} P' (N x_N^* (z)) \, \dot{x}_N^* (z) \, x_N^* (z) \, dz \geq 0,$$

which cannot hold for any $\theta' < \theta$ since $\dot{x}_N^* (\cdot) < 0$ implies that $x_N^* (\theta') > x_N^* (z)$ for every $z > \theta'$ and $P' (\cdot) \, \dot{x}_N^* (\cdot) > 0$. ■

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