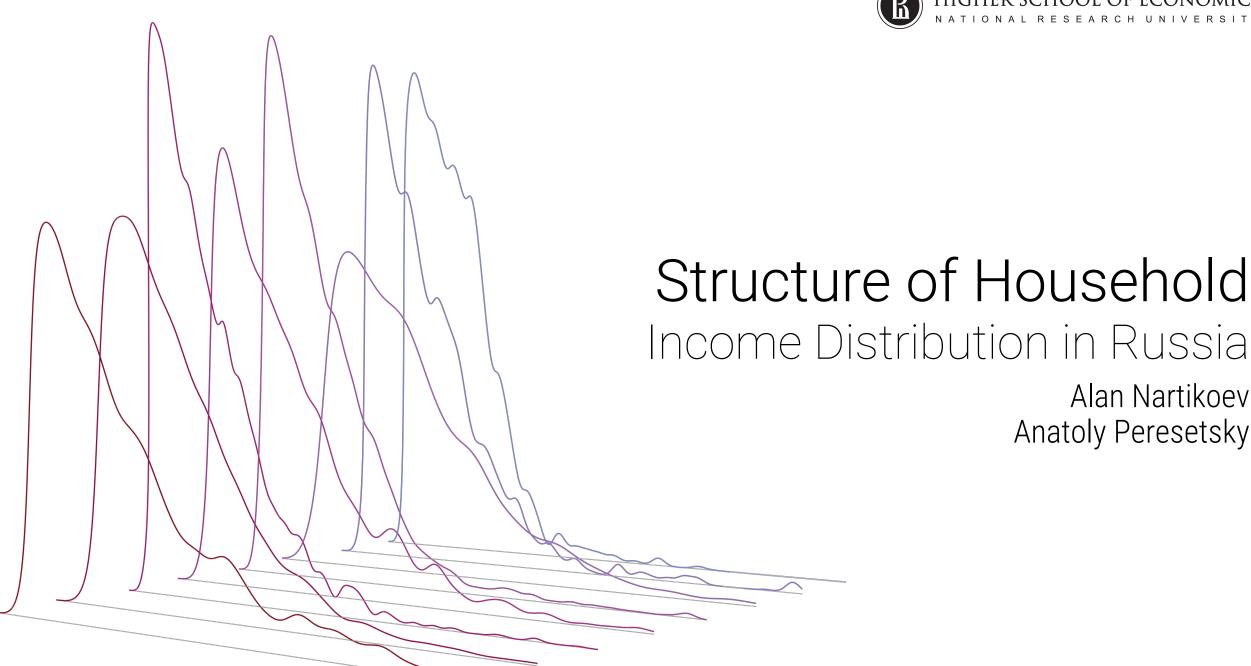


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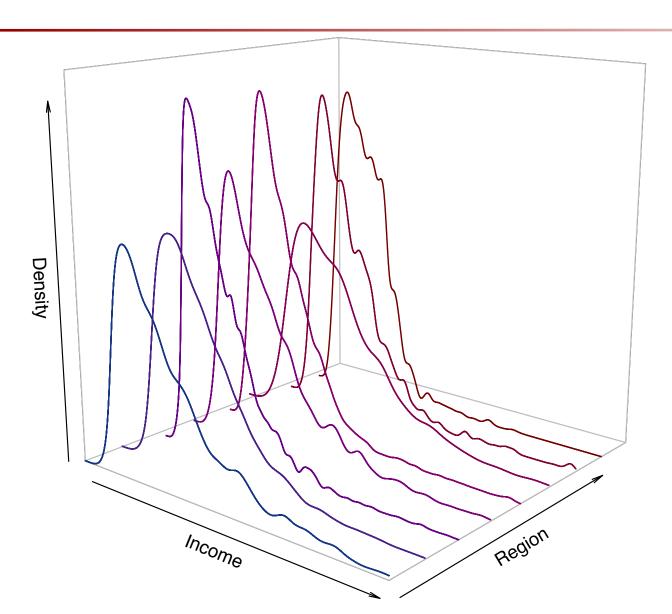
Premises

- o Income distribution is heterogeneous: it inevitably includes different groups of population;
- o It is possible to reveal relatively homogeneous groups inside the overall distribution;
- Group membership is determined by individual's characteristics;
- Income distribution also exhibits spatial heterogeneity across different regions;

Data

- Statistical Survey of Income and Participation in Social Programs 2017 (Federal State Statistics Service)
- o 160 thousand observed households across Russia;
- Data is weighted with an inverse probability of sample inclusion;
- OECD equivalence scale is applied to household disposable income

Kernel Density Estimation



Descriptive Statistics

	n obs.	Min.	1 st Quartile	Median	Mean	3 rd Quartile	Max.
Central	$40,\!560$	820	185,400	271,600	365,000	459,800	4,688,000
Northwestern	17,448	2,000	$215{,}600$	311,600	367,600	$459,\!800$	3,110,000
Volga	$31,\!536$	780	155,900	210,300	238,000	287,800	2,677,000
Ural	$13,\!152$	1,200	$169,\!200$	240,900	300,100	363,400	3,931,000
Siberian	21,936	1,150	151,600	213,100	250,300	$304,\!200$	5,513,000
Far Eastern	10,200	800	$215,\!600$	322,700	397,300	$502,\!400$	$3,\!500,\!000$
Southern	$16,\!584$	1,800	151,700	$206,\!500$	$235{,}100$	284,600	8,655,000
North Caucasian	8,592	521,700	116,300	$169,\!200$	188,600	$234,\!200$	3,560,000

Model Description (Flachaire and Nuñez, 2007)

Finite mixture of distributions (generalized linear model):

$$f(y; \boldsymbol{\beta}, \boldsymbol{\theta}) = \sum_{k=1}^{K} \pi_k(x; \boldsymbol{\beta}_k) f_k(y; \boldsymbol{\theta}_k)$$

Individual's i probability of belonging to group k, $\pi_{ik}(\cdot)$, is a concomitant variable, i.e.

$$\pi_{ik} = F_k(\boldsymbol{x}_i; \boldsymbol{\beta}_k),$$

Where $F_k(x_i; \beta_k)$ is a distribution function depending on the parameter vector β_k , such that

$$\sum_{k=1}^{K} F_k(\boldsymbol{x}_i) = 1$$

Model Description (Flachaire and Nuñez, 2007)

We choose k=3 and f_k being LogNormal, i.e.

$$f(y; \boldsymbol{\beta}, \boldsymbol{\theta}) = \sum_{k=1}^{3} \pi_k(x; \boldsymbol{\beta}_k) \frac{1}{y\sigma_k \sqrt{2\pi}} \exp\left(\frac{-(\ln y - \mu_k)^2}{2\sigma_k^2}\right),$$

where parameter vector θ is given by $(\mu_1, \mu_2, \mu_3, \sigma_1, \sigma_2, \sigma_3)'$. We also choose π_k to be modelled via multinomial logistic regression:

$$\pi_{ik} = \pi_k(\boldsymbol{x_i}; oldsymbol{eta}) = rac{e^{oldsymbol{x_i}^{\mathsf{T}} oldsymbol{eta}_k}}{\sum_{l=1}^K e^{oldsymbol{x_i}^{\mathsf{T}} oldsymbol{eta}_l}},$$

Estimation via EM-algorithm

j-th E-step:

$$\hat{p}_{ik}^{(j)} = \frac{\pi_k(\boldsymbol{x}_i; \boldsymbol{\beta}_k^{(j)}) f(y_i; \boldsymbol{\theta}_k^{(j)})}{\sum_{l=1}^K \pi_l(\boldsymbol{x}_i; \boldsymbol{\beta}_l^{(j)}) f(y_i; \boldsymbol{\theta}_l^{(j)})},$$

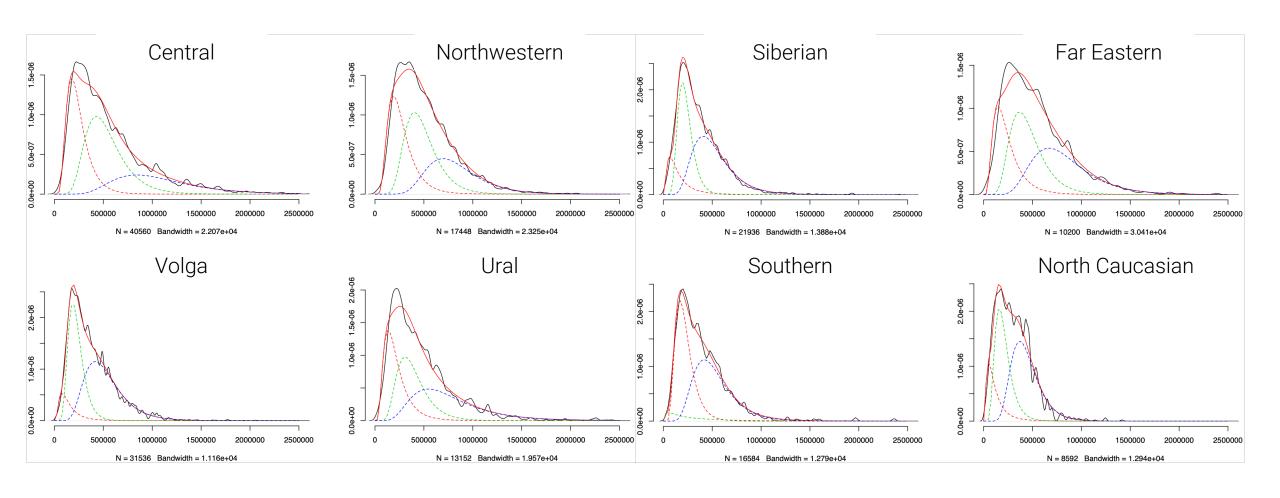
j-th M-step:

$$\hat{\boldsymbol{\Psi}}^{(j+1)} = (\hat{\boldsymbol{\beta}}^{(j+1)}, \hat{\boldsymbol{\theta}}^{(j+1)})^{\intercal} = \argmax_{\boldsymbol{\beta}, \boldsymbol{\theta}} Q(\boldsymbol{\beta}^{(j+1)}, \boldsymbol{\theta}^{(j+1)}; \boldsymbol{x}; \boldsymbol{y}),$$

where

$$Q(\boldsymbol{\beta}^{(j+1)}, \boldsymbol{\theta}^{(j+1)}; \boldsymbol{x}; \boldsymbol{y}) = \sum_{i=1}^{n} \sum_{k=1}^{K} \hat{p}_{ik}^{(j)} \ln(\pi_k(\boldsymbol{\beta}_k^{(j+1)}; x_i)) + \sum_{i=1}^{n} \sum_{k=1}^{K} \hat{p}_{ik}^{(j)} \ln(f(\boldsymbol{\theta}_k^{(j+1)}; y_i))$$

Distribution Estimates



Distribution Estimates

	Central		Northwestern		Volga		Ural	
	II	III	II	III	II	III	II	III
(Intercept)	-3.469	-11.103	-2.875	-7.627	3.088	-0.004	-1.838	-5.772
edu = sec.	-1.929	-4.173	-1.642	-3.397	-1.150	-2.937	-2.029	-5.595
edu = prim.	-3.255	-5.863	-3.683	-6.076	-1.331	-4.044	-3.151	-7.887
loc = med.	0.547	0.376	0.457	0.096	0.631	0.874	0.380	0.499
loc = big	2.531	6.107	1.977	3.913	1.082	2.059	1.071	1.263
loc = rural	-1.030	-2.400	-1.246	-2.683	-1.168	-1.322	-1.756	-4.161
employed ratio	8.534	15.421	8.200	13.540	0.288	7.257	8.295	15.800
	Siberian		Far Eastern		Southern		North Caucasian	
	II	III	II	III	II	III	II	III
(Intercept)	2.545	-0.367	-2.022	-5.583	-4.007	-2.986	1.457	-2.237
edu = sec.	-0.984	-2.781	-2.454	-4.856	-0.212	-1.728	-0.413	-2.069
edu = prim.	-1.408	-4.029	-4.234	-7.521	0.055	-2.922	-0.871	-4.559
loc = med.	0.576	1.392	0.870	0.136	1.867	0.228	0.906	1.478
loc = big	1.191	1.764	1.100	0.813	-1.254	0.984	-0.279	0.869
loc = rural	-1.260	-2.178	-1.073	-2.959	1.989	-0.426	-1.253	-1.343
employed ratio	0.127	7.350	11.039	17.997	0.982	7.164	4.372	14.042

Conclusion

- o It is possible to distinguish three income classes in the most economically well-off federal districts of Russia;
- o The least well-off districts tend to have more leptokurtic income density curves;
- The least well-off districts do not provide evidence of the existence of statistically significant middle class groups;
- o Higher education, inhabitancy in huge urban areas and higher ratio of employed household members are positive predictors of probability of belonging to a high-income class



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