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Drafting problem in team contests

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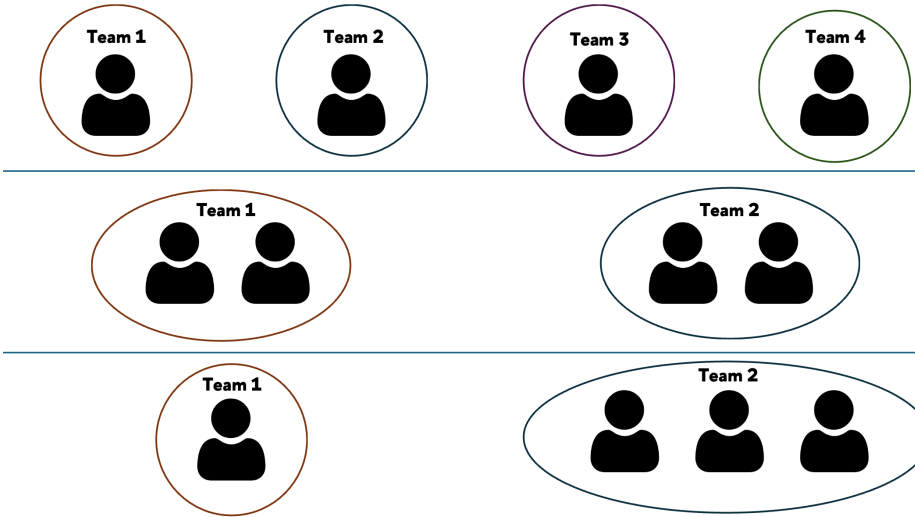
Theory of contests

An extension of game theory, where participants exert costly efforts in order to increase their probabilities of winning some resource.

Most popular applications:

- Bribery and lobbying
- R&D and arm races
- Oligopoly models
- **Group projects**
- Sport competitions

Team formation



Literature review

- **Lazear and Rosen (1981)**: two players competition in conditions of competition between workers
- **Lu and Lu (2020)** task arrangement in competitions between two teams, each with two players
- **Arbatskaya and Konishi (2021)** synchronous contests and asynchronous contests with publicly observable actions
- **Barbieri and Serena (2022)** sorting contests and allocation of a prize budget among there contests

Motivation

The main problem that arises is the existence of a third party — a contest designer — who sets the rules, making the process of team formation **entirely external**.

Definition of Drafting

Form of cooperation that naturally emerges in contests, resulting in forming teams.

Baseline Model: 4 players and 4 teams

- 4 players: I, II, III, IV
- 6 tasks: 1, 2, ..., 6

For each task (t), player i from team j chooses an effort level e_{ti} if the task is assigned to him

Outcome

The outcome is generated by Cobb-Douglas function.

$$f_j = e_{1i}^{a} e_{2i}^{b} e_{3i}^{c} e_{4i}^{d} e_{5i}^{f} e_{6i}^{g}, \tag{1}$$

where a, b, c, d, f, g — return on effort for the corresponding task // task complexity.

Payoff function

Probability

Probability is modeled by Tullock success function (Tullock, 1980).

$$P_j = \frac{f_j}{f_j + f_{-j}}, \tag{2}$$

Expected payoff

Each player i from team j maximize objective function.

$$V_i = P_j \frac{V}{N_j} - \sum_t e_{ti}, \tag{3}$$

where N_j — number of players in team j.

Assumptions

- A grand prize only
- Linear cost function $\sum_t e_{ti}$
- $\Psi = a + b + c + d + f + g < 1$
- $a \geq b \geq c \geq d \geq f \geq g$

Drafting problem - 2

Drafting mechanism

Agents may join another team, resulting in the division of both the prize and the tasks among them.

Configurations

- 1-1-1-1
- 2-2
- 1-3

Results - 1

	I	II	III	IV
1-1-1-1				
Relative effort (e_{it}/ψ_i)	$\frac{3}{16}V$	$\frac{3}{16}V$	$\frac{3}{16}V$	$\frac{3}{16}V$
Expected utility	$\frac{1-\Psi}{16}V$	$\frac{1-\Psi}{16}V$	$\frac{1-\Psi}{16}V$	$\frac{1-\Psi}{16}V$
2-2				
Relative effort (e_{ti}/ψ_i)	$\frac{1}{8}V$	$\frac{1}{8}V$	$\frac{1}{8}V$	$\frac{1}{8}V$
Expected utility	$\frac{4-2\psi_I}{16}V$	$\frac{4-2\psi_{II}}{16}V$	$\frac{4-2\psi_{III}}{16}V$	$\frac{4-2\psi_I V}{16}V$

Results - 2

A comparison of the symmetric configurations 1-1-1-1 and 2-2

The observed reduction in effort can be fully explained by the **competition effect**: it influences the effort choices of contestants through changes in three variables:

1. p_i — the probability of winning.
2. $V_i = V/N$ — the prize share.
3. TE_i — the total effort exerted by a given contestant.

Results - 3

	I	II	III	IV
1-3				
Relative effort (e_{ti}/ψ_i)	$\frac{3^{\Psi}}{(3^{\Psi}+1)^2}V$	$\frac{3^{\Psi-1}}{(3^{\Psi}+1)^2}V$	...	...
Expected utility	$\frac{3^{\Psi}(3^{\Psi}+\Psi-1)}{(3^{\Psi}+1)^2}V$	$\frac{3^{\Psi-1}(3^{\Psi}-\psi_{II})+1}{(3^{\Psi}+1)^2}V$	...	...

1-1-1-1		2-2	
Relative effort (e_{ti}/ψ_i)	Expected utility	Relative effort (e_{ti}/ψ_i)	Expected utility
$\frac{3}{16}V$	$\frac{1-\Psi}{16}V$	$\frac{1}{8}V$	$\frac{4-2\psi_i}{16}V$

Conclusion

- It is more profitable for players to form large teams to reduce competition
- Grouping into large teams reduces player effort
- The desire to change the team depends on the change in the task complexity

Future reseach

- fix task allocation procedure
- work on other formation (3-1-1, N players)

All questions and comments are welcome!

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