Diffusion and contangion on social networks

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Plan

- Social networks and relevant properties
- Network formation
- Diffusion on networks

Social Networks

- Network - nodes and edges between them
- They can be random or strategically formed
- Can be directed or undirected
- In context of disease spread either describe the interaction or contagion

Social Networks

- N nodes
- Graph (N, g)

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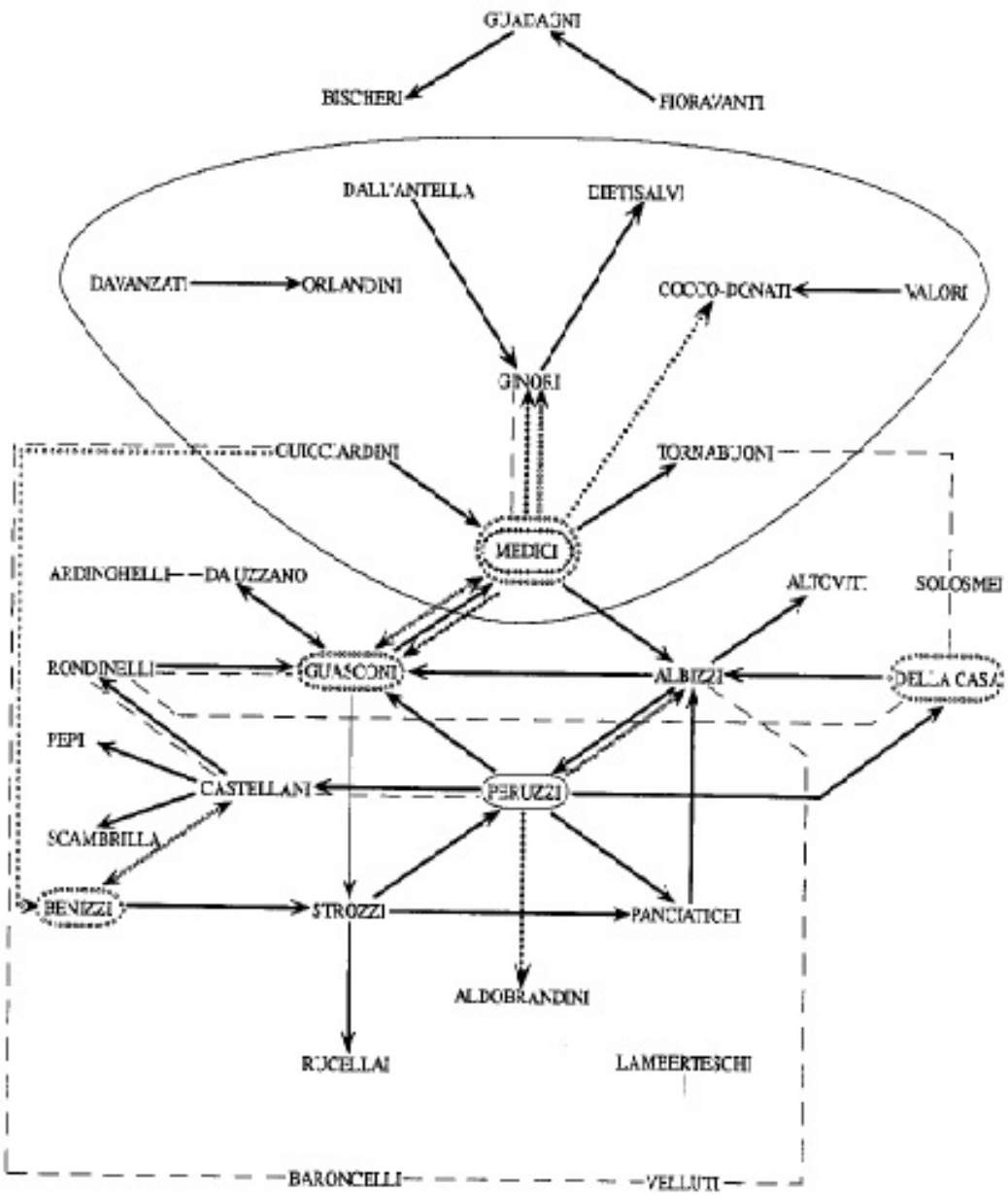
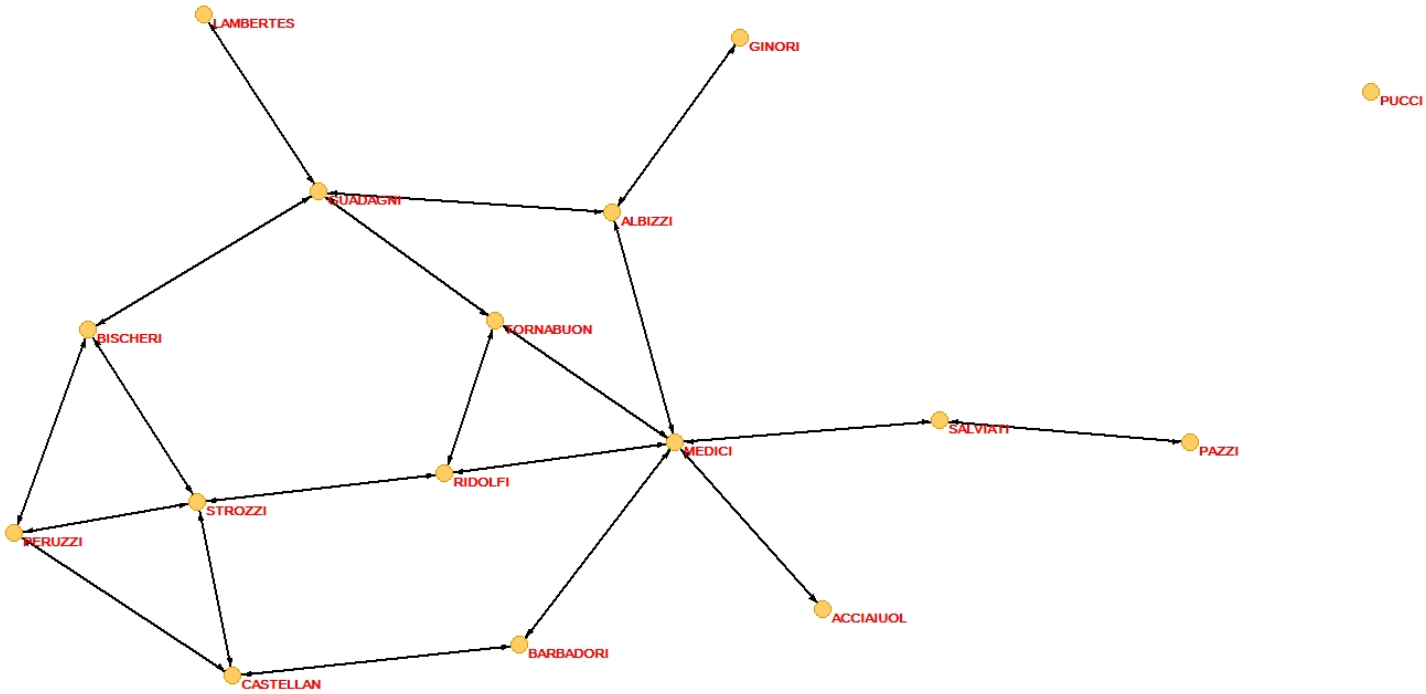
Florentine Families

- Padgett, Ansell 1993
- When Medici came to power
- Construct a network of families

Florentine Families



Florentine Families



- Medici have the similar degree to other families
- Count the shortest path between two families P_{ij}
- How many path goes through the family k
- Take the average - "centrality"
- Medici - 0.45, Guadagni - 0.22

Random Networks

- Erdos, Renyi
- N nodes, each link forms with p
- Binomial model of link formation

Random Networks

- Degree of node - the number of links in a node
- Average degree in the network

Random Networks

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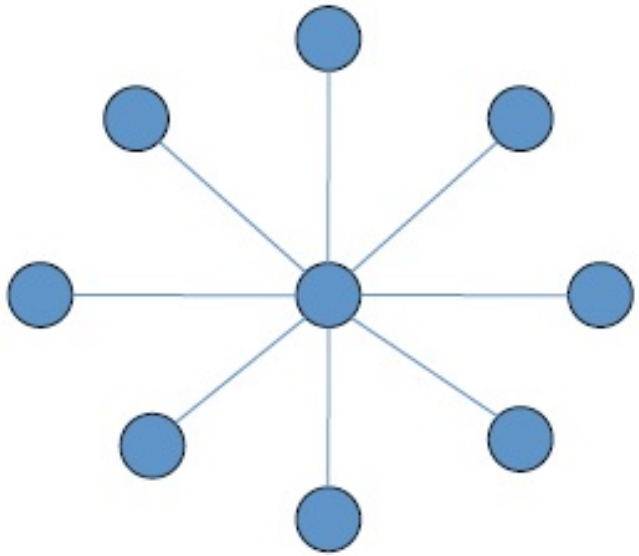
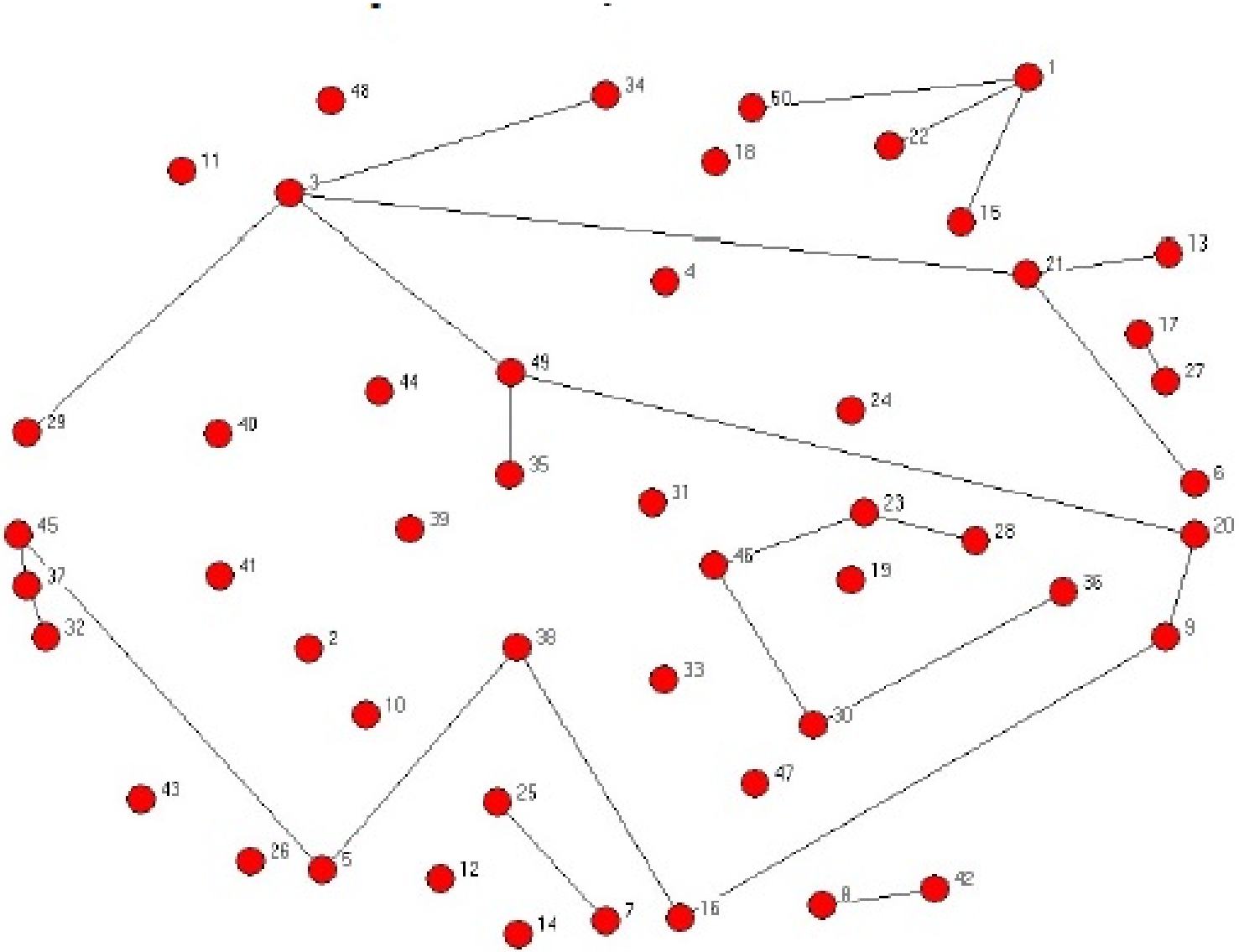


Random Networks

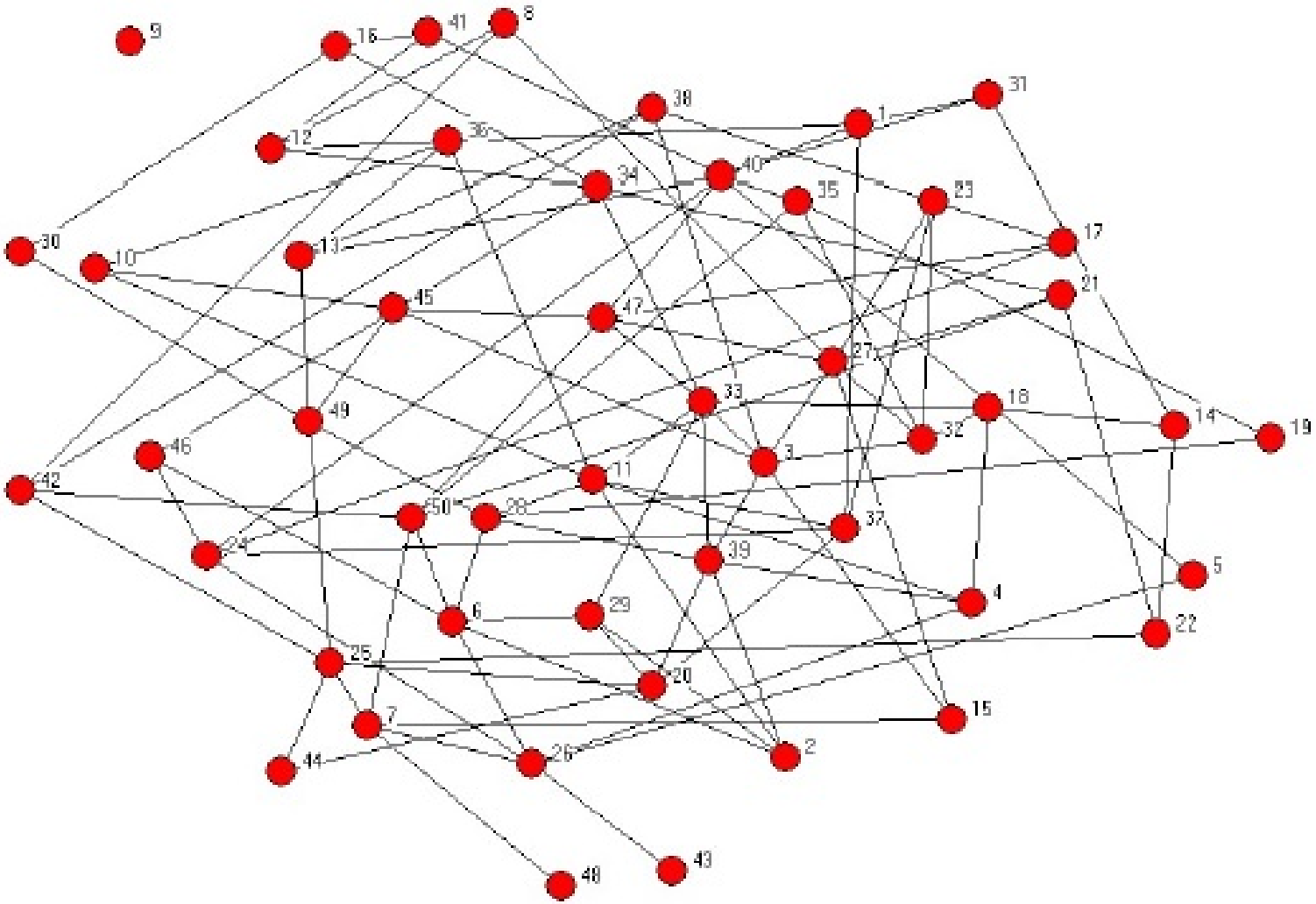
- Degree distribution
- $C_{n-1}^{d} p^{d} (1-p)^{n-1-d}$
- When n becomes large the degree distribution is approximated by Poisson distribution
- $\frac{e^{-(n-1)p}((n-1)p)^{d}}{d!}$

Random Networks



$n = 50$, $p = 0.02$, from Jackson

Random Networks

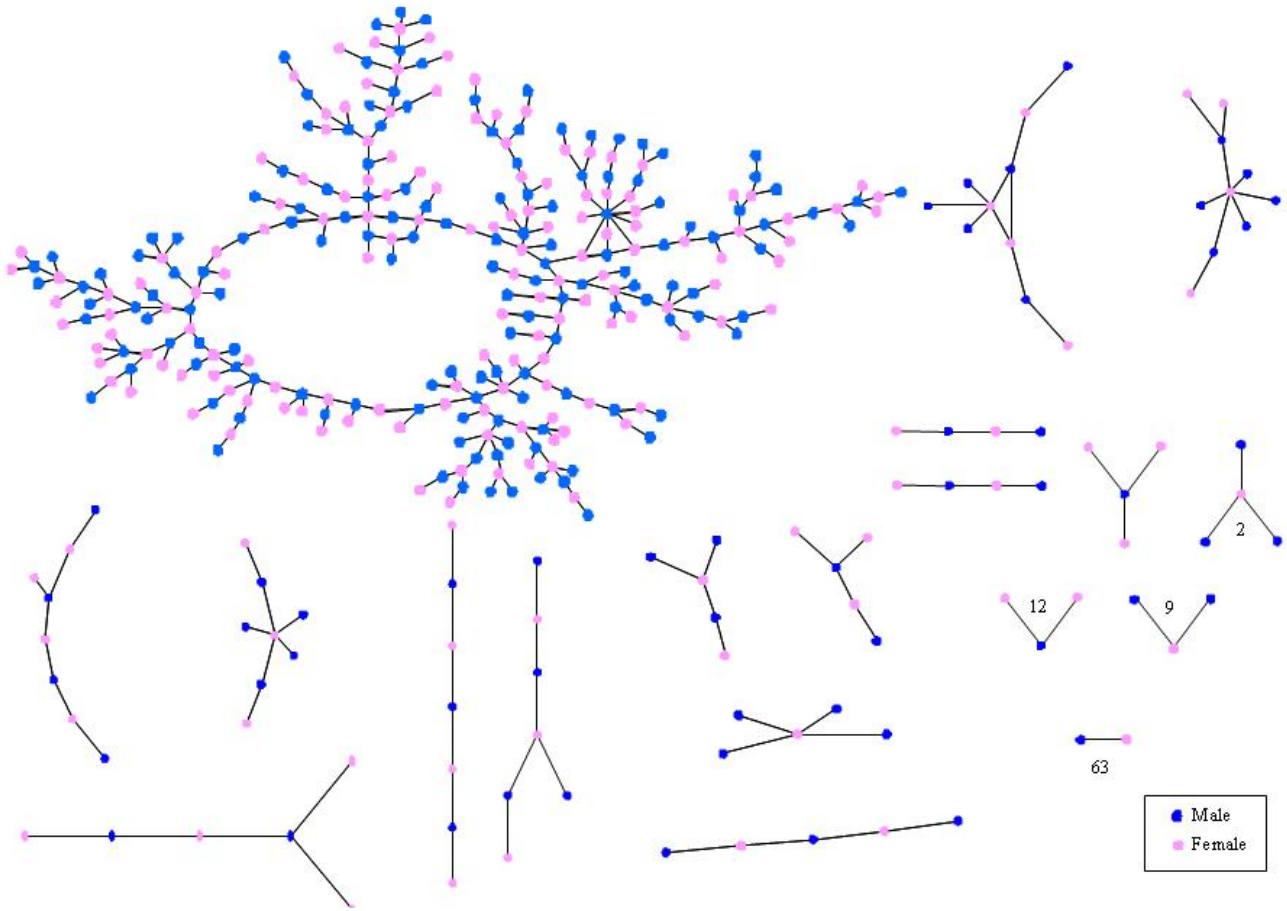


$n = 50$, $p = 0.08$, from Jackson

Social Networks

- Network can be fully connected or consist of separate components
- Giant component - nontrivial fraction of nodes
- If network has one component - it's connected

Social Networks

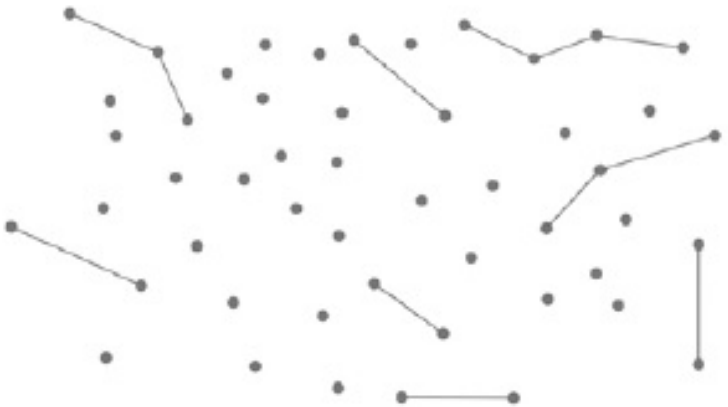


Bearman, Moody, Stovel

Network formation

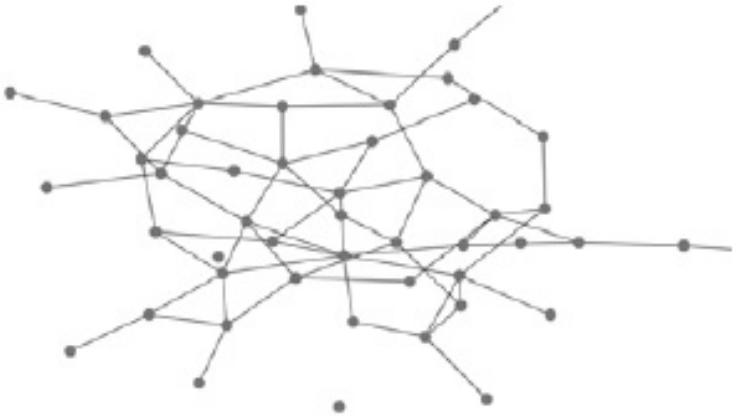
- Basic reproduction number - R0.
- When greater than one - there is a giant component
- Further increase - becomes connected
- Both disease properties and underlying network topology matter
- Vaccination - bring reproduction number below 1.

Social Networks

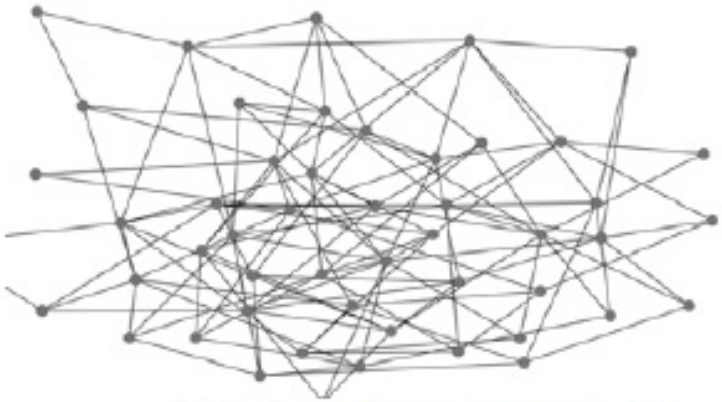


(a) A network with average degree .5.

(b) A network with average degree 1.5.



(c) A network with average degree 2.5.



(d) A network with average degree 5.

from Jackson

Real-world networks

- Well-connected
- Sparse
- Small world - small diameter and average path
- High clustering
- Fat tails of degree distribution

Contagion

- SIR/SI and SIS.
- SIR - how many nodes are infected in the process
- SIS - the prevalence of the infection in long-run

SI/SIR models

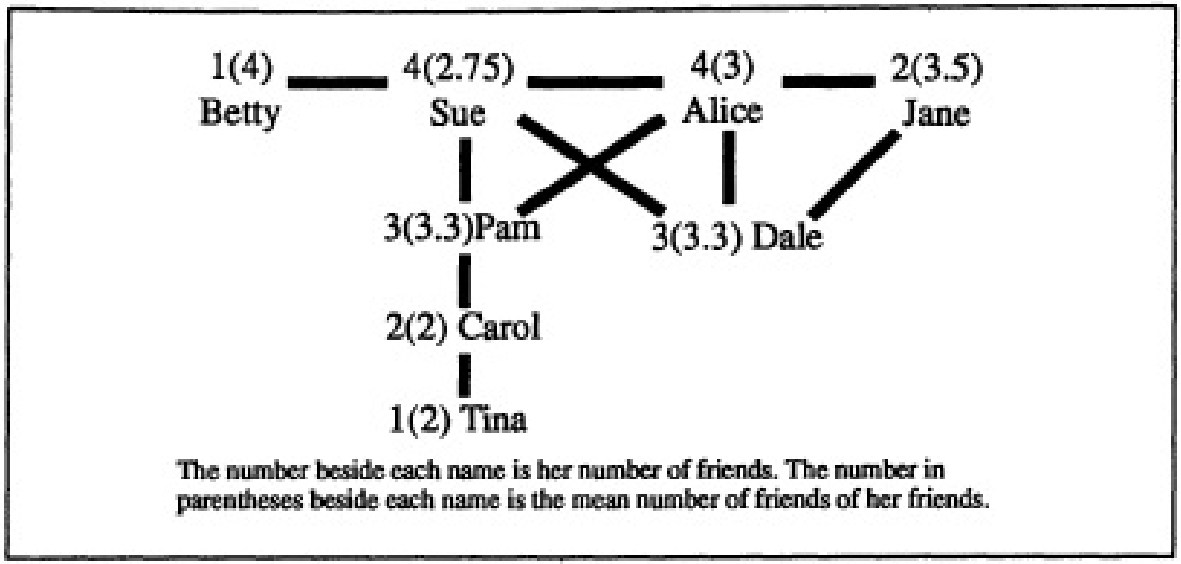
- In every t $S(t)$ may become infected at some rate for each neighbour.
- If $I(0)$ is a singleton - have a positive probability to reach unbounded number of nodes if $z_2 > z_1$

Contagion

- Centrality of a node - not vital for diffusion but can accelerate
- Immunization should be targeted towards high-degree nodes

Contagion

- Friendship paradox - most people have fewer friends than their friends have, on average.



The number beside each name is her number of friends. The number in parentheses beside each name is the mean number of friends of her friends.

Contagion

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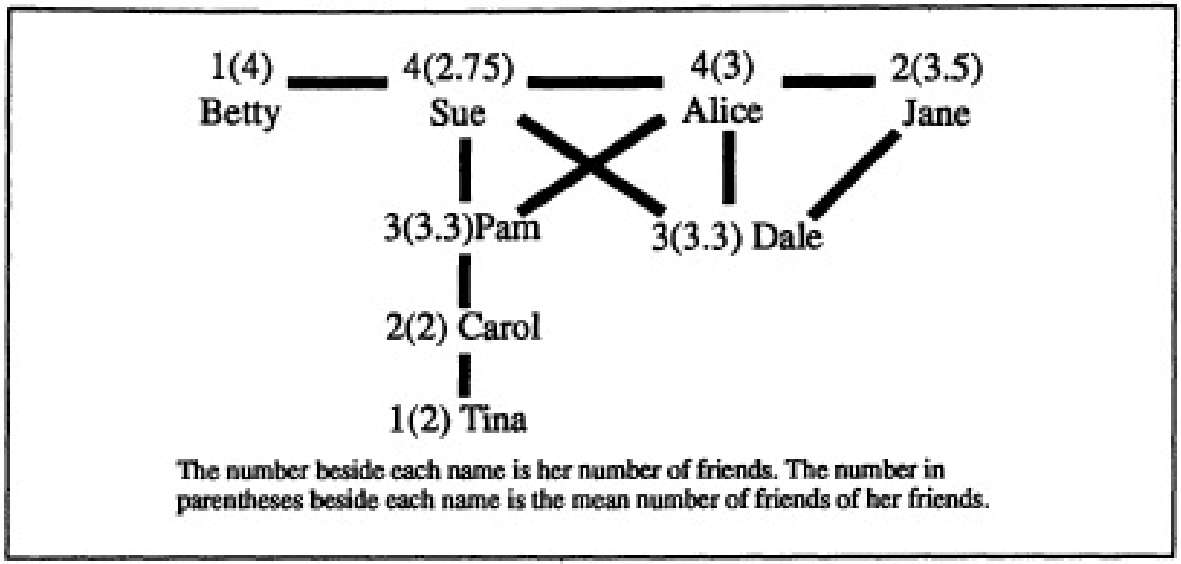

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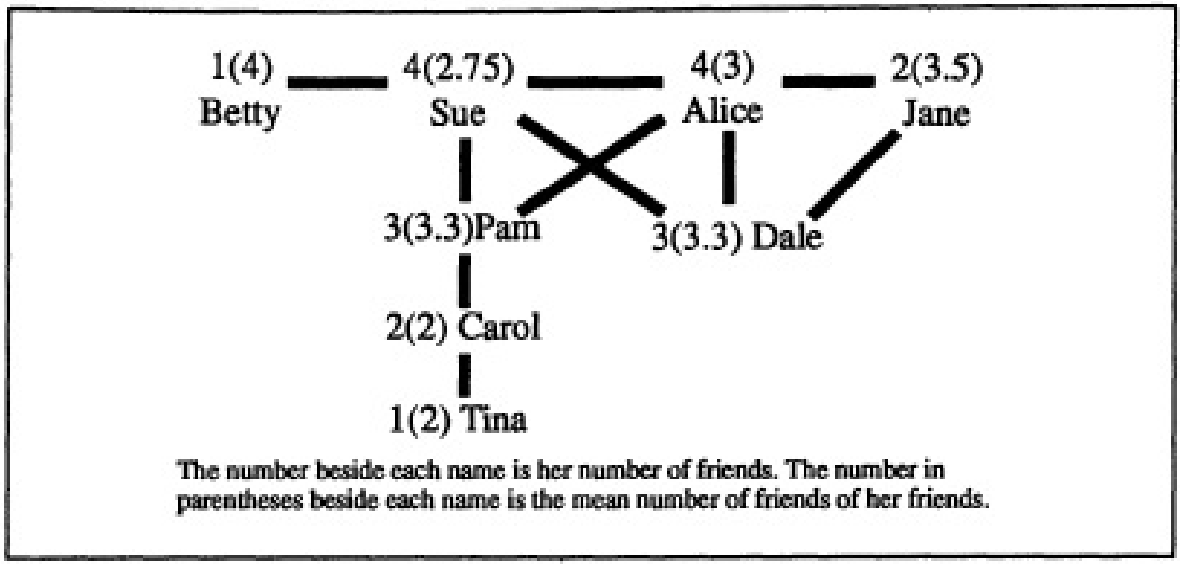

- Acquaintance immunization
- In SIS can get decrease the positive prevalence in LR if cure is biased towards high-degree nodes

Conclusions

- Social networks - powerful tool to model interactions
- Predicts global spread of diseases
- Vaccinations can eradicate the spread but should be centralized
- Can identify central nodes through the neighbours
- Effective policies should include network topology (and target central nodes)