Choking under pressure in relay races

Nikolai Avkhimovich

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Roberto Baggio misses decisive shoot-out during 1994 FIFA World Cup

1993 Wimbledon Championship

Steffi Graf	7	1	1	(15)
Jana Novotna	6	6	4	(40)

Steffi Graf	7	1	6
Jana Novotna	6	6	4

Literature review

1900s

Yerkes-Dodson law (1908): performance increases with (mental) arousal, but only up to a point; beyond this threshold, performance declines

1980s

Baumeister (1984): defined “choking under pressure”: the performance decrement happens as a result of “increased attention to one’s own performance”, which is typical reaction of agents being under pressure

2010s

Ariely (2009): through set of experiments concluded, that “high reward levels can have detrimental effects on performance; **Sanders and Walia (2012):** observed “shirking under pressure” effect, when higher stakes could lead to lower performance level in the presence of pressure

2000-2020

Economics (sport economics): finding evidence, quantifying, etc.

Friendly environment pressure:

- Soccer penalty shoot-outs (Dohmen (2008)), biathlon shooting stage (Harb-Wu & Krumer (2019)), basketball (Boheim (2019))

“Follower” pressure:

- Soccer penalties (Echenique & Rodriguez (2017)) vs. tennis tie-break (Cohen-Zada (2018))

Decisive moments pressure:

- Cao et al. (2011): being at final stage of a very close basketball game decreases shooting accuracy by 5-10%
- Hickman et al. (2019), Teeselink et al. (2020): evidence of performance decrements in decisive moments in darts, golf

Psychology: Why does choking happen? How to resolve it?

Choking mechanism and interventions:

- Hill et al. (2010), Gropel & Mesagno (2019): primary mechanisms of choking and specific interventions (distraction based, self-focus based, mindfulness intervention, etc.) to prevent performance decrement under pressure in sport


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Agenda

Theoretical modelling

Computer simulations

Model setup and “normal conditions” case

Model setup

2x2 relay race

- Team A: A1, A2, Team B: B1, B2

Players with differentiated skills:

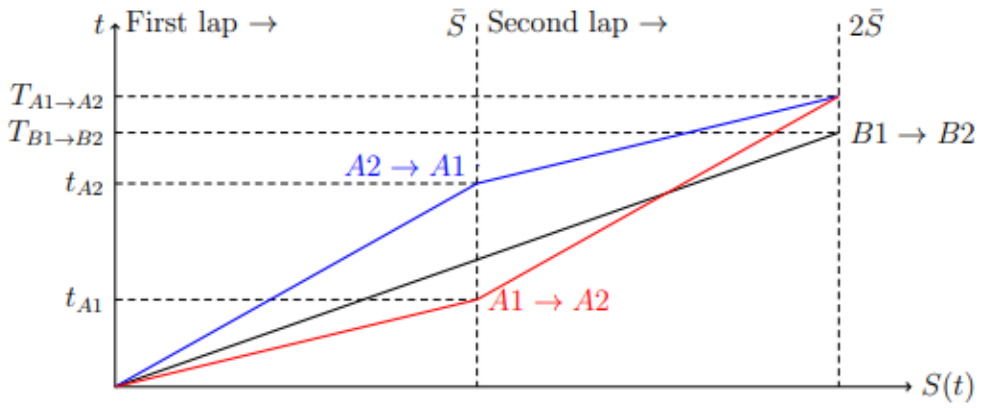
- Ranking: A1 ≥ B1 ≥ B2 ≥ A2
- Meaning t*(A1) ≤ t*(B1) ≤ t*(B2) ≤ t*(A2), where t* is time of the lap in “normal conditions”[1]

Coaches choose sequence of players (e.g. A1 → A2 or A2 → A1)

Lemma 0: Coaches are indifferent of sequence of their players under “normal conditions”

Individual sportsmen time of the lap is predetermined by his / her ranking and would be the same regardless the order in race

$$T_{A1\to A2} = t_{A1} + t_{A2} = \frac{\bar{S}}{\bar{v_{A1}}} + \frac{\bar{S}}{\bar{v_{A2}}} \quad = \quad T_{A2\to A1} = t_{A2} + t_{A1} = \frac{\bar{S}}{\bar{v_{A2}}} + \frac{\bar{S}}{\bar{v_{A1}}}$$



1. ”Normal conditions” means the absence of ”choking under pressure” effect, i.e. there is no negative impact on sportsmen performance from other participants of the race. It means, that in ”normal conditions” sportsmen shows his / her peak performance. Assume, that vi – peak speed of the sportsmen with ranking i in ”normal conditions”: that constant speed over the lap leads to performance in ”normal conditions” with t* time of the lap

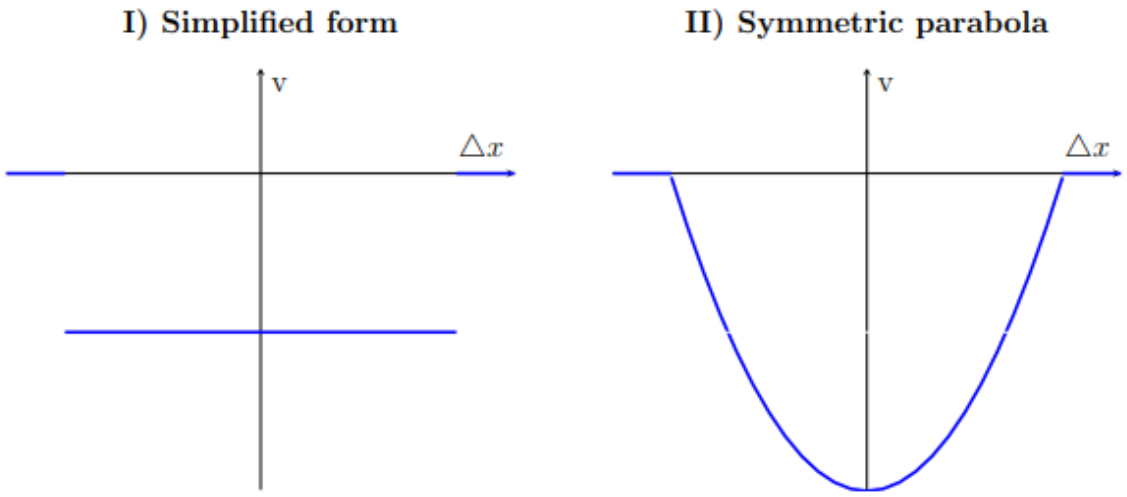
“Choking” function

Baumeister (1984) directionally suggested the form of choking function

- If one is far ahead, one can afford some errors without losing; pressure is minimal
- If one has only a slight lead, the pressure is increased, although an occasional or minor error will keep the contest still undecided
- Pressure would seem to be greatest if one is slightly to moderately behind. In that situation, one retains the possibility of success only if one performs very well; any further mistakes or setbacks may end one's chance of winning
- If one is far behind, pressure is presumably diminished

Two symmetric “choking” functions are introduced and considered in the paper

- “Choking”: negative effect on momentum speed in decisive moments (when the competitor is close)



Potential for future research: functional form of “choking” function (incl. asymmetric to account for Baumeister’s (1984) effects)

Scenario 1: Team A is stronger

Simplified model[1]

Simplified “choking function” for Team A only with negative effect “u”

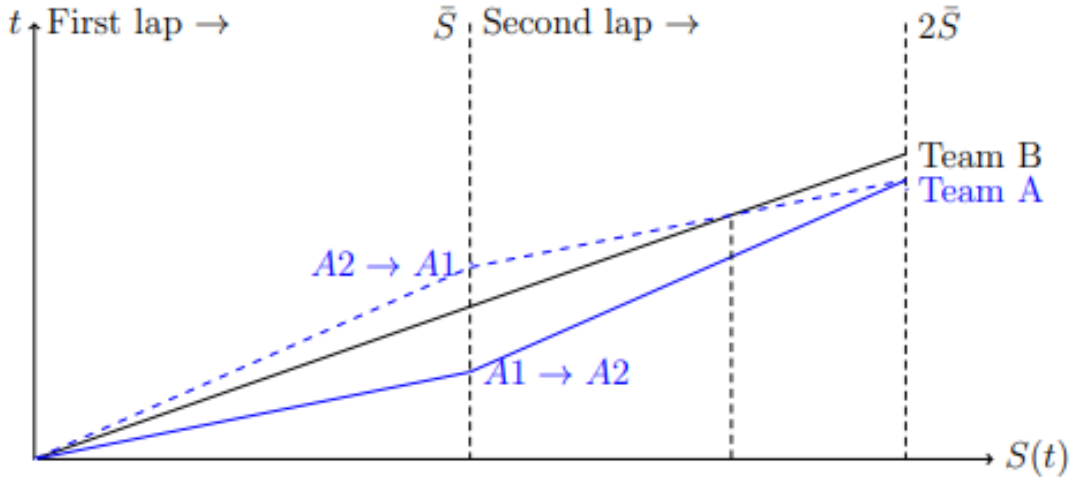
V(A1) = v + theta1

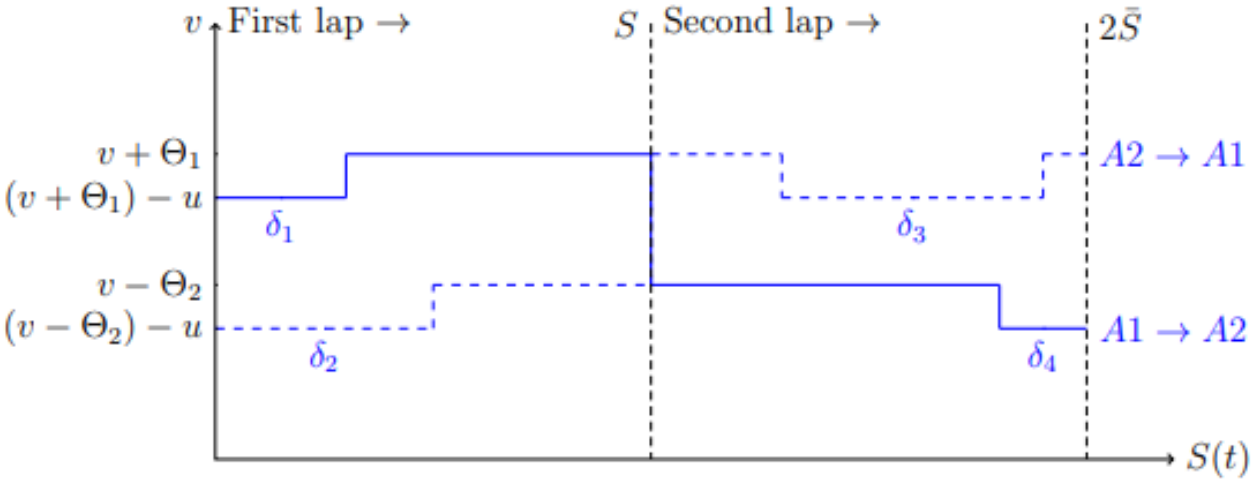
V(A2) = v – theta2

V(B1) = V(B2) = v

Theta1 > u

“Winning condition” on theta1, theta2, u





Lemma 1a:

- Coach would prefer A1→A2
- Faster to start, slower to finish

Proof (in a nutshell):

- T(A1→A2) < T(A2→A1)
- $\delta_1 - \delta_3 < \delta_2 - \delta_4$
- $\delta_1 - \delta_3 < 0$ (overtaking)
- $\delta_2 - \delta_4 > 0$ (no overtaking)

1. Assumptions: simplified "choking function"; "choking" effect is for Team A only; theta1 > theta2 > u, v(B1) = v(B2) = v

Scenario 1 [Proof]: Team A is stronger

Simplified model[1]

$$T_{A1\to A2} < T_{A2\to A1}$$

$$\frac{\delta_1}{v+\Theta_1-u}+\frac{S-\delta_1}{v+\Theta_1}+\frac{S-\delta_4}{v-\Theta_2}+\frac{\delta_4}{v-\Theta_2-u}<\frac{\delta_2}{v-\Theta_2-u}+\frac{S-\delta_2}{v-\Theta_2}+\frac{S-\delta_3}{v+\Theta_1}+\frac{\delta_3}{v+\Theta_1-u}$$

$$\frac{\delta_1-\delta_3}{v+\Theta_1-u}+\frac{S-\delta_1-(S-\delta_3)}{v+\Theta_1}<\frac{\delta_2-\delta_4}{v-\Theta_2-u}+\frac{S-\delta_2-(S-\delta_4)}{v-\Theta_2}$$

$$\frac{\delta_1-\delta_3}{v+\Theta_1-u}-\frac{\delta_1-\delta_3}{v+\Theta_1}<\frac{\delta_2-\delta_4}{v-\Theta_2-u}-\frac{\delta_2-\delta_4}{v-\Theta_2}$$

$$(\delta_1-\delta_3)\,\frac{v+\Theta_1-(v+\Theta_1-u)}{(v+\Theta_1-u)(v+\Theta_1)}<(\delta_2-\delta_4)\,\frac{v-\Theta_2-(v-\Theta_2-u)}{(v-\Theta_2-u)(v-\Theta_2)}$$

$$\frac{u\,(\delta_1-\delta_3)}{(v+\Theta_1-u)(v+\Theta_1)}<\frac{u\,(\delta_2-\delta_4)}{(v-\Theta_2-u)(v-\Theta_2)}$$

Since $u>0$, $v+\Theta_1>u$ and $v-\Theta_2-u>0$:

$$\delta_1-\delta_3<\delta_2-\delta_4$$

Lemma 1a:

- Coach would prefer A1 → A2
- Faster to start, slower to finish

1. Assumptions: simplified "choking function"; "choking" effect is for Team A only; theta1 > theta2 > u , v(B1) = v(B2) = v

Scenario 2: Team A is weaker

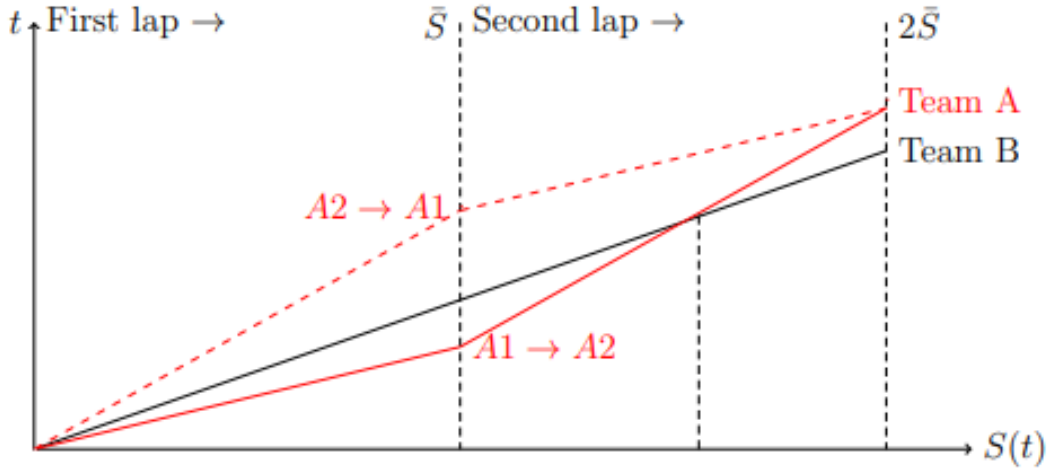
Simplified model[1]

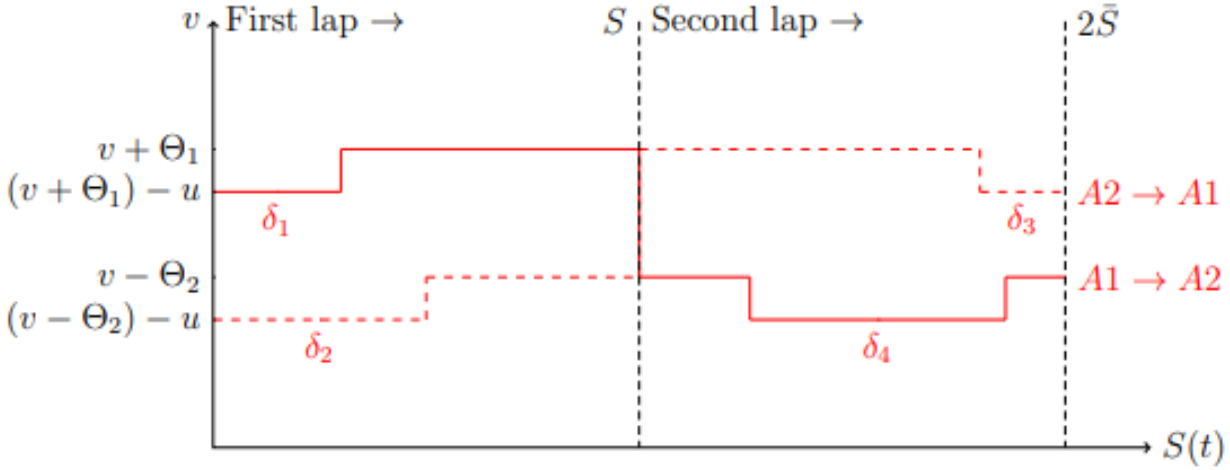
Simplified “choking function” for Team A only with negative effect “u”

V(A1) = v + theta1
V(A2) = v – theta2
V(B1) = V(B2) = v

Theta1 > u

“Loosing condition” on theta1, theta2, u





Lemma 1a:

- Coach would prefer A2→A1
- Slower to start, faster to finish

Proof (in a nutshell):

- T(A1→A2) > T(A2→A1)
- $\delta_1 - \delta_3 > \delta_2 - \delta_4$
- $\delta_1 - \delta_3 > 0$ (no overtaking)
- $\delta_2 - \delta_4 < 0$ (overtaking)

1. Assumptions: simplified "choking function"; "choking" effect is for Team A only; theta1 > theta2 > u , v(B1) = v(B2) = v

Agenda

Theoretical modelling

Computer simulations

Computer simulations setup

Main input

Speed in “normal conditions”

- v(A1), v(A2), v(B1), v(B2)

“Choking” function form:

- Quadratic function

Normal distribution of “choking” value

- Mean: “Choking” function value

Number of simulations

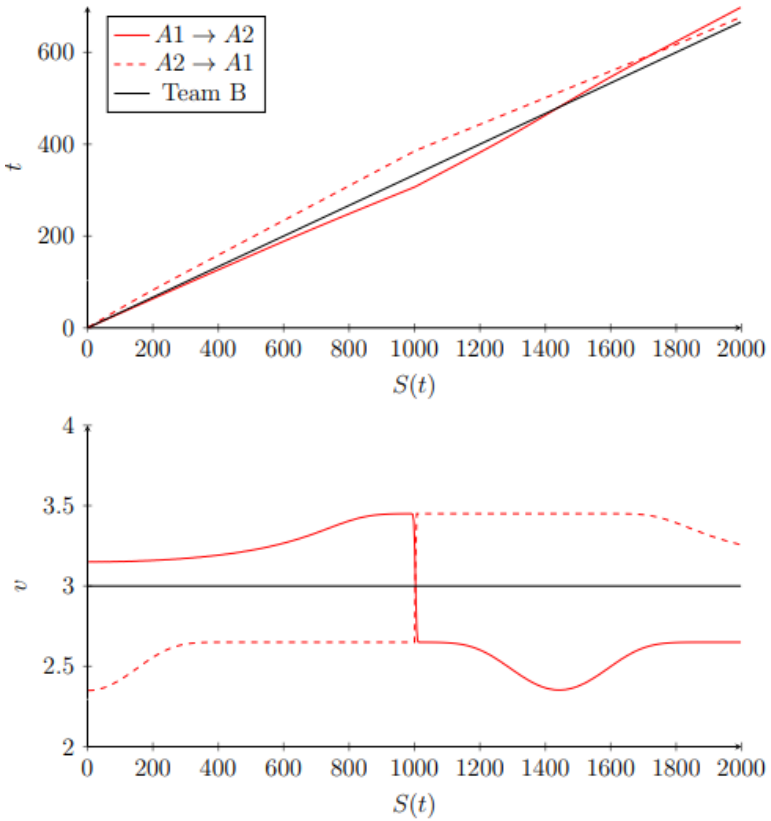
Modelling approach

1) At time 0 first sportsmen from both teams starts the race (initially distance between them is equal to 0)
2) “Choking” function is calculated for specific distance between sportsmen for both of them
3) Negative impact on sportsmen speed is randomly realized
4) Momentum speed for both sportsmen is determined
5) Sportsmen “move”, setting new distance between sportsmen
6) Return to step (2)

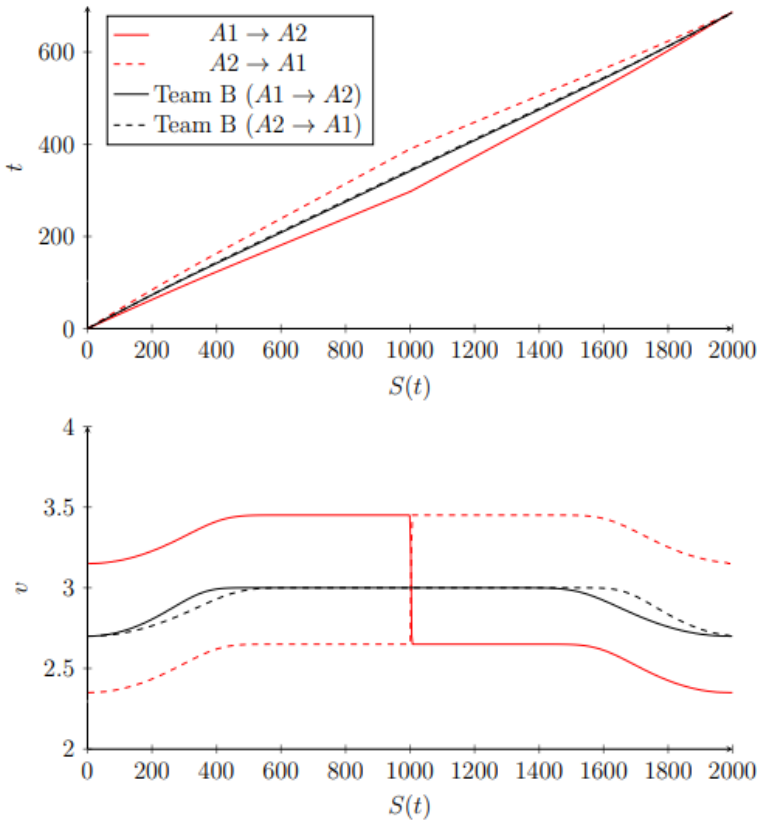
Computer simulations results

B1 → B2, v(B1) = v(B2)[1]

Computer simulations are in line with theoretical results, however…



… adding "choking" effect on Team B as well could bring new opportunities for further analysis



Based on 100 000 simulations:

Average time decreases in line with model:

- A1 → A2: 687.0 s
- A2 → A1: 685.9 s

However, distance under "choking" effect for Team B varies

Probability of winning:

- A1 → A2: 14%
- A2 → A1: 9%

1. v(B1) = 3, v(B2) = 3

Best response (1/3)

B1 → B2, v(B1) = v(B2)[1]

V0_A1 \ V0_A2	2.7	2.725	2.75	2.775	2.8	2.825	2.85	2.875	2.9	2.925	2.95	2.975	3	3.025	3.05	3.075	3.1	3.125	3.15	3.175	3.2	3.225	3.25	3.275	3.3
2.7																							Indifferent	Indifferent	Indifferent
2.725																							Indifferent	Indifferent	Indifferent
2.75																					Indifferent	Indifferent	Indifferent	Indifferent	A2->A1
2.775																			Indifferent	Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	Indifferent
2.8																		Indifferent	Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	Indifferent	Indifferent
2.825																	Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	A2->A1	Indifferent	Indifferent	
2.85																Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	Indifferent	Indifferent	Indifferent		
2.875															Indifferent	Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	A2->A1	Indifferent	Indifferent		
2.9													Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	A2->A1	Indifferent	Indifferent	Indifferent				
2.925												Indifferent	Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	Indifferent	Indifferent	Indifferent					
2.95											Indifferent	Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	Indifferent	Indifferent	Indifferent						
2.975										Indifferent	Indifferent	Indifferent	A2->A1	A2->A1	A2->A1	Indifferent	Indifferent	Indifferent							
3									Indifferent	Indifferent	Indifferent	A1->A2	Indifferent	A2->A1	Indifferent	Indifferent	Indifferent								
3.025								Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	A1->A2	Indifferent	Indifferent	Indifferent									
3.05							Indifferent	Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent	Indifferent	Indifferent										
3.075						Indifferent	Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent	Indifferent	Indifferent											
3.1					Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	A1->A2	Indifferent	Indifferent													
3.125					Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent	Indifferent														
3.15				Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent	Indifferent															
3.175			Indifferent	Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent																
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3.25	Indifferent	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent																			
3.275	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent																				
3.3	Indifferent	Indifferent	A1->A2	A1->A2	Indifferent																				

Team A coach would prefer faster to start, slower to finish (A1 → A2, if v(A1) > v(A2))

1. v(B1) = 3, v(B2) = 3

Best response (2/3)

B1 → B2, v(B1) < v(B2)[1]

V0_A1 \ V0_A2	2.7	2.725	2.75	2.775	2.8	2.825	2.85	2.875	2.9	2.925	2.95	2.975	3	3.025	3.05	3.075	3.1	3.125	3.15	3.175	3.2	3.225	3.25	3.275	3.3
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3.3	Indifferent	A1->A2	A1->A2	Indifferent																					

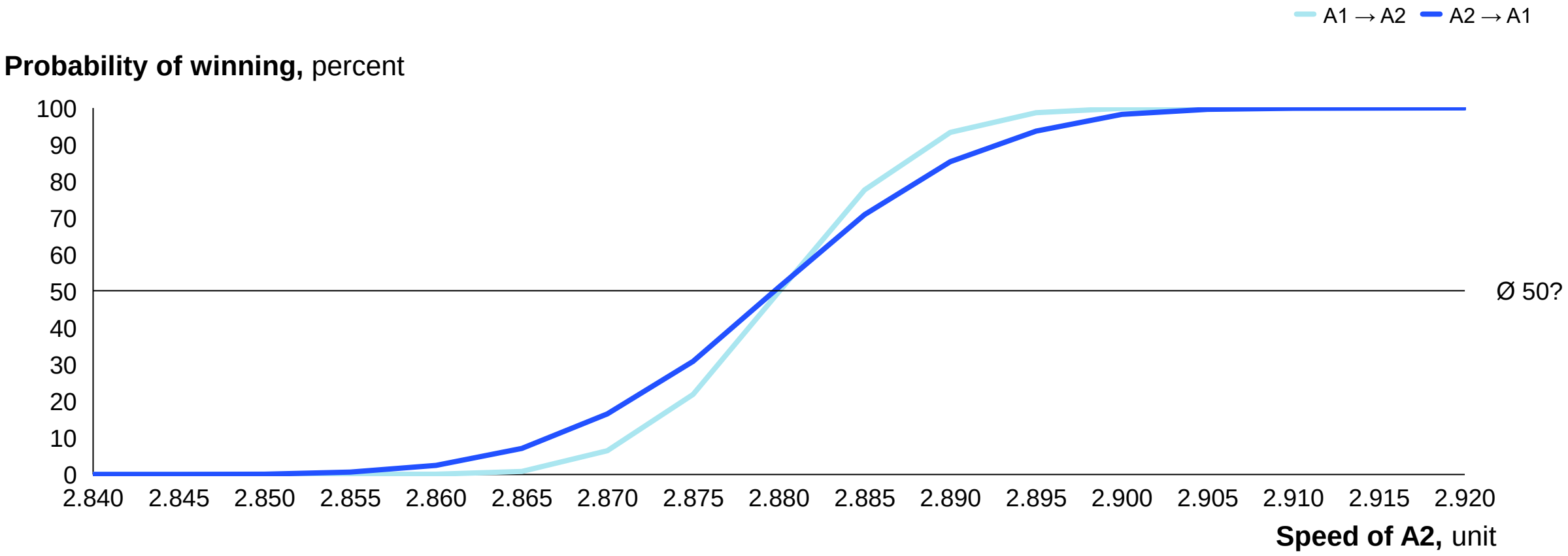
With lower probability of winning: Team A coach would prefer slower to start, faster to finish (A2 → A1, if v(A1) > v(A2))

With higher probability of winning: Team A coach would prefer faster to start, slower to finish (A1 → A2, if v(A1) > v(A2))

1. v(B1) = 2.8, v(B2) = 3.2

Best response (2/3) – deep dive for particular speed v(A1) = 3.1

B1 → B2, v(B1) < v(B2)[1]



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With higher probability of winning: Team A coach would prefer faster to start, slower to finish (A1 → A2, if v(A1) > v(A2))

1. v(B1) = 2.8, v(B2) = 3.2

Best response (3/3)

B1 → B2, v(B1) > v(B2)[1]

V0_A1 \ V0_A2	2.7	2.725	2.75	2.775	2.8	2.825	2.85	2.875	2.9	2.925	2.95	2.975	3	3.025	3.05	3.075	3.1	3.125	3.15	3.175	3.2	3.225	3.25	3.275	3.3
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With lower probability of winning: Team A coach would prefer faster to start, slower to finish (A1 → A2, if v(A1) > v(A2))

With higher probability of winning: Team A coach would prefer slower to start, faster to finish (A2 → A1, if v(A1) > v(A2))

1. v(B1) = 3.2, v(B2) = 2.8

Conclusions and next steps

Conclusions

Theoretical modelling:

B1 → B2, v(B1) = v(B2)

Team A is weaker: slower to start, faster to finish

Team A is stronger: faster to start, slower to finish

Computer simulations:

B1 → B2, v(B1) < v(B2)

With lower probability of winning: slower to start, faster to finish

With higher probability of winning: faster to start, slower to finish

B1 → B2, v(B1) > v(B2)

With lower probability of winning: faster to start, slower to finish

With higher probability of winning: slower to start, faster to finish

Opportunities for further research

Deep dive into border probabilities of winning:

- 50%?

Functional form of “choking” function:

- Other specifications (incl. asymmetric)

Differentiated “choking” function impact:

- E.g. strong players “choke” less

Simultaneous game and limited information case:

- Team A does not know Team B “choking function”

Nikolai Avkhimovich

13 July, 2021, EURO conference