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The optimal design of elimination tournaments with a superstar

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Introduction: Single-elimination tournaments

Definition

Single-elimination tournament (SET[1]) is a type of elimination tournament where the loser of each match-up is immediately eliminated from the tournament.

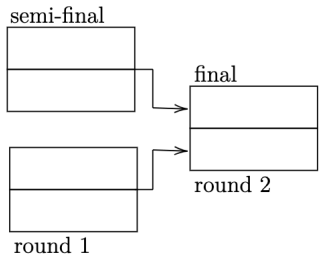

Figure: Tournament structure for single-elimination tournament for 4 players (SET4).

[1]Hereinafter, the abbreviation of the name of the tournament + N players.

Introduction: Double-elimination tournaments

Definition

Double-elimination tournament (DET) is a knockout (elimination) tournament in which a player is eliminated from the tournament after a second loss.

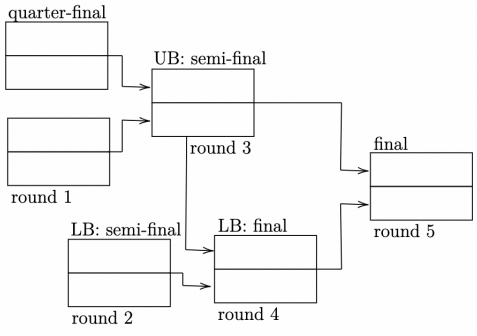

Figure: Tournament structure for double-elimination tournament for 4 players (DET4).

Introduction: tournaments

Example

- In 2016, for the first time, a series of major tournaments was held in Dota 2.
- The first two were held according to the standard rules for this discipline,
- and in the third the group stage was held according to the DET4 system.

- Huang (2016) did a comparison of single- and double-elimination tournaments for 4 homogeneous players and show that the second one brings higher total effort.

Motivation

Questions

- Which type of tournament brings higher revenue?

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- Which type of tournament brings higher revenue? Definitely not DET4
- Is it profitable to attract a superstar? Rather yes

Model

In each match (t), each player chooses an effort level e_{it}.

Probability

Probability is modeled by Tullock success function.

$$P_{it} = \frac{e_{it}}{e_{it} + e_{jt}}, \tag{1}$$

Expected payoff

Each player maximize objective function at each match.

$$\max_{e_{it}} V_{it} = P_{it} V^*_{it'} + (1 - P_{it}) V^*_{it''} - e_{it} \tag{2}$$

Assumptions

- First place prize only
- Linear cost function: $c_i(e_i) = e_i$

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Assumptions

- First place prize only
- Linear cost function: $c_i(e_i) = e_i$
- One superstar (s) and the rest of the players are regular (r)
- Players vary in strength (prize evaluation)
- The prize of regular players is normalized to 1
- The superstar values the prize α times more than the regular player

We compare SET4 and DET4.

Results: tournaments' comparison

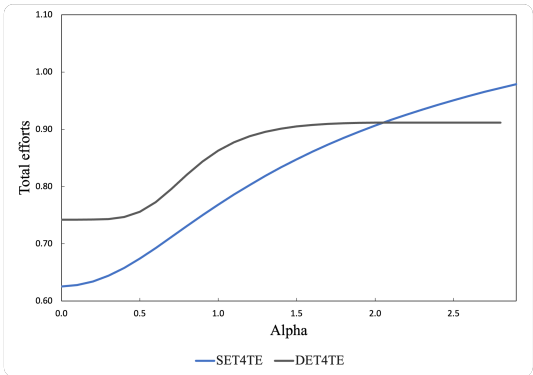

Figure: Total effort

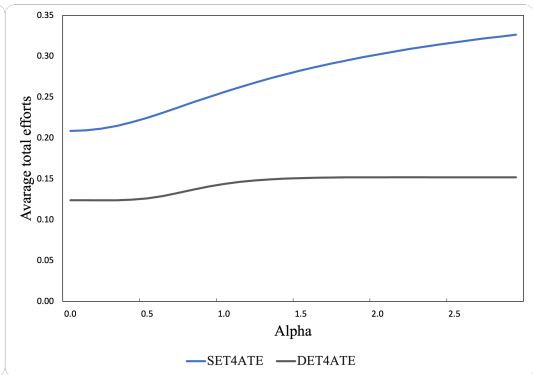

Figure: Average total effort

Results: tournaments' comparison

The superstar winning probability for a tournament is the reason why an organizer might choose a DET4.

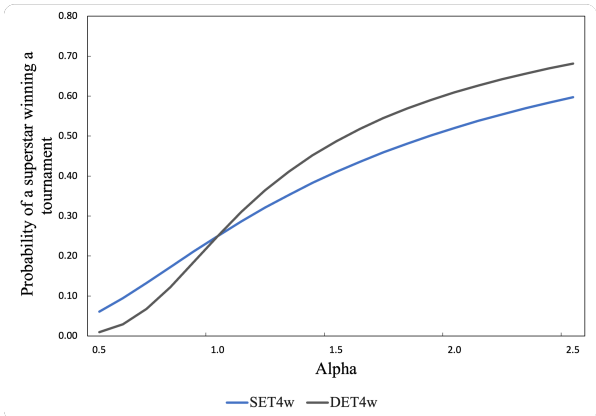

Figure: Probability of a superstar winning a tournament

Discussion: Single-elimination tournament for 8 players

Tournament	DET4	SET4	SET8
Number of players	4	4	8
Number of matches	6	3	7

Table: Motivation for tournaments choice.

Results for SET8: natural heterogeneity

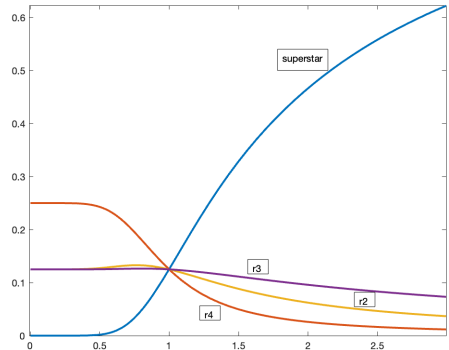

Figure: Probability of winning a SET8 for different types of players.

where i from r_i is a number of round at which player will meet or potentially meet the strongest player.

Results: tournaments' comparison with SET 8

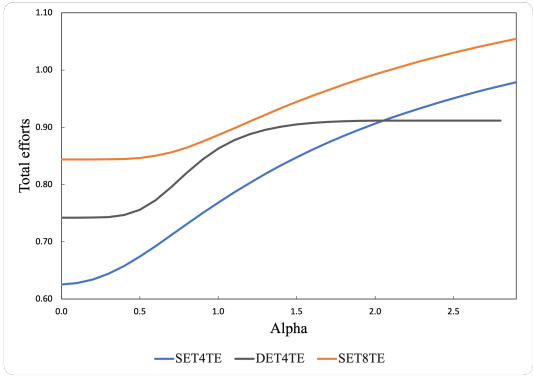

Figure: Total effort

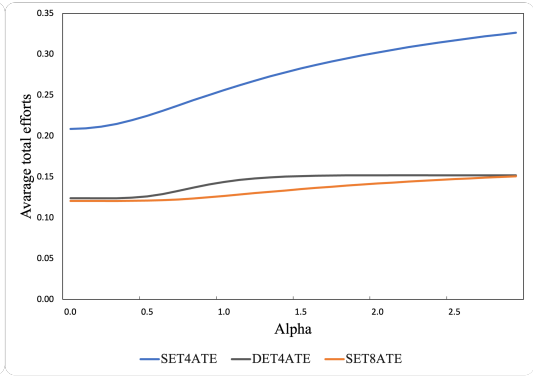

Figure: Average total effort

Results: tournaments' comparison with SET 8

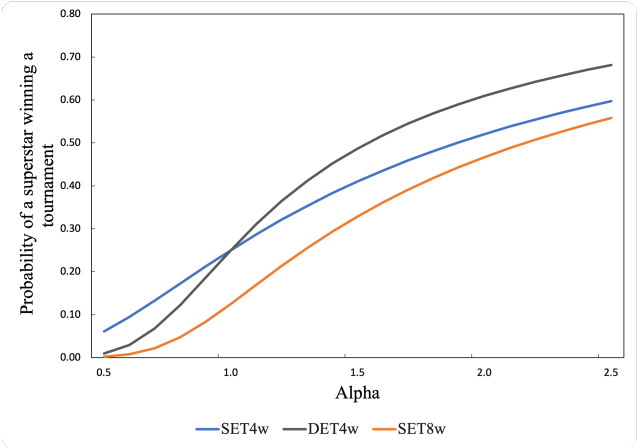

Figure: Probability of a superstar winning a tournament

Conclusion

- Single-elimination tournaments generate higher revenue.
- However, the probability of winning the tournament for the superstar is higher in the case of DET4.
- Seeding can be used to manipulate the probability of winning for different players.

- Thank you!